Further Estimates of the Input Limits of X-Ray Generators.
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Introduction.

The object of this paper is to complete the previous notes* on the input limits of X-ray generators. It deals essentially with the "line focus" on a moving and on a stationary target.

Energy Density in the Focus.

The target is assumed to be a large mass of metal with an infinite plane surface. The small area which is bombarded by the rays is called the focus. The temperature gradient normal to the plane surface is zero at any point in this surface with the exception of the focus. The other boundaries of the target are at distances which are large compared with the focus dimensions. It is owing to this assumption that the target dimensions do not enter into the expressions for the maximum input. The remote boundaries are kept at a constant temperature "$T_0$." The energy inflow at the focus is assumed to be

$$\kappa \frac{\partial T}{\partial z} = \frac{W}{\pi} \cdot \frac{\ln^2 2}{\delta_1 \cdot \delta_2} \cdot \exp \left( - \ln 2 \left[ \frac{x^2}{\delta_1^2} + \frac{y^2}{\delta_2^2} \right] \right).$$

"W" is the total energy flow per second. $2\delta_1$ and $2\delta_2$ are the width and the length of the focal line. The expression above is referred to a system of co-ordinates in which the focus centre is permanently at the origin. The XY plane lies in the surface of the target. $x$ and $y$ are the co-ordinates of a point in this surface. The Z axis is perpendicular to the XY plane. The formula above becomes identical with that of the previous notes (loc. cit.) if $\delta_1$ is made equal to $\delta_2$.

Results of the Calculations.

The final results are given below. The actual calculations which lead to these results are rather long. They were carried out on the same lines as those in the previous notes. It is not intended to reproduce the calculations here.

A. Stationary Target.

\[ W_{\text{max}} = 4 \cdot 25 \times \kappa \times (T - T_0) \times \mu(\delta_1\delta_2). \]

\( \kappa \), thermal conductivity of the target material in watt cm.\(^{-1} \) centigr.\(^{-1} \).

\( T \), melting temperature of the target material in centigr.

\( T_0 \), temperature of the cooled boundary of the target.

\( 2\delta_1 \) and \( 2\delta_2 \), width and length of the focus in centimetres.

\( W_{\text{max}} \), maximum input for the stationary target in watts.

\( \mu(\delta_1\delta_2) \) a function of \( \delta_1 \) and \( \delta_2 \) which is defined as follows:

\[ a_1 = \delta_1 + \delta_2; \quad b_1 = \sqrt{\delta_1\delta_2} \]

\[ a_2 = a_1 + b_1; \quad b_2 = \sqrt{a_1 \cdot b_1} \]

\[ \vdots \]

\[ a_n = a_{n-1} + b_{n-1}; \quad b_n = \sqrt{a_{n-1} \cdot b_{n-1}} \]

\[ \lim_{n \to \infty} a_n = \lim_{n \to \infty} b_n = \mu(\delta_1\delta_2). \]

In the special case when \( \delta_1 = \delta_2 = \delta \) we have

\( \mu(\delta\delta) = \delta \),

i.e., the formula for the maximum input turns into

\[ W_{\text{max}} = 4 \cdot 25 \times \kappa \times (T - T_0) \times \delta* \]

for a circular focus which has already been given in the previous notes.

B. Moving Target.—The following formula gives the input limit for a moving target. The focus and the target have now a constant relative velocity in the direction of \( \delta_1 \), i.e., the velocity is normal to the long axis of the focus.

\[ W_{\text{max}} = 4 \cdot 04 \times \kappa \times (T - T_0) \times \delta_2 \times \sqrt{\frac{2c}{\kappa}} v \cdot \delta_1, \]

the additional symbols are:

\( \rho \), density of the target material in gr. cm.\(^{-3} \).

\( c \), specific heat in watt sec. gr.\(^{-1} \) centigr.\(^{-1} \).

\( v \), relative velocity of focus and target in cm. sec.\(^{-1} \).

\( W_{\text{max}} \), maximum input in watts.

* The numerical constant in the previous paper is 17.8 instead of 4.25. This is due to the fact that \( \kappa \) is expressed in gr. cal. sec.\(^{-1} \) cm.\(^{-1} \) centigr.\(^{-1} \) in the previous paper, and in watt cm.\(^{-1} \) centigr.\(^{-1} \) in the present note.

