Deviation from the Coulomb law for the proton

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1. Introduction

In a recent paper R. C. Williams (1938) has found that the fine structure of the $H_\alpha$ line in the spectrum of the hydrogen atom is not quite in agreement with the theoretical predictions (Sommerfeld’s fine structure formula). In discussing these experiments, Pasternack (1938) has pointed out that these deviations can be described by a simple shift of the $2^3S$ level of hydrogen by an amount of 0.03 cm$^{-1}$ in the direction of higher energies.
At the present state of our knowledge it seems conceivable that such a departure from the theory may be ascribed to a deviation from the Coulomb law of force at small distances rather than to a breakdown of the relativistic wave equation for the electron.

A departure from the Coulomb law of force has often been discussed in connexion with the anomalous scattering of heavy particles. We know now, however, that this anomalous scattering is due to the internuclear forces and has no direct connexion with a possible departure from the Coulomb law. In view of the above experiments a new examination of the validity of the Coulomb law seems to be desirable.

Such a discussion has now been made possible by the recent theories of the proton and the neutron based on the existence of the mesotron (Yukawa 1935; Yukawa, Sakata and Taketani 1938; Fröhlich, Heitler and Kemmer 1938; Kemmer 1938 a and b; Bhabha 1938; Stückelberg 1938; and other papers). This theory has led to the conception that the proton spends a certain fraction of time in a dissociated state as a neutron and a positive mesotron distributed around the neutron in a volume with linear dimensions of the order of the electronic radius. It is obvious that this would lead to a departure from the Coulomb law at small distances, i.e. to a decrease of the Coulomb attraction and hence to an additional repulsion, as required by the experiments.

In the following we shall work out the departure of the Coulomb law according to Yukawa's theory. The result will be that, instead of the Coulomb attraction, we obtain a strong repulsion beginning at a distance of about one-fifth of the electronic radius. This is rather surprising since the picture of a charge distribution smeared out around the proton could only lead to a diminished attraction but never to a repulsion. The repulsion is connected with the question of the self-energy and will be discussed in § 2. We want to emphasize, however, that such a repulsion is required by the experiments mentioned above whereas a diminished attraction would not explain the order of magnitude of the observed shift.

The results of our theory should not, however, be taken too literally. Yukawa's theory, like any quantized field theory, breaks down for small wave-lengths, and no result can be relied upon for distances considerably smaller than the electronic radius. But this is just the region where our departure takes place. In all probability the effect lies just at the limit of the validity of quantum mechanics. It is for this reason that the effect may claim some interest.
2. DEPARTURE FROM THE COULOMB LAW

We shall first calculate the energy of a proton in the field of a negative point charge $e$ at the distance $R$. We are, however, only interested in the deviation from the Coulomb energy $e^2/R$ which arises from the fact that the proton spends a fraction of its time in a dissociated state. We have, therefore, to calculate that part of the total energy of the system which contains the interaction between the mesotron and the point charge. The interaction energy of the system consists of two parts, $H = H^{(1)} + H^{(2)}$, the interaction $H^{(1)}$ between the mesotron and the proton, and the interaction $H^{(2)}$ between the mesotron and the point charge. $H^{(1)}$ gives rise to the emission and re-absorption of mesotrons by the proton.

Applying perturbation theory, the lowest order in which our effect occurs is the third: (i) the mesotron is emitted by the proton, due to the interaction $H^{(1)}$, (ii) the mesotron interacts with the point charge resulting in a deflexion of the mesotron, (iii) the mesotron is reabsorbed. An alternative sequence is: (i) a pair of mesotrons is created by the field of the point charge, (ii) the negative mesotron is absorbed by the proton, (iii) the positive mesotron is absorbed. The latter process can also occur in the reverse way. The general formula for the interaction energy of the third order is then

$$W = \sum \frac{H_{0n} H_{n'n} H_{n'0}}{(E_0 - E_n)(E_0 - E_{n'})},$$

where the sum has to be extended over all states $n$, $n'$ and the three types of processes mentioned above. $H_{0n}$, etc., are the matrix elements of the total interaction $H^{(1)} + H^{(2)}$ for the transitions in question and $E_n$, $E_{n'}$ are the energies in the intermediate states. In our case $E_0$ can be put equal to zero. For each transition either $H^{(1)}$ or $H^{(2)}$ gives a contribution. Thus, in our case we obtain for the contribution from the first type of process

$$W_1 = \sum_{k, k'} \frac{H_{0k}^{(1)} H_{k'k}^{(2)} H_{k0}^{(1)}}{\epsilon_k \epsilon_{k'}},$$

(1)

where $k$ is the wave number of the mesotron and $\epsilon_k = \hbar c \sqrt{(k^2 + \lambda^2)}$ its energy.† The matrix elements of $H^{(1)}$ for the emission and the absorption of a mesotron are given by Fröhlich, Heitler and Kemmer (1938, eq. (32)). For the calculation of the matrix elements of $H^{(2)}$ we use the expression for the interaction energy of a mesotron with the field of a point charge.

