

*Boundary Conditions for the Wave Equation.*

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A single electron in the field of two fixed nuclei, constituting the idealized hydrogen molecular ion, provides the simplest case for the application of wave mechanics to molecular, as distinct from atomic, problems. The most extensive theoretical discussion of the corresponding wave equation has been given by A. H. Wilson\* in these 'Proceedings.' He was led to conclude that this equation possesses no eigen-solutions satisfying the usual boundary conditions for an atomic problem. Subsequent investigators† have succeeded, however, in obtaining by numerical methods eigen-values in good agreement with observed values of the energy. But, with the exception of Teller, they appear not to have taken account of Wilson's result. It is therefore worth while to investigate the existence of their solutions and to clear up, if possible, any doubt as to the applicability of the familiar boundary conditions to this type of problem.

The usual existence theorems‡ for eigen-values apply only to boundary conditions at ordinary points of the differential equation. The difficulty in cases like Wilson's equation is that the conditions are given at singular points.

We have proved§ that, *if  $a, b$  are two successive singularities of the differential equation*

$$\frac{d}{dx} \left( K \frac{dy}{dx} \right) - Gy = 0, \quad (1)$$

where  $K, G$  are functions of  $x$  and of a parameter  $\lambda$ , satisfying, in every closed interval  $(\alpha, \beta)$ ,  $a < \alpha \leq x \leq \beta < b$ , the conditions usually imposed (Ince, *loc. cit.*) in the whole interval  $a \leq x \leq b$ , and if the indicial equations for  $a, b$  have each the single non-negative root zero, then there exist real values of  $\lambda$  such that (1) possesses a solution which, with its derivatives, is finite and continuous throughout  $a \leq x \leq b$ . We extend this to cases where only normal solutions exist

\* 'Proc. Roy. Soc.,' A, vol. 118, p. 617 (1928).

† Morse and Stueckelberg, 'Phys. Rev.,' vol. 33, p. 932 (1929); Teller, 'Z. Physik,' vol. 61, p. 458 (1930); Hylleraas, *ibid.*, vol. 71, p. 736 (1931), where a fuller list of references is given.

‡ Cf. Ince, "Ordinary Differential Equations," chaps. 10, 11 (1926).

§ In a paper to be published by the London Mathematical Society.

at the end-points. Roughly speaking, it means that if equation (1) possesses one solution behaving in the required manner near  $x = a$ , and another solution behaving in the required manner near  $x = b$ , then it is possible so to choose  $\lambda$  that these solutions join up and become identical. The theorems themselves are of mainly mathematical interest, and will be published elsewhere.

This theory covers the important cases occurring in wave mechanics, and in particular it shows that Wilson's equation must possess eigen-solutions of the usual kind. These solutions seem to have evaded his search when he compares his general asymptotic forms of solution, on p. 625 of his paper, with a particular case (p. 624) in which certain coefficients do not vanish. This is sufficient to show that the corresponding coefficients in the general form do not vanish *identically*. But it does not follow that they cannot vanish, as required by the boundary conditions, for particular values (eigen-values) of the parameters, and in fact our work has shown that they must actually vanish in this way.

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### *The Adsorption of Iodine by Potassium Iodide.*

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Opinion has been divided for some years as to whether the adsorption of a vapour on a solid surface is confined to a monomolecular layer until the vapour pressure reaches the saturation value, or whether multimolecular layers are formed.

Experimental work has generally failed to provide an unequivocal answer to this point, often because the true area of the surface of the adsorbent could not be estimated. It is now recognized that the true area of a surface may be many times the apparent area, and the ratio of these will depend on the previous treatment of the surface. The position has been complicated further by the difficulty of determining the amount of adsorption on a small area of a plane surface when the adsorption is so small as to be undetectable without sensitive apparatus; consequently, the bulk of the experimental work has been carried