

Viscosity Measurements with Glass.

By VAUGHAN H. STOTT, M.Sc., EDITH IRVINE, B.Sc., and D. TURNER, B.Sc.Tech.
(of the National Physical Laboratory).

(Communicated by Dr. W. Rosenhain, F.R.S.—Received January 19, 1925.)

(Work conducted at the National Physical Laboratory, mainly on behalf of the Glass Research Association.)

The paucity of data on the viscosity of glass is due to serious experimental difficulties. At the lower temperatures, the main difficulty is the very rapid variation of the viscosity with temperature. At the higher temperatures, the corrosive action of the glass on its container is added to the grave difficulties inherent in most work at such temperatures.

The present work has been carried out with two types of apparatus—one applicable to the measurement of viscosities between $10^{16.6}$ and 10^6 C.G.S. units or *poises*, and the other to the measurement of viscosities between 10^4 and rather less than 10^2 poises. Measurements have not been made of viscosities between 10^6 and 10^4 poises, owing to the readiness with which many glasses devitrify in this range. Apart from devitrification, the apparatus used for viscosities of 10^6 poises could easily be modified for measuring viscosities of 10^4 poises.

The apparatus employed for the measurement of the higher viscosities may be used in two different ways, according to the magnitude of the viscosity to be measured. Very high viscosities (from 10^9 to 10^{17} poises) are measured by applying torsional forces to the two ends of a circular rod of the glass under examination.* For viscosities lower than 10^9 poises the rod would be too soft. In this case the glass is held in a cylindrical container and a cylindrical co-axial rod is plunged into the glass and rotated. In each case the time required for the moving portion of the apparatus to rotate through a given angle under the action of a known torque is a measure of the viscosity of the glass.

Fig. 1 shows the arrangement as used for the determination of very high viscosities. The lower end of a vertical rod of glass is fixed, and its upper end twisted by means of pulleys and falling weights. The specimen, over a length of $1\frac{1}{2}$ inches at the middle, has a diameter of $\frac{1}{4}$ inch. The remainder of the rod has a diameter of $\frac{1}{2}$ inch. These dimensions ensure that most of the

* Trouton and Andrews: "On the Viscosity of Pitch-like Substances." 'Proc. Phys. Soc.,' London, vol. 19, p. 47.

twisting takes place at the narrow part of the rod, where its temperature is known.

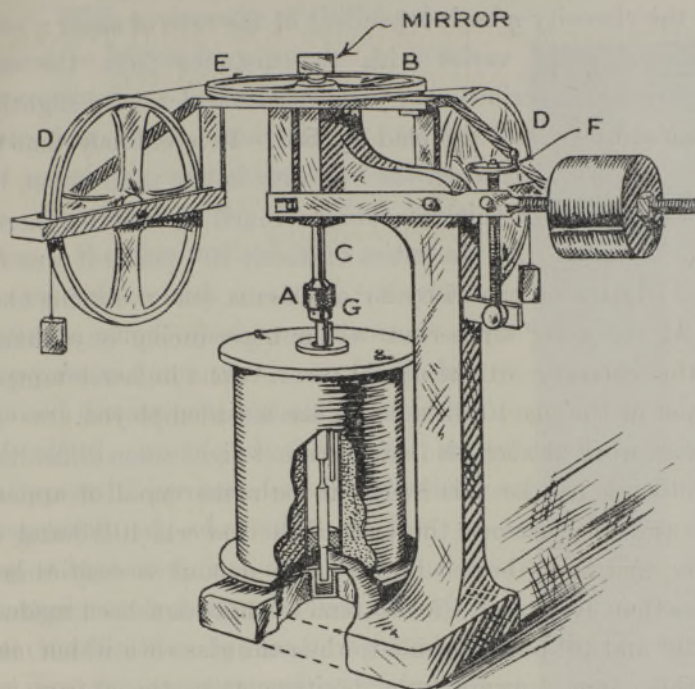


FIG. 1.

The lower end of the specimen is cemented into a steel sleeve; this can move with a sliding fit in a vertical hole in the bottom of the main casting, and is provided with a key to prevent rotation. The steel sleeve is supported underneath by means of a spring, which takes up the thermal expansion of the glass. The upper end of the rod is likewise cemented into a holder, which is fixed in the chuck A (fig. 1). The chuck and the wheel B are mounted on the shaft C and rotate freely in spherical ball-bearings. A known torque can be applied to the upper end of the specimen by means of weights attached to threads passing over the pulleys D. These pulleys are provided with pivot-bearings of hardened steel. A scale of degrees is engraved on the upper surface of the wheel B, and can be read by means of the pointer E. For greater precision optical magnification has been used. The chuck and the three pulleys form a unit which may be moved vertically by means of the hand-wheel, F.

The specimen is heated in an electric resistance furnace wound with nichrome wire, the temperature being measured by means of a platinum, platinum-rhodium thermocouple, placed with the junction close to the centre of the specimen within a protecting tube G.

In the present experiments a rod of length l and radius a has one end fixed, and the other end rotates with angular velocity ω , under the action of a couple C . If the viscosity η be independent of the rate of shear $\eta = 2C/\pi\omega a^4$. Since the rate of shear varies with the distance from the centre, the method of experiment described may yield results which vary slightly with the diameter of the rod used and the load applied. It is unlikely that the results obtained in the experiments when the viscosity is less than about 10^{14} poises, and the rod about $\frac{1}{4}$ inch thick, would be much altered by increasing the maximum rate of shear. At viscosities of about 10^{11} poises it was found that the couple could be varied from 6 by 10^5 dyne-cms. (the rod being about 6 mm. in diameter) to 6 by 10^6 dyne-cms. without producing any change in the viscosity greater than the experimental error. At the lower temperatures a constant couple of 6.17 by 10^6 dyne-cms. has been employed.

The measurement of the length l offers at first sight some difficulties, as it is advantageous to arrange for this length to be rather small in order to avoid variations of temperature along the part of the rod which is being twisted.

The method which has been adopted is to deposit a very thin layer of platinum on the narrow part of the rod, and to engrave three longitudinal lines thereon. At the end of the experiment, while the glass is still hot, the divided wheel B is rotated to a known angular position, as measured from its position at the beginning of the experiment. (A rotation of two revolutions is convenient for rods $1\frac{1}{2}$ inches long by $\frac{1}{4}$ inch diameter.) The pitches of the three helices thus formed by the previously engraved lines are measured by means of a travelling microscope after the rod has cooled, and the effective value of l can then be deduced. The value found is about 8 per cent. greater than the actual length of the narrow part of the rod. This is due to the slight amount of flow which necessarily takes place in the thicker parts of the rod. The effective length of the rod depends on the temperature distribution in the furnace, and will increase slightly at low temperatures. The variation is too small to be of importance. The form of the helices gives a very sensitive indication of the combined effects of irregularities of diameter and temperature along the rod. (Fig. 2 shows the appearance of a rod after an experiment.)