† We shall use the same notations as are used by Fröhlich, Heitler and Kemmer (1938). $\hbar c \lambda = $ rest energy of the mesotron.
For the purpose of reference we give here the general expression for the density of the interaction energy with an electromagnetic field as given by Kemmer (1938a, eqns. (27) and (28)). In the notation of Fröhlich, Heitler and Kemmer (1938) this can be written in the form

\[ H^{(2)} = \frac{i e}{4 \pi \hbar c} \left[ \text{curl} \, \phi, \left[ A, \phi^* - \frac{1}{c \lambda} \phi^* \right] \right] + \text{div} \left( A, \phi^* + \frac{1}{c \lambda} \phi^* \right) \\
+ \frac{V}{c} (\phi^*, \phi) + \frac{V}{c} (\phi^*, \phi) - \text{conj. complex} \\
+ \frac{e^2}{4 \pi \hbar^2 c^2} \left[ |[A, \phi]|^2 + \frac{1}{c^2 \lambda^2} |(A, \phi)|^2 + \frac{1}{c \lambda} (A, \phi^*) (A, \phi) \\
+ \frac{1}{c \lambda} (A, \phi) (A, \phi^*) + \frac{1}{c \lambda} (A, \phi^*) (A, \phi) + \frac{1}{c \lambda} (A, \phi) (A, \phi^*) \\
+ |(A, \phi)|^2 + \frac{1}{c^2 \lambda^2} |[A, \phi^*]|^2 \right] \].

Here \( V \) is the scalar potential and \( A \) the vector potential of the electromagnetic field. \( \phi \) and \( \phi^* \) are the transverse and the longitudinal parts of the wave function of the mesotron respectively.

In our case we have \( A = 0, \quad V = -\frac{e}{|R - r|} \),

where \( r \) is the co-ordinate of the mesotron. Therefore

\[ H^{(2)} = -\frac{ie^2}{4 \pi \hbar c^2} \frac{1}{|R - r|} \left\{ (\phi^*, \phi) - (\phi, \phi^*) + (\phi^*, \phi) - (\phi, \phi^*) \right\}. \]  (2)

We are now prepared to calculate the matrix elements \( H^{(2)}_{kk'} \). The transition can be described as the annihilation of a positive mesotron in the state \( k \) and the creation of a positive mesotron in the state \( k' \). We thus obtain from (2), using Fröhlich, Heitler and Kemmer, eqns. (9), (12), (19), (20), (8), (25), (29),

\[ H^{(2)}_{kk'} = -\frac{e^2}{2} (j, j') \left( \sqrt{\epsilon_{kk}} + \sqrt{\epsilon_{k'}} \right) \int d^3 r e^{i(k - k', r)} \left| \frac{1}{|R - r|} \right| . \]

Here \( j \) and \( j' \) are the unit vectors of polarization in the states \( k \) and \( k' \). The equation holds for longitudinal as well as for transverse waves, \( j \) being parallel to \( k \) in the former case and perpendicular to \( k \) in the latter case.
Since transitions from longitudinal to transverse waves do not occur we obtain for $W_1$ eqn. (1), using the above expressions for the matrix elements

$$W_1(R) = -\frac{e^2}{\lambda^2\hbar c} \sum_{k, k'} \left( \frac{1}{(k^2 + \lambda^2)\sqrt{(k'^2 + \lambda^2)}} + \frac{1}{(k'^2 + \lambda^2)\sqrt{(k^2 + \lambda^2)}} \right) \times \left\{ \begin{array}{l} g^2(k, k') + f^2 \sum_{j, j'} \langle \sigma, [j, k] \rangle \langle j', k' \rangle \langle \sigma, [j', k'] \rangle \end{array} \right\} \int \frac{d\tau}{|R - r|} e^{i(k-k', r)}.$$  

