from the wire is made small. When this ratio is not small, λ cannot be determined unless the surface emissivity, h, be known independently.

A simple form of conductivity apparatus was used, and although long thick wires were mounted in it, no "end" corrections were necessary. A small lateral loss of heat by radiation which involved a correction in λ of the order of 1 per cent. was allowed for by calculation.

The thermal conductivity, λ, the electrical conductivity, κ, and the Lorenz coefficient, $\lambda/\kappa T$, T being the temperature in degree K., are given in Table V, p. 331, of this paper for silver, gold, molybdenum and tungsten.

Material and Radiational Waves.

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§ 1. In what follows the Maxwellian equations of electromagnetic and electron theory are derived from one set of basic relations, in a manner which throws some light on the relationship between material and radiational waves, and accounts for the existence of exactly three types of physical entities, namely, positive electricity, negative electricity, and radiation. We assume the existence of two vectors **A** and **n** and of a scalar quantity θ and identify **n** with a unit normal to the wave surface associated with the observable entity (whether material or radiational), and θ with the normal speed of propagation of the surface, so that

$$\theta \mathbf{n} = \frac{d\mathbf{n}}{dt}. \tag{1}$$

If E_1, E_2, E_3 are the components of the electric vector **E** in a Cartesian system of co-ordinates (x_1, x_2, x_3) the set of relations from which all equations are derivable may be written

$$\left.\begin{aligned} \frac{\partial E_\mu}{\partial x_\nu} &= \cos(nx_\nu)\, A_\mu, \qquad && \mu, \nu = 1, 2, 3 \\ \frac{\partial E_\mu}{\partial t} &= \theta A_\mu, && \mu = 1, 2, 3 \end{aligned}\right\}, \tag{2}$$

where $\cos(nx_\nu)$ stands for the cosine of the angle between $\mathbf{n}$ and the axis of x_ν.

§ 2. First, let it be supposed that the vector $\mathbf{A}$ is normal to the wave surface so that

$$\mathbf{A} = \pm |\mathrm{A}| \mathbf{n}, \tag{3}$$

where $|\mathrm{A}|$ is the absolute value of $\mathbf{A}$, the positive or negative sign to be taken according as $\mathbf{A}$ and $\mathbf{n}$ are in the same or in opposite directions. We then have from (2)

$$\operatorname{div} \mathbf{E} = \rho, \tag{4}$$

where

$$\rho = \pm |\mathrm{A}|. \tag{5}$$

We identify ρ with the electric density. Further from (2) we get

$$\begin{aligned}\frac{\partial \mathrm{E}_3}{\partial x_2} - \frac{\partial \mathrm{E}_2}{\partial x_3} &= \mathrm{A}_3 \cos(nx_2) - \mathrm{A}_2 \cos(nx_3) \\ &= \pm [\cos(nx_3)\cos(nx_2) - \cos(nx_2)\cos(nx_3)] |\mathrm{A}| \\ &= 0,\end{aligned}$$

and similarly for the other two components of curl $\mathbf{E}$, so that

$$\operatorname{curl} \mathbf{E} = 0. \tag{6}$$

Again from (2) we have

$$\frac{\partial \mathbf{E}}{\partial t} = \theta \mathbf{A},$$

which, on account of (1), (3) and (5) may be written

$$\frac{\partial \mathbf{E}}{\partial t} + \rho \mathbf{V} = 0, \tag{7}$$

where

$$\mathbf{V} = -\theta \mathbf{n} = -\frac{d\mathbf{n}}{dt}, \tag{8}$$

and we identify $\mathbf{V}$ with the velocity of the charge. From (7) we get, on operating with div and using (4)

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{V}) = 0. \tag{9}$$

§ 3. Next, let it be supposed that $\mathbf{A}$ is tangential to the wave surface so that

$$\sum_1^3 \mathrm{A}_\mu \cos(nx_\mu) = 0, \tag{10}$$

we then have from (2)

$$\operatorname{div} \mathbf{E} = 0, \tag{11}$$

denoting the absence of electric charge. Further we get

$$\frac{\partial E_3}{\partial x_2} - \frac{\partial E_2}{\partial x_3} = [\cos (Ax_3) \cos (nx_2) - \cos (Ax_2) \cos (nx_3)] \mid A \mid,$$

with two similar equations for the other two components of curl **E**. Let **n′** be a unit vector perpendicular both to **n** and to **A**. It is seen that

$$\text{curl } \mathbf{E} = \mid A \mid \mathbf{n}'$$

so that if we define a vector **H** by means of the equation

$$\mathbf{n}' \mid A \mid = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \tag{12}$$

where c is the fundamental velocity, we have

$$\text{curl } \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}. \tag{13}$$