In the conduct of an experiment the most important precaution is to obtain a steady temperature before making the measurements. The measurements of time are obtained by means of a stop-watch. It is necessary to be certain that the wheel has had time to reach its steady velocity before the readings are taken. Except at the highest viscosities, a very small time is required for this purpose. When the viscosity is greater than about 10^{12} poises, it is necessary

to consider the variation of viscosity with time and to allow a sufficient time to elapse to obtain the limiting value. (The initial value is lower than the final value.) With a viscosity of $10^{16.5}$ poises, the load must be left on for a number of hours before the readings are taken.

Reference has been made to a second method of measuring viscosities, by rotating a rod in a cup filled with the glass under examination. For this method it is necessary to find a material which is strong, which is not appreciably acted upon by glass or air at high temperatures, and which can be worked to the required dimensions. An alloy consisting chiefly of nickel, iron, and chromium, and known as "reactol," has been found satisfactory for measurements of viscosities as low as 10^6 poises. China-clay fired to $1,500^\circ$ C. has also been used successfully, although the rods are more troublesome to make from this material. For the measurement of viscosities lower than 10^6 poises the use of clay rods becomes necessary. The absolute values of the viscosities measured in this way are found by comparison with the values determined by the method previously described, since the two methods overlap to a considerable extent. The experimental error is of the same order in both methods. A few measurements only have been made in this way.

The temperature viscosity curves of thirteen glasses are shown in fig. 3.

The shape of the curves can be determined with satisfactory accuracy, but the absolute position depends largely on the circularity and uniformity of diameter of the rods. In this respect the rods originally used, which were drawn by hand, were not very good. As an example of the variations which may arise from this cause, glass St. 59 may be cited (see fig. 4). The portion of the curve for temperatures above 490° C. is derived from observations on three different rods. (The experimental points are shown by crosses.) The curves derived from the three different rods were found to be parallel. The experimental points have therefore been brought on to the single curve shown



FIG. 2.

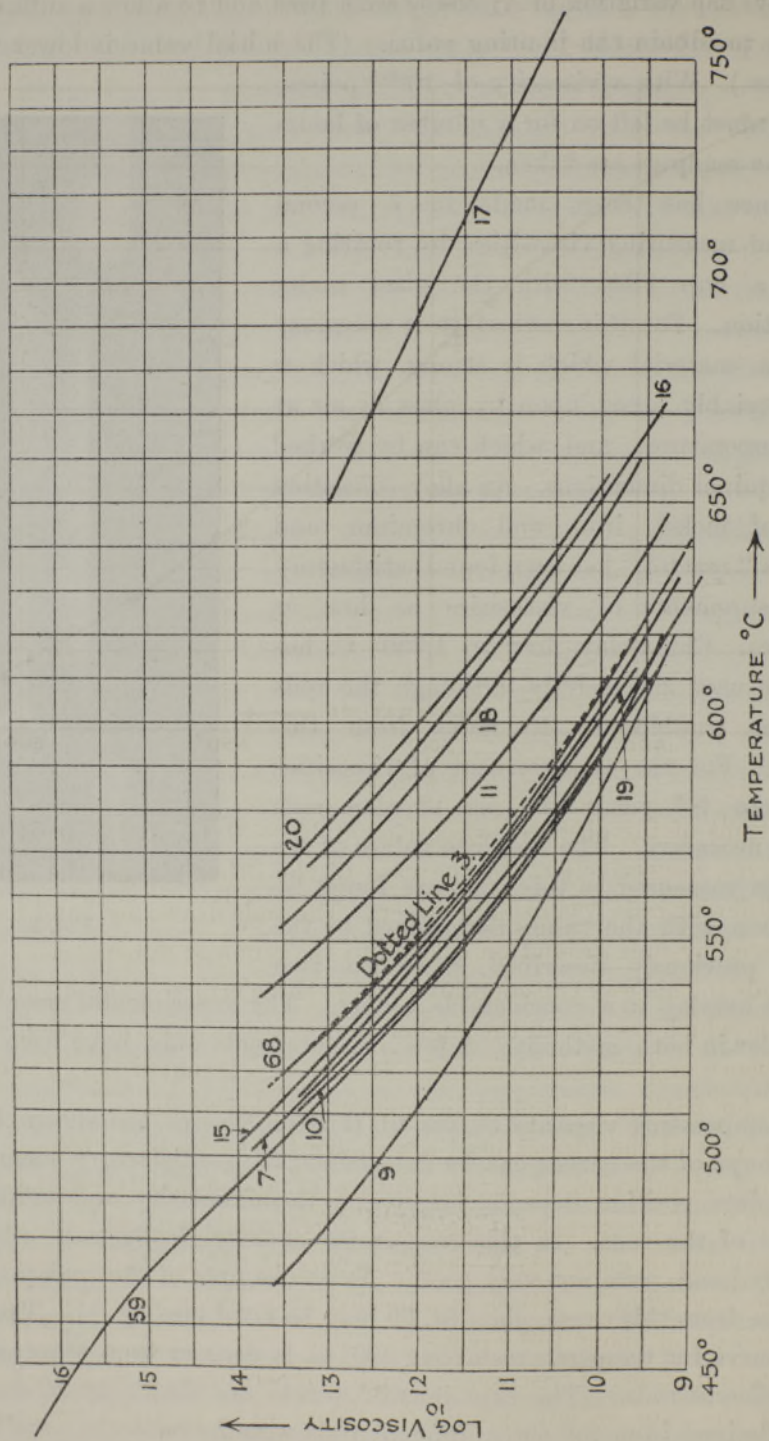


Fig. 3.

in fig. 4 by displacing vertically the three separate curves so as to bring each as close as possible to their mean position. This has been done by adding to