We shall now transform the sums over $k$ and $k'$ into integrals in the usual way. Carrying out the summation over $j$ and $j'$ we obtain (making use of the fact that the sum is symmetrical in $k$ and $k'$)

$$W_1 = -\frac{e^2}{32\pi^2\lambda^2\hbar c} \int k^2 dk d\Omega k'^2 dk' d\Omega' \int \frac{d\tau}{|R - r|} e^{i(k-k', r)} \times \frac{1}{(k^2 + \lambda^2)\sqrt{(k'^2 + \lambda^2)}} \{g^2(k, k') + f^2[3(k, k') - (\sigma, k)(\sigma, k')]\}. \quad (3)$$

The integration over the angles $\Omega, \Omega'$ in the $k$ and $k'$ space leads to

$$W_1 = -\frac{e^2(g^2 + 2f^2)}{2\pi^2\lambda^2\hbar c} \int \frac{dk dk' k^3 k'^3}{(k^2 + \lambda^2)\sqrt{(k'^2 + \lambda^2)}} \int \frac{d\tau}{|R - r|} \times \left( \begin{array}{c} \cos kr \\ \sin kr \end{array} \right) \left( \begin{array}{c} \cos k'r \\ \sin k'r \end{array} \right).$$

Here use has been made of the fact that

$$(\sigma, r)^2 = r^2.$$  

Performing the integration over $k$ and $k'$ we obtain

$$W_1(R) = \frac{e^2(g^2 + 2f^2)}{8\pi^2\hbar c} \int \frac{d\tau}{|R - r|} iH^m_2(i\lambda r) \frac{e^{-\lambda r}}{r} \left( \begin{array}{c} \lambda \frac{1}{r} \end{array} \right). \quad (4)$$

Here $H^m_2$ is the Hankel function of the first kind and the second order. The function $iH^m_2(i\lambda r)$ is always real and negative.

For $R \gg r$ we see immediately that

$$W_1 \sim 1/R.$$  

This means that for large $R$ the charge distribution of the mesotron may be represented by a point charge in the origin. Since we are only interested in the deviation from a $1/R$ law we have to replace in (4)

$$\frac{1}{|R - r|} \text{ by } \frac{1}{|R - r|} - \frac{1}{R'}$$

We denote the expression obtained in this way by $W'_1(R)$. 

It should be noted that both $W'_1(R)$ as well as the integral which is to be subtracted from it, are divergent, whereas the deviation $W'_1(R)$ has a finite value for each $R > 0$. These divergencies are connected with the divergent self-energies which occur in every quantized field theory.\(^\dagger\)

We obtain now, after carrying out the remaining angular integration in the $r$-space and putting $\lambda r = x$,

$$W'_1(R) = \frac{e^2(g^2 + 2f^2)\lambda}{2\hbar c} I(R\lambda),$$

where

$$I(R\lambda) = i\int_{R\lambda}^\infty dx \left(1 - \frac{x}{R\lambda}\right) H_2^{(0)}(ix) e^{-x} \left(\frac{1}{x} + \frac{1}{x^2}\right).$$

Since the mass of the mesotron is equal to about 150 electronic masses $m$, $1/\lambda$ is roughly equal to the electronic radius $r_e$,

$$\frac{1}{\lambda} \approx r_e = \frac{e^2}{mc^2}. \quad (5)$$

Furthermore, it has previously been shown (cf. Kemmer 1938b) that

$$\frac{g^2}{\hbar c} \approx 2, \quad \frac{f^2}{\hbar c} \approx \frac{1}{7}. \quad (6)$$

Thus

$$W'_1(R) \approx \frac{1}{6 r_e} I(R\lambda). \quad (7)$$

The function $I(R\lambda)$ behaves like $1/R^3\lambda^3$ for $R\lambda \ll 1$ and decreases rapidly to zero for $R\lambda > 1$.