† Similarly here.
The formula is an approximation which only holds if \( \frac{\varepsilon \rho}{\kappa} v \delta_1 \) is large compared with unity. The expression under the square root has the dimension of a pure number.

**Discussion.**

The calculations show how the input limit depends upon the focus dimensions and the velocity of the target. In many applications of X-rays it is found necessary to keep at least one of the focus dimensions small, say of the order of 1 mm. An increase of input can in this case be obtained by an increase of length of the focal line, or, if the target is moving, by an increase of the velocity or by a combination of both. We take, for example, a stationary target and compare the maximum input for a focal line of a certain width with the maximum input for a circular focus the diameter of which is equal to the width of the line focus.

Supposing that the width of the focal line is \( 2\delta_1 = 0.1 \text{ cm.} \) and its length \( 2\delta_2 = 1 \text{ cm.} \) It is found that \( \mu (0.5, 0.05) = 0.212. \) The ratio \( W_{\text{max.}} \) line focus to \( W_{\text{max.}} \) circular focus is \( 0.212/0.05 = 4.3. \) In a previous note it was mentioned that the maximum input for a stationary copper target and a focus of 0.05 cm. radius is about 750 watts. The maximum input with the line focus is therefore \( 4.3 \times 750 = 3.2 \text{ kw.} \) Supposing the same input were to be obtained, but now with a circular focus and a moving target. The formula requires that

\[
\frac{\varepsilon \rho}{\kappa} v \delta_1 = \frac{4.25}{4.04} \times 4.3
\]

\( \frac{\varepsilon \rho}{\kappa} \sim 1 \) for copper. \( \delta_1 = 0.05 \text{ cm.} \) Therefore

\[ v \sim 400 \text{ cm./sec.} \]

This velocity would have to be given to an infinite target. In practical work the target consists of a rotating disc. Any point in the path of the focus is consequently subjected to a periodically repeated heating. The formula in this paper cannot be expected to give the correct value for the input limit unless the disc is large. If the temperature of a point in the focus has dropped to a very small fraction of its maximum value before it is heated again, then the minimum temperature will not rise much above the temperature of the cooling liquid. The calculation should in this case prove to be approximately correct.

An experiment was made with a rotating copper target of 8 cm. diameter. The focus was circular. The path of the focus was a circle of 5.9 cm. diameter.
The disc rotated with 1950 revolutions per minute, i.e., the velocity of the focus was approximately 600 cm. per second. The total input was measured by the amount and the rise of temperature of the cooling water. The focus was observed directly. At 2 kw. the first signs of wear were observed on the target, at 2.2 kw. the melting of the target material could be seen quite distinctly. The radius of the focus was measured on the trace which the cathode rays left on the target surface. This trace was approximately \( \frac{3}{4} \) mm. wide, i.e., \( \delta_1 = 0.037 \) and \( \delta_2 = \delta_1 \) for a circular focus. The data required for the calculation are:

For copper:
\[
\kappa \times (T - T_0) = 3.6 \times 10^3 \text{ watt/cm.} \quad \frac{\rho \sigma}{\kappa} \sim 1,
\]
\[
\delta_1 = \delta_2 = 0.037 \text{ cm.} \quad v = 6 \times 10^2 \text{ cm./sec.}
\]

The formula therefore gives
\[
W_{\text{max.}} = 4.04 \times 3.6 \times 0.037 \times 10^2 \times \sqrt{\frac{6 \times 0.037 \times 10^2}{\kappa}} = 2.6 \text{ kw.}
\]

The observed value 2 kw. is about three-quarters of the calculated. The generator ran on biphas e rectified alternating current of 50 cycles. No condensers were used, i.e., the maximum instantaneous input was higher than 2 kw. The experiment thus shows that the formula gives the right order of magnitude even for a comparatively small disc.

**Summary.**

Further estimates are made for the input limit of X-ray generators. The results of calculations dealing with a line focus on a stationary and on a moving target are given. An experiment with a rotating anode shows that the theory gives the right order of magnitude for the input limit.

**Conclusion.**

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