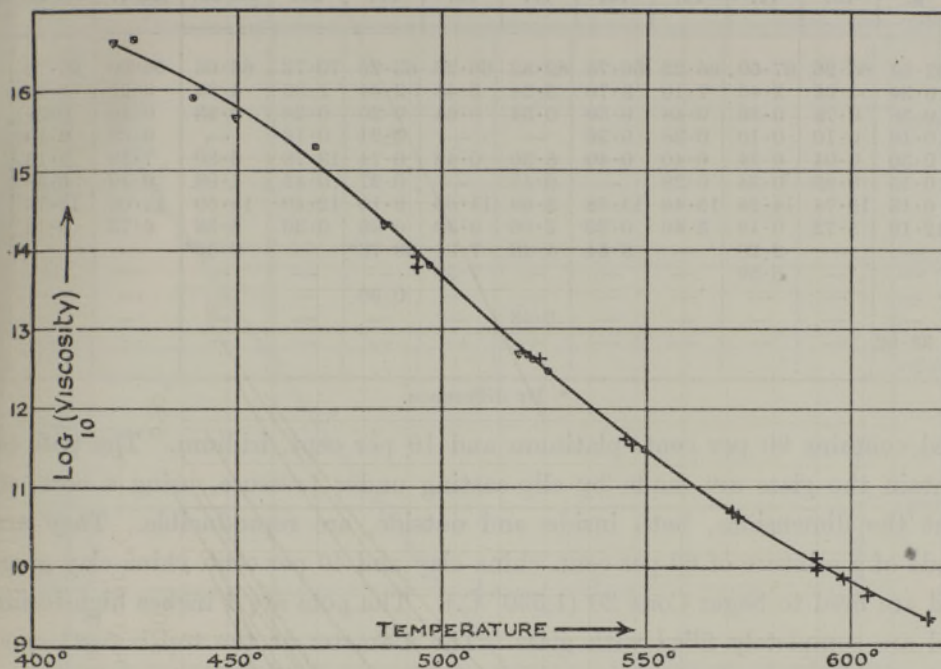


FIG. 4.

the logarithms of the observed viscosities 0.10, — 0.11, and — 0.04 on each of the curves respectively. In the case of the other glasses the agreement between different rods was usually closer. The rods drawn by hand have now been replaced by rods ground on a lathe to the required dimensions. The use of rods made in this way considerably reduces these errors. The slight bending of the rods which occurs at the higher temperatures does not seem to have much influence on the measurements. It may also be noted that a very small correction for friction is applied in calculating the results.

The analyses of the glasses used are shown in the table, p. 160.

The point of inflexion at a temperature of about 500° C. found for the glass St. 59 is of interest, as it probably corresponds with the discontinuity in the expansion of glass which usually occurs at temperatures a little below the "annealing point." (Glass is annealed at a temperature corresponding with a viscosity of about $10^{13.5}$ poises.)

The determinations of viscosity at high temperatures have been made by measuring the rate of fall through the glass of a partially counterpoised platinum-iridium ball suspended on a wire of the same material. The alloy

Table.

	9.	10.	11.	15.	16.	17.	18.	19.	20.	St. 59.	St. 7.	St. 3.	St. 68.
SiO ₂	52.54	65.96	67.60	66.22	66.78	69.82	66.26	63.26	70.72	66.66	66.50	66.78	66.50
Al ₂ O ₃	0.34	4.98	3.40	7.10	9.70	5.34	5.48	3.44	1.86	4.33	5.22	5.22	6.85
Fe ₂ O ₃	0.38	0.78	0.26	0.48	0.50	0.54	0.64	0.20	0.26	0.23	0.16	0.24	0.29
MnO.....	0.16	0.10	0.10	0.38	0.36	—	—	0.24	0.16	—	0.05	0.10	0.30
CaO.....	0.30	6.04	6.24	6.40	0.40	8.30	0.42	0.74	13.78	6.30	7.18	7.16	6.00
MgO.....	0.15	0.93	0.34	0.28	—	0.48	—	0.27	0.41	1.08	0.19	0.38	0.40
Na ₂ O.....	0.18	15.74	14.24	15.46	13.38	3.69	11.66	9.10	12.40	15.50	14.00	11.76	15.64
K ₂ O.....	12.10	5.72	0.18	3.46	0.30	2.06	0.30	0.98	0.30	5.58	6.72	8.18	2.90
B ₂ O ₃	—	—	2.28	—	8.54	4.48	7.91	20.78*	—	0.32*	—	—	1.09
ZnO.....	—	—	5.59	—	—	—	7.30	—	—	—	—	—	—
Sb ₂ O ₃	—	—	—	—	—	—	—	0.99	—	—	—	—	—
BaO.....	—	—	—	—	—	5.48	—	—	—	—	—	—	—
PbO.....	33.82	—	—	—	—	—	—	—	—	—	—	—	—

* By difference.

used contains 90 per cent. platinum and 10 per cent. iridium. The pots to contain the glass are made by slip-casting under pressure, using a core, so that the dimensions, both inside and outside, are reproducible. They are made of a mixture of 60 per cent. china-clay and 40 per cent. china-clay grog, and are fired to Seger Cone 20 (1,530° C.). The pots are 4 inches high inside and are completely filled with glass. The diameter at the top is 8.23 cms. and at the bottom 6.33 cms.

The apparatus was calibrated by means of "Lyle's Golden Syrup," which was placed in a brass vessel of similar dimensions immersed in a thermostat. The calibrating vessel was slightly higher than the clay pots, and was provided with a water-tight cover, carrying in the centre a long brass tube through which could be passed the platinum ball and wire. In this way the vessel could be deeply immersed in the thermostat. The top of the brass tube was covered, except for a very small opening to take the wire, and no effects due to slight evaporation of the syrup could be detected even after prolonged experiments. The vessel was filled with syrup up to a level corresponding with the height of the clay pots. The viscosity of the syrup was independently measured at various temperatures by dropping steel balls, of diameter 3/64 inch, through a cylinder of the liquid, and measuring the rate of fall over a distance of 5.93 cms. in the middle of the cylinder. The height of the cylinder was 30 cms. and the diameter 4.7 cms. In calculating the viscosities, Ladenburg's corrections for the finite size of the vessel were applied to Stokes' formula. The measurements were made in the same thermostat and simultaneously with the measurements conducted with the large platinum ball. The results are shown in fig. 5.