$W'_1(R)$ represents the deviation from the Coulomb interaction energy due to the first sequence of processes described above, i.e. emission, deflexion and reabsorption of a positive mesotron. As already mentioned, there exist

\(^\dagger\) In the theory of the mesotron the time which the proton spends in the dissociated state is $g^2/\hbar c \times K$ where $K$ is a diverging integral. If we cut off at wavelengths of the order $1/\lambda$, $K$ becomes of the order of magnitude unity. The contribution to the Coulomb field at large distances, i.e. the part which behaves as $1/R$ for $R \to \infty$, from the mesotron in the dissociate state is, therefore, $g^2/\hbar c \times e^2/R \times K$. On the other hand, the contribution to the Coulomb field of the proton in the non-dissociated state is not $e^2/R$ but $e^2/R \times [1 - g^2/\hbar c \times K]$ because $\int$ is the fraction of time during which the proton is not dissociated. Formally, the latter term would occur in our theory in the second order of approximation for the self-energy of the proton taking into account the fact that the energy difference in the intermediate state (denominators in the expression for the second order perturbation energy) contains an additional term $e^2/R$. In the text, we subtract all contributions behaving like $1/R$ for $R \to \infty$ in order to obtain the deviation from the Coulomb field. Later on (eqn. (10)) we shall again insert the Coulomb field $e^2/R$. 
two further types of sequence connected with the creation or the annihilation of a pair of mesotrons by the field of the point charge. Both processes give an equal contribution $W'_\Pi(R) + W'_\text{III}(R) = W'_1(R)$ to the deviation from the Coulomb interaction. The calculation is in principle very similar to the calculation of $W'_1(R)$. The result may be written in the form

$$W'_\Pi(R) + W'_\text{III}(R) = W'_1(R) - \frac{e^2(g^2 + 2f^2)}{hc} J(R\lambda),$$

where $J(R\lambda) = \int_R^\infty dx x^3 \left(1 - \frac{x}{R\lambda}\right) \int_0^\infty dt \frac{e^2}{(t^2 + x^2)^2} \{H_2^1(i \sqrt{t^2 + x^2})\}^2$.

Therefore the total deviation from the Coulomb energy is, using (5), (6), (7) and (8), given by

$$W'(R) = W'_1 + W'_\Pi + W'_\text{III} \approx \frac{1}{3} \frac{e^2}{r_0} (I(R\lambda) - J(R\lambda)).$$

In fig. 1 we have plotted the function $W'(R)$ (curve II) and the total interaction energy between the proton and the negative point charge (curve I + II), i.e.

$$V(R) = -\frac{e^2}{R} + W'(R).$$

The departure from the Coulomb field begins at a distance of about $r_0/2$, resulting in a diminished attraction. At still smaller distances, $R < r_0/6$, the attraction goes over into a strong repulsion, as $1/R^3$. Thus a proton behaves approximately like a rigid sphere of radius $r_0/6$. It is surprising that we have obtained a repulsion since a smeared out charge distribution would only lead to a constant attractive potential for small $R$'s. The repulsion is due to the fact that also the self-energy of the proton (due to emission and reabsorption of mesotrons) is influenced by the presence of the point charge. This gives a further contribution to the total interaction energy between a proton and a point charge which we have calculated.

3. The Energy Shift

In the previous section we have calculated the interaction between a proton and a negative point charge. This interaction turned out to be a Coulomb attraction for large distances down to about $R = r_0/2$. For $R < r_0/2$ this attraction is less than $e^2/R$ and changes into a repulsion at $R \approx r_0/6$ (fig. 1).
We shall now apply this interaction law to the interaction between the proton and the electron in the hydrogen atom. Since the repulsion mentioned above increases very rapidly ($\sim 1/R^3$) with decreasing $R$, we may replace the interaction energy $V(R)$ by

$$V(R) = -\frac{e^2}{R} \quad \text{for } R > \frac{r_0}{6},$$

$$V(R) = +\infty \quad \text{for } R < \frac{r_0}{6}. \quad (11)$$

This deviation of $V(R)$ from the Coulomb law leads to a shift of the energy levels of the hydrogen atom. This shift will be very small since $r_0 \ll a$ where $a$ is the Bohr radius.

In order to calculate this shift we have to solve the Schrödinger equation for the potential $V(R)$ eqn. (11). This is equivalent with the solution of the
Schrödinger equation for an ordinary Coulomb potential if we impose upon the wave function \( \psi \) the boundary condition

\[
\psi = 0 \quad \text{for} \quad R = r_0/6. \tag{12}
\]

Let \( \psi^{(0)} \) be the ordinary wave function for the hydrogen atom, i.e. for the pure Coulomb interaction. Then it is obvious that by applying the boundary condition (12) the S-terms will be changed much more than the P, D, ... terms because the wave functions \( \psi^{(0)} \) for these latter terms are practically zero for \( r = r_0 \) and, therefore, have to undergo only a very slight change to fit the new boundary condition. The wave function \( \psi^{(0)} \) for the S-terms, however, are different from zero in the origin and have, therefore, to be changed considerably to fit the boundary condition.