We identify H with the magnetic vector. Operating on both sides of (13) with the operator curl, we have

$$\text{curl curl } \mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t} (\text{curl } \mathbf{H}),$$

or since

$$\text{curl curl } \mathbf{E} \equiv \text{grad div } \mathbf{E} - \nabla^2 \mathbf{E}$$

we obtain an account of (11)

$$\nabla^2 \mathbf{E} = \frac{1}{c}\frac{\partial}{\partial t} (\text{curl } \mathbf{H}),$$

hence substituting from (2) we have for the x_1 components

$$\left[\cos (nx_1) \frac{\partial}{\partial x_1} + \cos (nx_2) \frac{\partial}{\partial x_2} + \cos (nx_3) \frac{\partial}{\partial x_3}\right] A_1$$
$$+ A_1 \left[\frac{\partial \cos (nx_1)}{\partial x_1} + \frac{\partial \cos (nx_2)}{\partial x_2} + \frac{\partial \cos (nx_3)}{\partial x_3}\right] = \frac{1}{c}\frac{\partial}{\partial t}[\text{curl } \mathbf{H}]_{x_1}, \tag{14}$$

if now we suppose the vector **n** to be solenoidal so that

$$\frac{\partial \cos (nx_1)}{\partial x_1} + \frac{\partial \cos (nx_2)}{\partial x_2} + \frac{\partial \cos (nx_3)}{\partial x_3} \equiv \text{div } \mathbf{n} = 0, \tag{15}$$

we may write

$$\frac{d\mathbf{A}}{dn} = \frac{1}{c}\frac{\partial}{\partial t} (\text{curl } \mathbf{H}),$$

which on introducing the speed $\theta = dn/dt$ gives

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{\theta}{c}\frac{\partial}{\partial t} (\text{curl } \mathbf{H}). \tag{16}$$

If now we put

$$\theta = \pm c \tag{17}$$

we get from (2)

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{1}{\theta}\frac{\partial^2 \mathbf{E}}{\partial t^2},$$

which in conjunction with (16) and (17) yields

$$\frac{\partial \mathbf{E}}{\partial t} = c \operatorname{curl} \mathbf{H}. \tag{18}$$

§ 4. Equations (4), (6), (7) and (9) are seen to be the Maxwellian equations in the absence of radiation,* whereas equations (11), (13) and (18) are the equations in the absence of charge. The first set is obtained on the assumption that the vector **A** is longitudinal and the velocity dn/dt of wave propagation variable and equal to minus the velocity of the charge, the second set is obtained on the assumption that **A** is transverse and dn/dt constant and equal to the fundamental velocity. Thus a material entity is associated with the propagation of a longitudinal vector, whereas a radiational entity is associated with the propagation of a transverse vector. Since a longitudinal vector may have either of two directions of opposite signs, we have exactly two types of material entities, namely, positive and negative electricity. Radiation corresponds to a mid-way position, so to speak, between positive and negative electricity.

§ 5. In the general case, the vector **A** will possess a longitudinal and a transverse component, corresponding to the existence of matter and radiation. If we assume the longitudinal component to be propagated with a variable speed $-v$, and the transverse component with a constant speed c, so that θ is given one or the other of these two values according as it is associated with the one type of component or with the other, the Maxwellian equations may now be obtained in their general form

$$\left.\begin{aligned} \operatorname{div} \mathbf{E} &= \rho && \text{(i)} \\ c \operatorname{curl} \mathbf{E} &= -\frac{\partial \mathbf{H}}{\partial t} && \text{(ii)} \\ c \operatorname{curl} \mathbf{H} &= \frac{\partial \mathbf{E}}{\partial t} + \rho \mathbf{V} && \text{(iii)} \\ \frac{\partial \rho}{\partial t} + \operatorname{div} (\rho \mathbf{V}) &= 0 && \text{(iv)} \end{aligned}\right\} . \tag{20}$$

* All radiation is characterised by the existence of a magnetic vector. A so-called "steady" electromagnetic field is an idealistic conception of a limiting form of radiation, namely, when the frequency is supposed to be vanishingly small.

Of these (i) and (ii) follow as in (4) and (13) above, (iii) is obtained from (ii) by operating with curl as in deriving (18) above, but with div $\mathbf{E} = \rho$ instead of 0, and (iv) follows from (iii) on operating with div.

Summary.

The Maxwellian equations of electromagnetic and electric theory are derived from one set of basic relations, in a manner which throws some light on the relationship between material and radiational waves, and accounts for the existence of exactly three types of physical entities, namely, positive electricity, negative electricity and radiation. It is shown that a physical entity may be associated with the propagation of a vector $\mathbf{A}$ in a direction $\mathbf{n}$. If $\mathbf{A}$ and $\mathbf{n}$ are in the same direction, the entity is recognised as positive electricity, if in opposite directions as negative electricity, and if mutually perpendicular as radiation. In the general case $\mathbf{A}$ will have a longitudinal and a transverse component corresponding to the existence of matter and radiation.

The Chemical Constant of Chlorine Vapour and the Entropy of Crystalline Chlorine.

By T. E. Stern, Trinity College, Cambridge.

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Introduction.

In a recent paper* the writer calculated the vapour pressure of hydrogen crystals, using the Einstein-Bose statistics for the gaseous phase. The work was an extension of that of R. H. Fowler,† who had used the slightly less accurate classical statistics for the hydrogen gas.

In this paper we propose to apply similar methods of investigation to chlorine. The investigation will have to be different in some respects, however. Hydrogen was considered to consist of a mixture of two gases, para- and ortho-hydrogen, which retained their individuality over long periods of time at low temperatures. Due to the existence of two isotopes of chlorine, we shall here have five gases to consider instead of two. Further, hydrogen molecules almost

* 'Proc. Roy. Soc.,' A, vol. 130, p. 367 (1931).

† 'Proc. Roy. Soc.,' A, vol. 118, p. 52 (1928).