The determinations at the different temperatures were made in an irregular order, and the smoothness of the curve shows that the viscosity of the syrup

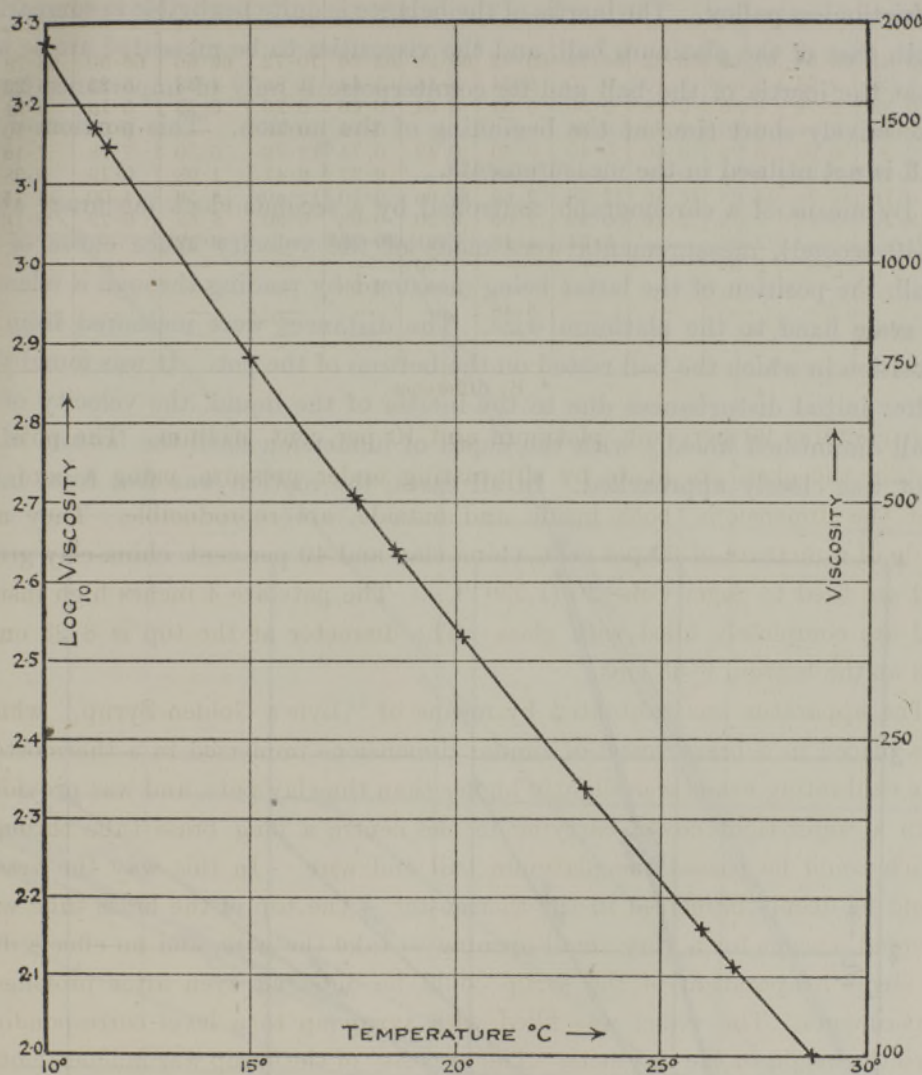


FIG. 5.

does not depend on its past thermal history, within the range of temperature employed. The platinum ball was suspended from an extremely light form of balance constructed in aluminium alloy. The balance rests upon two steel balls, of diameter $3/64$ inch, in lieu of knife-edges, and turns to a milligramme. Sectors concentric with the axis of rotation are fitted to the arms of the balance, and over these sectors hang light threads carrying the scale pans and the

platinum ball. The centre of gravity of the balance is adjustable, and is made to lie on the axis of rotation. The arrangement constitutes virtually a frictionless pulley. The inertia of the balance is quite negligible in comparison with that of the platinum ball, and the viscosities to be measured are so high that the inertia of the ball and its counterpoise is only of importance for an excessively short time at the beginning of the motion. This portion of the fall is not utilised in the measurements.

By means of a chronograph controlled by a seconds clock (accuracy about $1/40$ second), measurements were made of the velocity space curve of the ball, the position of the latter being measured by reading through a telescope a scale fixed to the platinum wire. The distances were measured from the position in which the ball rested on the bottom of the pot. It was found that, after initial disturbances due to the inertia of the liquid, the velocity of the ball diminished lineally with the depth of immersion until the bottom of the pot was closely approached. In all cases, the motion was free from initial

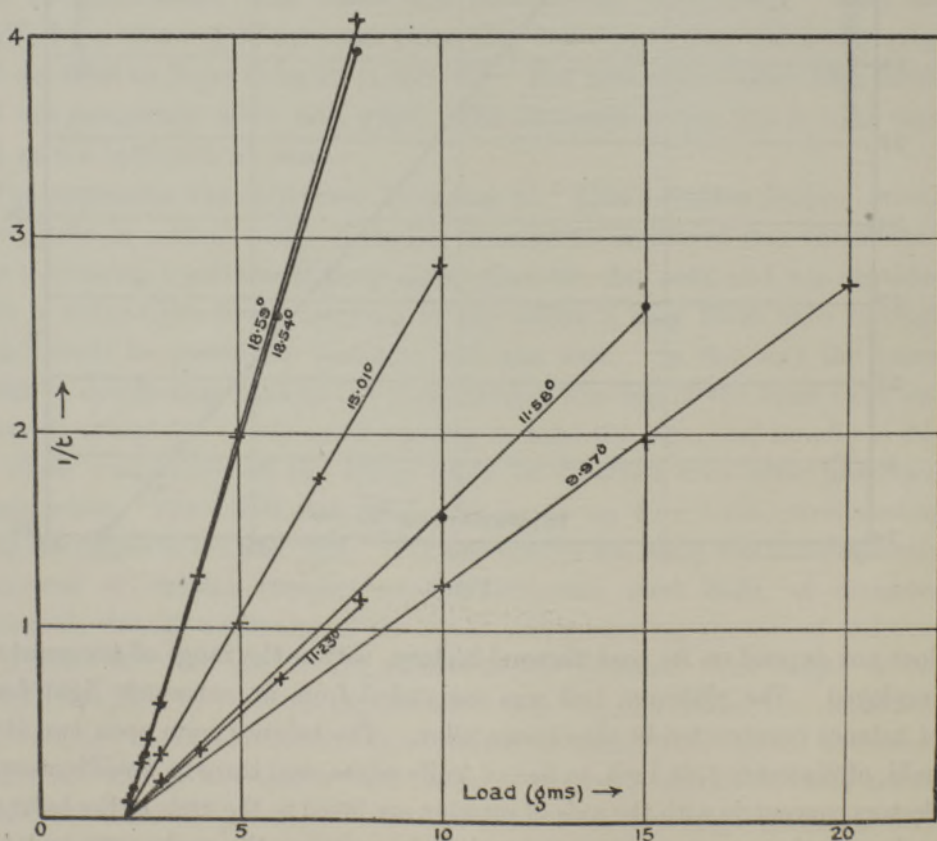


FIG. 6.