A method for calculating the change in the energy if the boundary condition is perturbed has been given by Fröhlich (1938). According to Fröhlich's eqn. (5a) the difference \( \epsilon = E - E^{(0)} \) between the non-degenerate eigen-values \( E \) and \( E^{(0)} \) corresponding to the wave functions \( \psi \) and \( \psi^{(0)} \) is given by

\[
\frac{2m}{\hbar^2} N \epsilon = - \int_S \psi^{(0)} \frac{\partial \psi}{\partial n} d\sigma \left. \right| \begin{array}{c} \psi^{(0)} \\ \psi \end{array} 
\]

\[
N = \int \psi^{(0)} \psi d\tau,
\]

if \( \psi \) vanishes at the surface \( S \). \( \frac{\partial \psi}{\partial n} \) denotes a differentiation in the outward direction perpendicular to \( S \). The volume integral \( N \) has to be extended over the total volume bounded by \( S \). In eqn. (13) no approximations have been made. In our case \( S \) is a sphere with radius \( r_0/6 \) and \( \frac{\partial \psi}{\partial n} = - \frac{d\psi}{dR} \).

Introducing the functions \( \phi = \psi R \) and \( \phi^{(0)} = \psi^{(0)} R \) we obtain, since for S-states \( \psi \) and \( \psi^{(0)} \) depend on \( R \) only and since \( \phi(r_0/6) = 0 \):

\[
\frac{2m}{\hbar^2} \epsilon = \left. \int_{r_0/6}^\infty dR \phi \phi^{(0)} \right| _{R=r_0/6}.
\]

The two functions \( \phi^{(0)} \) and \( \phi \) will be very similar, the main difference being that \( \phi^{(0)} \) drops to zero at \( R = 0 \), whereas \( \phi \) becomes zero at \( R = r_0/6 \). Assuming that we can replace \( \phi \) by \( \phi^{(0)} \) in (14) we obtain†

\[
\epsilon = \frac{\hbar^2}{2m} \left. \int_{0}^{\infty} dR \phi \phi^{(0)} \right| _{R=r_0/6}.
\]

† The approximation made in replacing the lower limit of integration \( r_0/6 \) by 0 is of a higher order. We should mention that it would be wrong to replace the wave function \( \psi \) by \( \psi^{(0)} \) in eqn. (13), since \( \psi^{(0)} \neq 0 \) for \( R = 0 \) whereas \( \psi = 0 \) for \( R = r_0/6 \). \( \psi' \) and \( \psi'^{(0)} \) are therefore very different for small \( R \).
We shall apply this formula to the 1S and the 2S terms. Since the respective wave functions are

\[ \phi_{1S}^{(0)} = R e^{-R/a}, \]
\[ \phi_{2S}^{(0)} = \left( R - \frac{R^2}{2a} \right) e^{-R/2a}, \]

we obtain for the respective energy shifts

\[ \epsilon_{1S} = 4 \frac{r_0}{6a} \text{Ry}, \]  
\[ \epsilon_{2S} = \frac{1}{2} \frac{r_0}{6a} \text{Ry}, \]
\[ \text{Ry} = e^2/2a, \]

(15)  
(16)

neglecting higher terms in \( r_0 \).

The approximation made above (i.e. the replacement of \( \phi \) by \( \phi^{(0)} \)) may seem to be doubtful in view of the fact that the wave functions \( \psi \) and \( \psi^{(0)} \) behave very differently for \( R \sim r_0/6 \). We, therefore, have calculated \( \epsilon \) also by a variational method. As wave functions we have used the functions \( \phi_{1S}^{(0)} \) and \( \phi_{2S}^{(0)} \), replacing \( R \) by \( R - r_0/6 \), in order to fit the boundary condition, and furthermore replacing the Bohr radius \( a \) by a parameter. This parameter was determined by the variational methods. The results were identical with (15) and (16). Since the variational method gives always too high a value for the energy of the ground level, our value (15) for \( \epsilon_{1S} \) represents an upper limit for the energy shift. We may expect, however, that the order of magnitude is correctly given by (15) and (16). Furthermore, we have also calculated the energy shift for the 1S term by taking the exact wave functions for small and large distances (which are obtained easily from the differential equation) and fitting the two parts together at \( R = a \). The order of magnitude for \( \epsilon \) was the same, but the numerical factor was smaller (2 instead of 4). We are, therefore, certain to have obtained the right order of magnitude for the energy shift.

