On the Relations of the Tensor-calculus to the Spinor-calculus

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1—Introduction

In this paper it is first shown (§ 2) that a six-vector which has the property of being identical with its own dual six-vector, and which, moreover, has its invariant null, has properties equivalent to those of a spinor. It is then shown (§ 3) that the correspondence thus set up between tensor-analysis and spinor-analysis enables us to replace some complicated tensor-operations by simple spinor-operations. In § 4 the correspondence is applied to the tensorization of Dirac's relativistic equation of the electron, in connexion with its generalization to the space-time of general relativity. It is shown that Dirac's equations are equivalent to the vanishing of an ordinary vector.

2—A Special Type of Six-vector, and its Equivalence to a Spinor

We shall first suppose that space-time is Galilean, so that the metric is given by $ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$, where $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$. It is well known that in Galilean space-time, to every six-vector X^{pq} there corresponds a *dual* six-vector Y^{pq} such that

$$Y_{pq} = iX^{rs}, \qquad X_{pq} = iY^{rs},$$

where $pqrs$ is an even permutation of 0123 and i is $\sqrt{-1}$.

Consider now a complex six-vector R^{pq} which has the property of being identical with its own dual; this property is invariant under transformations of coordinates. We have then

$$R_{01} = iR_{23}, \qquad R_{02} = iR_{31}, \qquad R_{03} = iR_{12}. \tag{1}$$

The two invariants of this six-vector, namely

$$(R_{01}{}^2 + R_{02}{}^2 + R_{03}{}^2 - R_{23}{}^2 - R_{31}{}^2 - R_{12}{}^2)$$

and

$$(R_{01}R_{23} + R_{02}R_{31} + R_{03}R_{12}),$$

are therefore not distinct, each of them being a multiple of

$$(R_{01}^2 + R_{02}^2 + R_{03}^2);$$

we shall now suppose further that

$$R_{01}^2 + R_{02}^2 + R_{03}^2 = 0, \tag{2}$$

so that the invariants of R^{pq} are zero. Self-dual six-vectors whose invariant is null, such as R^{pq}, play a prominent part in what follows.

Now let the general Lorentz transformation, *i.e.*, the linear transformation which satisfies the equation

$$\grave{x}_0^2 + \grave{x}_1^2 - \grave{x}_2^2 - \grave{x}_3^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2,$$

be

$$\grave{x}_i = \sum_{k=0}^{3} a_{ik} x_k. \tag{3}$$

Then the usual equations of transformation of a six-vector, combined with equations (1), give for the quantities R_{01}, R_{02}, R_{03}, the equations of transformation

$$\left.\begin{aligned}
\grave{R}_{01} = {} & (a_{00}a_{11} - a_{01}a_{10} - ia_{02}a_{13} + ia_{03}a_{12})\, R_{01} \\
& + (a_{00}a_{12} - a_{02}a_{10} - ia_{03}a_{11} + ia_{01}a_{13})\, R_{02} \\
& + (a_{00}a_{13} - a_{03}a_{10} - ia_{01}a_{12} + ia_{02}a_{11})\, R_{03} \\
\grave{R}_{02} = {} & (a_{00}a_{21} - a_{01}a_{20} - ia_{02}a_{23} + ia_{03}a_{22})\, R_{01} \\
& + (a_{00}a_{22} - a_{02}a_{20} - ia_{03}a_{21} + ia_{01}a_{23})\, R_{02} \\
& + (a_{00}a_{23} - a_{03}a_{20} - ia_{01}a_{22} + ia_{02}a_{21})\, R_{03} \\
\grave{R}_{03} = {} & (a_{00}a_{31} - a_{01}a_{30} - ia_{02}a_{33} + ia_{03}a_{32})\, R_{01} \\
& + (a_{00}a_{32} - a_{02}a_{30} - ia_{03}a_{31} + ia_{01}a_{33})\, R_{02} \\
& + (a_{00}a_{33} - a_{03}a_{30} - ia_{01}a_{32} + ia_{02}a_{31})\, R_{03}
\end{aligned}\right\}. \tag{4}$$

Now if α, β, γ, δ are any complex numbers satisfying the equation $\alpha\delta - \beta\gamma = 1$, then the transformation of two complex variables (ϕ_1, ϕ_2) represented by

$$\left.\begin{aligned}
\grave{\phi}_1 &= \alpha\phi_1 + \beta\phi_2 \\
\grave{\phi}_2 &= \gamma\phi_1 + \delta\phi_2
\end{aligned}\right\} \tag{5}$$

is called a binary unimodular transformation; and it is well known that to every binary unimodular transformation there corresponds a definite Lorentz transformation, the coefficients a_{pq} of the Lorentz transformation

being given in terms of the coefficients of the binary unimodular transformation by the matrix-equation