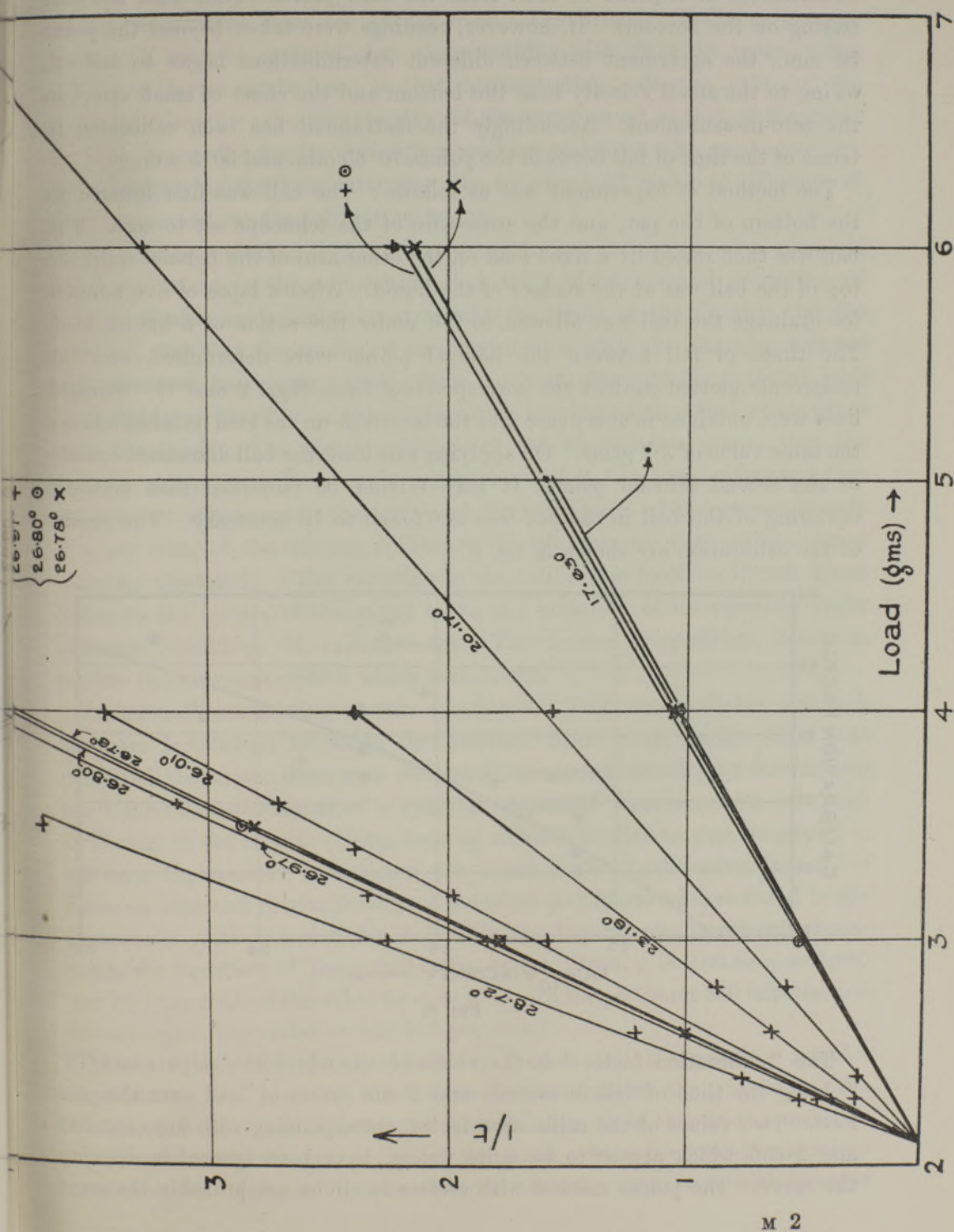


Fig. 7.

disturbances at a point 70 mm. from the zero position (*i.e.*, with the ball resting on the bottom). If, however, readings were taken beyond the point 50 mm., the agreement between different determinations began to fall off, owing to the small velocity near the bottom and the effect of small errors in the zero-measurement. Accordingly the instrument has been calibrated in terms of the time of fall between the points 70·65 mm. and 50·3 mm.

The method of experiment was as follows: The ball was first lowered to the bottom of the pot, and the cross-wire of the telescope set to zero. The ball was then raised by a fixed load on the other arm of the balance until the top of the ball was at the surface of the liquid. After a lapse of five minutes for drainage the ball was allowed to fall under the action of a known load. The times of fall between the fiducial points were determined, and the reciprocals plotted against the corresponding loads (figs. 6 and 7). Straight lines were obtained in every case and the intercept on the load axis had always the same value of 2·1 grms. On applying this load, the ball descended exactly to the second fiducial point. It may further be remarked that accurate centering of the ball in the pot was not found to be necessary. The results of the calibration are shown in fig. 8.

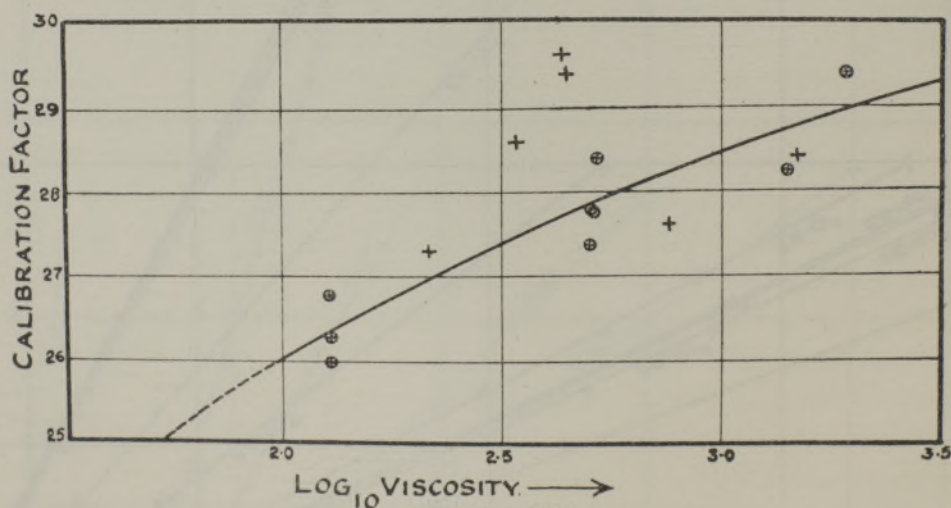


FIG. 8.