4. Discussion

In the preceding section we have found that the departure from the Coulomb law leads to a shift of the S-levels of hydrogen towards higher energies of the order of \( 3 \text{ cm}^{-1} \) for the 1S-level and \( \frac{1}{2} \text{ cm}^{-1} \) for the 2S level.†

† These figures have only significance as far as the order of magnitude and the ratio \( \epsilon_{2S}/\epsilon_{1S} \) are concerned. Not much weight should be attached to the numerical values.
We now compare these results with the experiments by Williams (1938). These experiments show that the fine structure of the $D_\alpha$ line of deuterium is not quite in agreement with Sommerfeld’s fine structure formula. Pasternack (1938) has analysed these results and has found that they can be understood by assuming a shift of the 2S level by 0.03 cm$^{-1}$ towards higher energies, whereas the P and D levels are not shifted.

For hydrogen no exact measurements are available yet. From more or less indirect arguments Williams has concluded that the departure from the true fine structure formula for $H_\alpha$ is of the same order of magnitude as for deuterium. This would be quite in agreement with our theory as far as the order of magnitude is concerned, although the theoretical figure is rather larger. For a more conclusive check of the qualitative predictions of our theory it would be necessary to have more accurate measurements for hydrogen.

For deuterium our theory cannot be applied directly, since the deuteron consists of two nuclear particles. We want to emphasize, however, that the mere fact that in the deuteron the charge of the proton is smeared out over a volume of the order of $r_0^3$ does not lead to any measurable shift of the energy levels, if it is assumed that the proton is a point charge. Such a charge distribution would lead to a shift of the 1S-level of the order of magnitude

$$\delta E_{1S} = \frac{e^2 r_0^2}{a a^2}, \quad (17)$$

which is smaller by an order $r_0/a \sim 10^{-4}$ than the effect discussed in the preceding section and far too small to account for the experiments. We conclude, therefore, that the departure from the Coulomb law for the deuteron must be due to similar effects as for the proton, i.e. to a departure from the Coulomb law for the proton itself. In this connexion we want to mention that our theory also leads to an interaction between a neutron and a point charge. Numerically, this interaction is the same as the departure from the Coulomb law for the proton, but has the opposite sign. For a negative point charge we obtain, therefore, an attractive potential, which is very narrow and very deep.† This fact will also have to be taken into account in any discussion of the potential of the deuteron.

The departure from the Coulomb law takes place at distances just smaller than $r_0$. From previous theoretical discussions it is known that the theory of the mesotron begins to break down at distances smaller than $r_0$ or at

† The question whether such a potential would lead to a discrete energy level lies certainly outside the limits of validity of the theory.
correspondingly high energies. We do not believe, therefore, that the quantitative results of our theory could claim exact validity, although we think it probable that a departure of the type outlined above exists. Presumably, the question of the departure from the Coulomb law and of the exact fine structure of hydrogen just reaches the limits of validity of our present quantum mechanics. This makes a thorough experimental investigation of this question and a comparison with our present theories most desirable. The information obtained from exact measurements of the fine structure of hydrogen (and especially of the exact value of the 1S level, for which the shift should be largest) will therefore probably be important for the development of future quantum electrodynamics.

**Summary**

We have calculated the deviation from the Coulomb law of force for the proton on grounds of the mesotron theory of the nuclear particles. The effect is due to the fact that the proton spends a certain fraction of its life in a dissociated state as a neutron and a positive mesotron, the latter having an average distance from the neutron of the order of magnitude of the electronic radius $r_0$. The result is the following: The Coulomb attraction (for a negative point charge) goes over into a strong repulsion for distances less than about $1/6r_0$.

This would lead to a shift of the hydrogen S-levels towards higher energies of the order of 1 cm.$^{-1}$ for the 1S-level and one-eighth of this figure for the 2S-level. This is compared with recent experiments by R. C. Williams on the fine structure of the $D_a$ and $H_a$ line which, according to Pasternack, can be explained by assuming a shift of the 2S-level by 0.03 cm.$^{-1}$.

How far these results lie within the limits of validity of present quantum mechanics is discussed.

**References**

Fröhlich 1938 *Phys. Rev.*, 54, 945.