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{pmatrix} = \tfrac{1}{2} \begin{pmatrix} \alpha & \beta & \gamma & \delta \\ \gamma & \delta & \alpha & \beta \\ -i\gamma & -i\delta & i\alpha & i\beta \\ \alpha & \beta & -\gamma & -\delta \end{pmatrix} \begin{pmatrix} \alpha^* & \beta^* & -i\beta^* & \alpha^* \\ \beta^* & \alpha^* & i\alpha^* & -\beta^* \\ \gamma^* & \delta^* & -i\delta^* & \gamma^* \\ \delta^* & \gamma^* & i\gamma^* & -\delta^* \end{pmatrix}, \quad (6)$$

where α^*, β^*, γ^*, δ^* are the conjugate-complex quantities to α, β, γ, δ. The group of Lorentz transformations† is isomorphic to the group of binary unimodular transformations, so that if two binary unimodular transformations B_1, B_2 correspond respectively in accordance with equation (6) to two Lorentz transformations L_1, L_2, then the binary unimodular transformation B_1B_2 corresponds to the Lorentz transformation L_1L_2. A pair of quantities (ϕ_1, ϕ_2) which undergo the binary unimodular transformation (5) when the coordinates are subjected to the Lorentz transformation defined by (3) and (6), is called a spinor.‡

Now substitute in equations (4) the values of the a_{pq} given by (6): we obtain

$$\left.\begin{aligned} \bar{R}_{01} &= \tfrac{1}{2}(\alpha^2-\beta^2-\gamma^2+\delta^2)\,R_{01} + \frac{i}{2}(\alpha^2+\beta^2-\gamma^2-\delta^2)\,R_{02} \\ &\qquad\qquad + (-\alpha\beta+\gamma\delta)\,R_{03} \\ \bar{R}_{02} &= \frac{i}{2}(-\alpha^2+\beta^2-\gamma^2+\delta^2)\,R_{01} + \tfrac{1}{2}(\alpha^2+\beta^2+\gamma^2+\delta^2)\,R_{02} \\ &\qquad\qquad + i(\alpha\beta+\gamma\delta)\,R_{03} \\ \bar{R}_{03} &= (-\alpha\gamma+\beta\delta)\,R_{01} + i(-\alpha\gamma-\beta\delta)\,R_{02} + (\alpha\delta+\beta\gamma)\,R_{03} \end{aligned}\right\}. \quad (7)$$

These equations give at once

$$\bar{R}_{01} + i\bar{R}_{02} = \alpha^2(R_{01}+iR_{02}) + \beta^2(-R_{01}+iR_{02}) - 2\alpha\beta R_{03}$$
$$-\bar{R}_{01} + i\bar{R}_{02} = \gamma^2(R_{01}+iR_{02}) + \delta^2(-R_{01}+iR_{02}) - 2\gamma\delta R_{03},$$

which by (2) are the squares of the equations

$$\left.\begin{aligned} (\bar{R}_{01}+i\bar{R}_{02})^{\frac{1}{2}} &= \alpha(R_{01}+iR_{02})^{\frac{1}{2}} + \beta(-R_{01}+iR_{02})^{\frac{1}{2}} \\ (-\bar{R}_{01}+i\bar{R}_{02})^{\frac{1}{2}} &= \gamma(R_{01}+iR_{02})^{\frac{1}{2}} + \delta(-R_{01}+iR_{02})^{\frac{1}{2}} \end{aligned}\right\}. \quad (8)$$

† More strictly speaking, not the *complete* Lorentz group, but the *restricted* Lorentz group, *i.e.*, the group of four-dimensional rotations of determinant +1.

‡ B. L. van der Waerden, 'Nachr. Ges. Wiss. Gött.,' p. 100 (1929). The name *spinor* is given more generally to quantities which bear the same kind of relation to (ϕ_1, ϕ_2) that tensors of rank greater than unity bear to vectors in ordinary tensor-analysis.