The "calibration factor" is the value of ηs , where the slope $s = 1/TP$, T being the time of fall in seconds and P the excess of load over the zero load. Two values of the calibration factor, corresponding with $\log \eta = 2.645$ and 2.636 , which appear to be quite wrong, have been ignored in drawing the curve. The points marked with crosses in circles are probably the most

reliable, being the results of the latest determinations, which were made with greater precautions than the earlier ones. The density of the syrup was 1.433. It may be assumed that when working with glass the value of the calibration factor to be used is that corresponding with the value of the kinematic viscosity (viscosity/density) of the syrup equal to that of the glass. For this purpose the density of the glass need not be known with great accuracy, since multiplying the kinematic viscosity by about 32 makes a difference of only 13 per cent. to the calibration factor.

It should be remarked that the above assumption is open to some question, being equivalent to the supposition that the hydrodynamical properties of the two liquids are completely expressible in terms of the density and the viscosity, and that the results of the experiments with the platinum ball are not influenced by certain minor effects, such as phenomena connected with the draining, or the effect of the inertia of the moving parts of the system other than the liquid. The direct evidence of the experiments shows that the drainage errors have been made negligible, and it has been calculated that the greatest effect due to the inertia of the solid parts of the system is only 0.4 per cent. of the viscous resistance (in the extreme case of the lowest viscosity measured). The variation in the calibration factor is therefore due either to the inertia of the syrup or to the variation of its viscosity under different conditions of measurement. The former supposition, however, implies the very assumption which is in doubt.

Additional data bearing on this question are, however, available, although they are insufficient to settle the matter. Some work, earlier than that described above, was done on a method of measuring viscosity by determining the logarithmic decrement of a cylinder suspended from a torsion wire, and oscillating in the liquid. Using a syrup solution similar to that employed in the later experiments, a variation was found in the "calibration factor" of the same sign and approximately of the same magnitude as that found in the experiments with the platinum ball. (In the logarithmic decrement experiments the variation of the "calibration factor" from $\eta = 100$ to $\eta = 1,000$ was 10.7 per cent. of the value for $\eta = 100$. In the platinum ball experiments the corresponding variation was 9.2 per cent.)

Since in both the platinum ball experiments and the rotating cylinder experiments, the rates of shear were independent of the viscosities, the variation in the "calibration factor" would seem to be due to the variation of viscosity with the rate of fall of the small sphere used for the absolute measurement. This would mean that the viscosity diminishes with increase

in the rate of shear. That no change of the logarithmic decrement with amplitude could be observed may be due to the fact that the effect would probably be greatest when the viscosity, and therefore the logarithmic decrement, were greatest. In this case the variation of the latter with amplitude becomes very difficult to measure.

The maximum circumferential velocities of the oscillating cylinder used in the determinations of the decrements were actually of the same order as the velocities of the small falling spheres. In the case of the experiments with the platinum ball, an effect should theoretically also be observable, since the lines of figs. 6 and 7 should be slightly curved. Since, however, the effect is probably inappreciable, except at very small velocities, comparable with the rates of all of the small spheres, the experiments are again indecisive. With regard to the supposition that the variation of the "calibration factor" with viscosity is due to the inertia of the liquid, such a hypothesis seems very improbable in the case of the oscillation experiments.

Fawsitt ('Roy. Soc. Proc.,' A, 1908, vol. 80, p. 290) has measured the logarithmic decrements of a disc oscillating in liquids of very greatly varying viscosities and densities. If we assume that the hydrodynamical properties of the liquids can be completely expressed in terms of the density and the viscosity, and, further, that the resistance to the motion of the disc can be expressed as the sum of a couple proportional to its acceleration and a couple proportional to its velocity, it may be shown by a consideration of the dimensions of the quantities involved that for the same apparatus λ/η is a function of ρ/η , where λ is the logarithmic decrement of the disc, η the viscosity of the liquid, and ρ its density.

The experimental results obtained by Fawsitt are in strict conformity with the above theory, and it has been deduced that for a value of ρ/η corresponding with $\rho = 1.43$ and $\eta = 100$ (the lowest value of η used in the present experiments), the value of λ/η is within one ten-thousandth part of its limiting value for infinite viscosity.

Since the dimensions of the authors' oscillating system did not differ very greatly from those of the system used by Fawsitt, these considerations suggest very strongly that the variation in the value of λ/η is not merely the inertia effect of the liquid. It would seem, then, that in spite of failure to observe a variation of the logarithmic decrement with amplitude, it is probable that the viscosity of syrup does depend upon the rate of shear. At the present time the above discussion may appear to be of academic rather than of practical interest, but it must be remembered that the interpretation of experiments on

the viscous properties of glass is by no means a simple matter, and it is not desirable unnecessarily to increase the inherent difficulties by the introduction of the unknown properties of other liquids used for calibration.

There is one more point which may be mentioned in this connection. It might be supposed that the variation of the viscosity of syrup, if it exists, could be measured by dropping through the liquid small spheres of varying sizes. Such experiments would not give the desired information, because the effect to be observed would not be of greater magnitude than the uncertainty of the corrections for the finite size of the containing vessel, which would have to be applied in the case of the larger balls.

Owing to the great variation of the viscosity of glass with temperature, a very carefully designed furnace and accurate means of measuring the temperature are necessary. The furnace which has been constructed is similar in principle to that described by Rosenhain and Coad-Pryor in the 'Transactions of the Faraday Society,' vol. 14, Part 3. The main features of the new design are shown in fig. 9. The heating elements, as before, consist of a number of graphite rings arranged in a pile so as to form a tube. The lower portion of the furnace is conical, so as to reduce the heat losses at the end, and very uniform heating has been obtained from this arrangement. A block in the shape of an inverted cone is fixed to the bottom of the furnace and serves to support a cylindrical muffle. (The muffle is shown in fig. 10.) The first block and stem were made in one piece out of a mixture of 60 per cent. carborundum and 40 per cent. china-clay, and the stem was fixed into a sheath at the bottom of the furnace by means of Portland cement. The arrangement proved satisfactory, but graphite has since been used instead of carborundum and clay in order to lessen the danger of damage through accidental overheating.

The muffle, which is made by slip-casting, contains 60 per cent. carborundum and 40 per cent. china-clay, and is fired to Seger Cone 26 (1,560° C.). This temperature is high enough to produce thorough vitrification of the bond, which is necessary in order to prevent carbon particles from penetrating the walls of the muffle. The joint between the muffle and the upper electrode is made with a mixture of clay, carborundum and sodium silicate. This cement sets hard at room temperatures, and the shrinkage is sufficient to allow the muffle to move freely with respect to the electrode.

When performing an experiment, the pot is placed in the centre of the muffle on a layer of sand. Above the pot a second muffle of china-clay is suspended from the top edge of the main muffle by means of a flange. The bottom of the china-clay muffle is about $\frac{1}{2}$ inch above the top of the pot. A horizontal disc

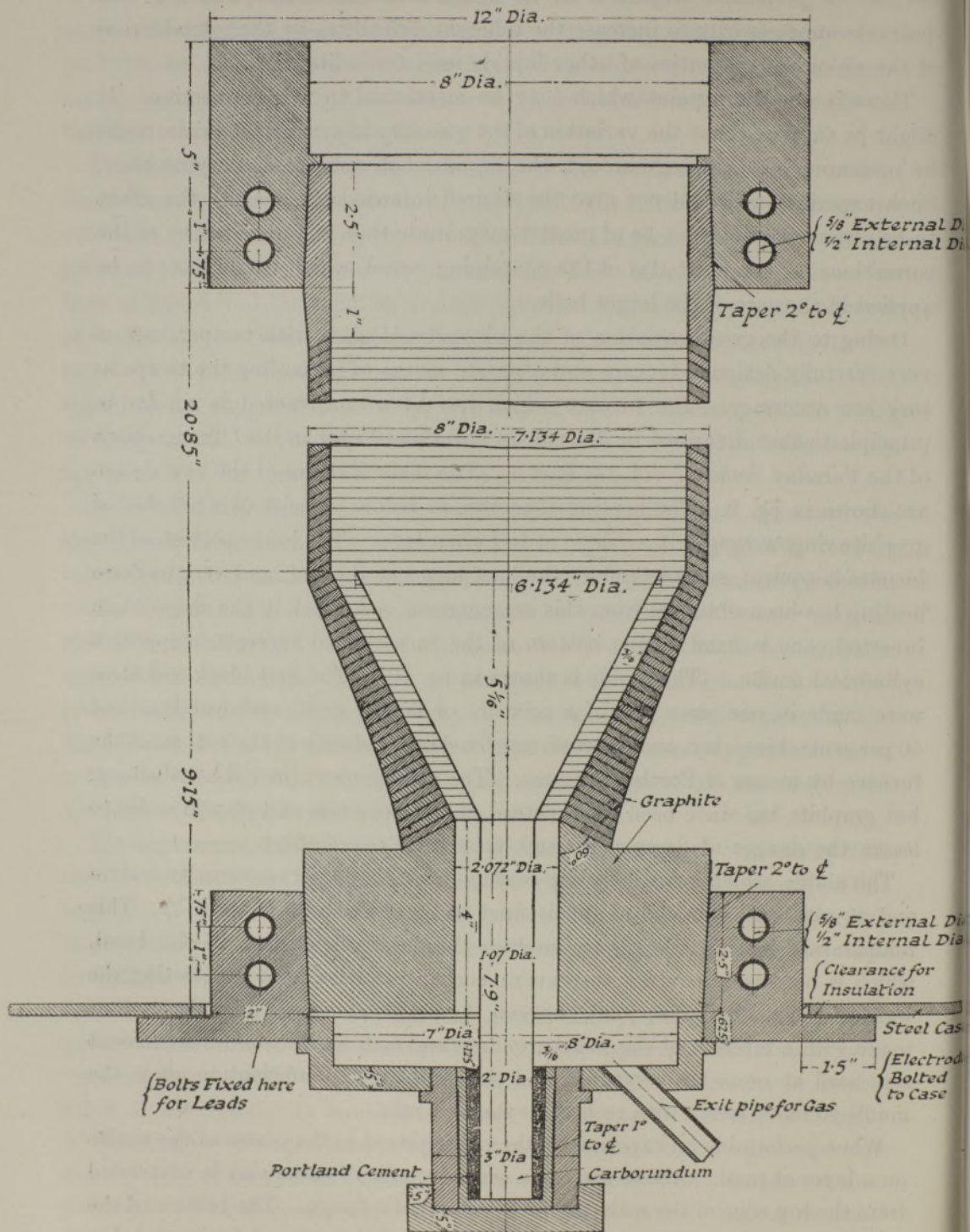


FIG. 9.

of china-clay is supported on a flange inside the china-clay muffle 6 inches above the bottom. This disc, and also the bottom of the china-clay muffle are provided with central holes 1 inch in diameter to allow the passage of the platinum ball, and a $\frac{1}{2}$ -inch diameter hole is also provided in each, through which an optical pyrometer is sighted. The normal line of sight of the latter passes about half-way between the centre of the pot and the outside. It has been found that the enclosure between the bottom of the carborundum muffle and the bottom of the china-clay muffle behaves as a perfect black body.

On sighting the optical pyrometer on a pot of glass, no difference in temperature could be detected between the centre of the pot and a point 1 inch away from the centre. As the radius of the pot used in the measurements is only $1\frac{1}{2}$ inches, the error in the temperature measurements may be considered as little greater than that due to the pyrometer itself. The latter has been specially designed for these experiments and is of the disappearing filament type. The precision of the pyrometer is of the order of one or two degrees in the hands of a skilled observer.

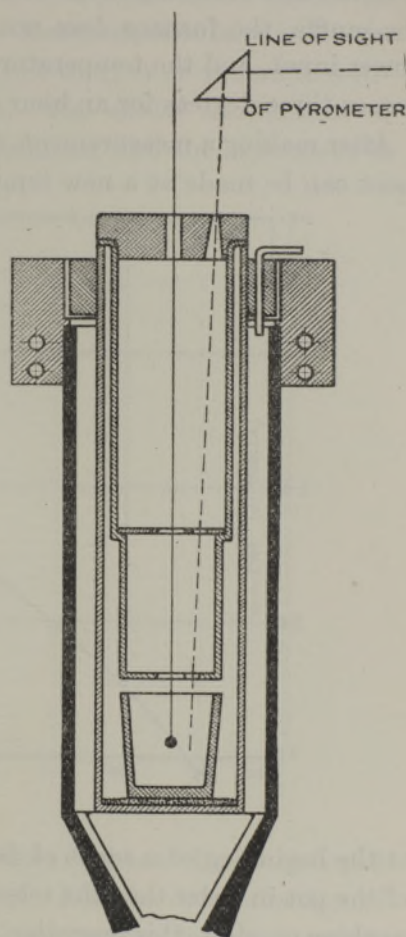


FIG. 10.

The general method of measurement has already been outlined. Certain precautions, however, remain to be mentioned. Care must be taken to prevent the platinum wire from fouling the apertures in the baffles through which it passes. For this reason it has been found desirable to make the necessary adjustments before starting-up the furnace, so that afterwards no adjustments need be made. Hence the whole of the charge of glass must be in the pot at the beginning of the experiment when the furnace is cold. It is necessary to prepare such a charge of glass by first casting a piece of glass of the same volume as that of the pot, but slightly smaller in diameter, so as to allow for expansion, and also because the pot may be slightly warped. The piece of glass is cast in

an iron mould and allowed to cool very slowly in a large furnace. To increase the life of the carbon parts of the furnace, a current of nitrogen is passed into it from below. Owing to the heat capacity of the conical block which supports the muffle, the furnace does not respond readily to small fluctuations of the power input, and the temperature can easily be maintained constant to within two or three degrees for an hour or two.

After making a measurement, about two hours must elapse before a measurement can be made at a new temperature. It has already been explained that

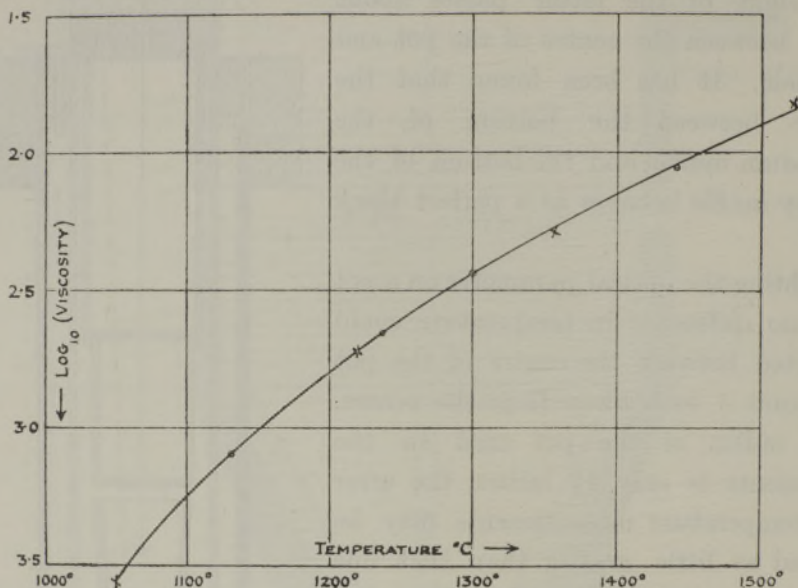


FIG. 11.

at the beginning of a series of determinations the ball is lowered to the bottom of the pot in order that the telescope may be set in its proper position. When working on glass, this operation is done only once, at the beginning of the series of measurements, in order to avoid stirring up the viscous layer of glass at the bottom of the pot. When pulling up the ball by hand, it is quite easy to feel the very much increased resistance experienced when the ball is at the bottom of the pot. After the telescope has been adjusted, a stop in the balance case is used, which arrests the balance when the ball has just passed the second fiducial mark. The necessity of using a ball made of a material which is not attacked by glass is manifest, and it is clear that, in general, methods of measuring the viscosity of glass at high temperatures, in which ordinary refractories are employed for *all* the parts, can be satisfactory only when the apparatus is of considerable size.

The results of some measurements which have been made on glass St. 59

are shown in figs. 11 and 12. It will be observed that the low-temperature part of the curve shown in fig. 12 is rather longer than that shown in previous figures. This extension represents the results of measurements made by rotating a rod in the glass as already explained. Measurements have not been made in the region indicated by the dotted line, but there is no reason to anticipate discontinuities in this part of the curve. The circles and crosses of fig. 11 represent two independent sets of measurement, and the agreement is satisfactory.

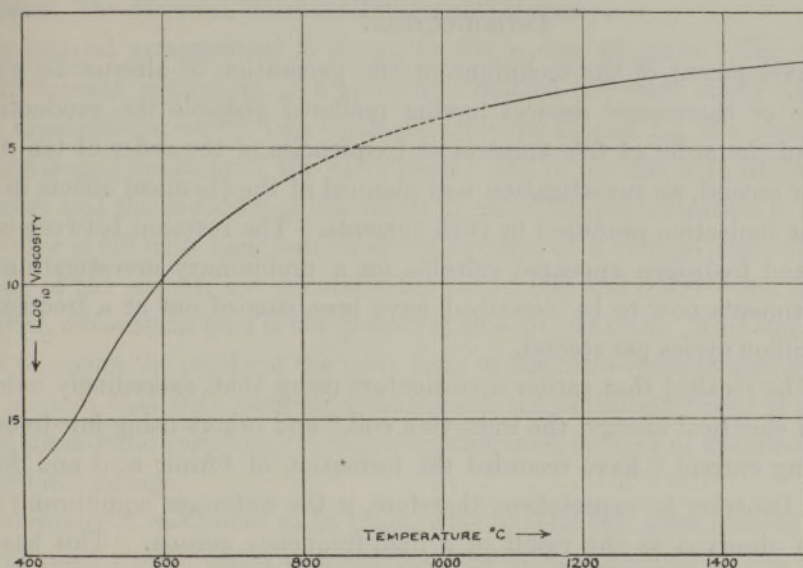


FIG. 12.

The methods of measurement which have been described have been developed as fundamental methods yielding absolute results. For the determination of the effects of very small changes in composition, or possibly, in certain cases, changes in thermal treatment of the glasses, it is desirable to develop comparative methods of measurement. Many of the experimental difficulties attending the absolute measurements are eliminated in the case of a differential measurement.

The work has been carried out under the direction of Dr. Rosenhain, F.R.S., in the Department of Metallurgy and Metallurgical Chemistry of the National Physical Laboratory, partly in connection with research on Optical Glass, and partly under the programme of the Glass Research Association. The authors are indebted to the Council of the Association for permission to publish the results. Their thanks are also due to their colleagues, Mr. W. H. Withey, B.A., and Miss H. Millar, who carried out the numerous analyses.