These equations show that *the pair of quantities* $(R_{01} + iR_{02})^{\frac{1}{2}}$, $(-R_{01} + iR_{02})^{\frac{1}{2}}$ *constitute a spinor*. Denoting this spinor by $(\sqrt{2}\phi_1, \sqrt{2}\phi_2)$, we have then

$$\sqrt{2}\phi_1 = (R_{01} + iR_{02})^{\frac{1}{2}}, \quad \sqrt{2}\phi_2 = (-R_{01} + iR_{02})^{\frac{1}{2}} \tag{9}$$

and

$$\left.\begin{array}{lll} R_{01} = \phi_1^2 - \phi_2^2, & R_{02} = -i(\phi_1^2 + \phi_2^2) & R_{03} = -2\phi_1\phi_2 \\ R_{23} = -i(\phi_1^2 - \phi_2^2), & R_{31} = -(\phi_1^2 + \phi_2^2) & R_{12} = 2i\phi_1\phi_2 \end{array}\right\} . \tag{10}$$

Equations (9) and (10) set up a correspondence between self-dual six-vectors of vanishing invariant, R_{rq}, on the one hand, and spinors (ϕ_1, ϕ_2) on the other hand. Thus *the calculus of spinors is included in the calculus of tensors*†; in the sense that to every spinor there corresponds a tensor, and to every relation between spinors there corresponds a relation between tensors. When, however, we study this correspondence more closely, we find some remarkable features which will now be examined.

3—The Tensor-operations Corresponding to Certain Spinor-operations

In the spinor-calculus it is shown that ordinary vectors can be constructed by combining spinors with their complex conjugates : thus if (ϕ_1, ϕ_2) and (ψ_1, ψ_2) are spinors, and if

$$D^0 = \phi_1\psi_1^* + \phi_2\psi_2^*, \quad D^1 = \phi_1\psi_2^* + \psi_1^*\phi_2,$$
$$D^2 = i\phi_1\psi_2^* - i\psi_1^*\phi_2, \quad D^3 = \phi_1\psi_1^* - \phi_2\psi_2^*, \tag{11}$$

then D^p is a vector. Let R_{pq} and S_{pq} be the self-dual six-vectors corresponding to the spinors (ϕ_1, ϕ_2) and (ψ_1, ψ_2) respectively; the question now arises: what tensor-operations, performed on the tensors R_{pq} and S_{pq}, will yield this vector D^p? The answer is remarkable. Let us form the tensor

$$V^{pq} = \sum_k R^{p}_{\cdot k} S^{*qk}, \tag{12}$$

where S^* denotes the complex-conjugate of S. Then we find that V^{pq} is a particular kind of tensor of rank two, namely, it is the outer product of a vector with itself; and, in fact,

$$V^{pq} = -2D^pD^q, \tag{13}$$

† It will be remembered that at present we are considering only behaviour with respect to Lorentz transformations.

where D^p is the vector defined in (11). Thus when $\sum_k R^{p\cdot}_{\cdot k} S^{*qk}$ has been formed, the vector D^p can be determined. But we observe that *this involves passing from the tensor* D^pD^q *to the vector* D^p, an operation which resembles extracting a square root in algebra. Thus while in the spinor-calculus the vector D^p is formed from the spinors (ϕ_1, ϕ_2) and (ψ_1, ψ_2) by simple direct operations, in the tensor-calculus D^p can be formed from the six-vectors R^{pq} and S^{pq} only by performing operations one of which is an inverse operation. This is characteristic of the relation between the tensor-calculus and the spinor-calculus: *the spinor-calculus enables us to construct by direct operations* (*addition, multiplication, contraction*) *new tensors which in the tensor-calculus can be constructed only by solving tensor equations.* We are reminded of the way in which the square roots that occur in trigonometry $(\cos\theta = \sqrt{(1-\sin^2\theta)})$ can be evaded by making use of the substitution $\cos\theta = \dfrac{1-t^2}{1+t^2}$, $\sin\theta = \dfrac{2t}{1+t^2}$.

A still more striking example is the following. In the spinor-calculus it is shown that if (ϕ_1, ϕ_2) is a spinor defined over a domain in space-time, then

$$H_p = \phi_2 \frac{\partial \phi_1}{\partial x^p} - \phi_1 \frac{\partial \phi_2}{\partial x^p} \tag{14}$$

is an ordinary vector. H_p depends only on (ϕ_1, ϕ_2), that is, wholly on the tensor R_{pq}, so we now inquire what operations performed on the tensor R_{pq} will yield this vector H_p. The answer is that if we form the tensor $\sum_k R_{ks}(D^k)_p$, where D^k is the vector already found, and the notation $(\ )_p$ is used to denote covariant differentiation, which in our Galilean space-time is merely ordinary differentiation, then this tensor is the outer product of two vectors, and, in fact,

$$\sum_k R_{ks}(D^k)_p = 2D_sH_p. \tag{15}$$

The vector H_p is thus obtained by tensor-analysis, but only by making use of the vector D^p, which itself was obtained by combining the tensor R_{pq} with the complex-conjugate of another arbitrary tensor S_{pq}.† The vector H_p, however, depends solely on R_{pq} and not at all on S_{pq}; the part played by S_{pq} in the matter resembles that played by a catalytic agent in chemistry; we cannot perform the calculation without using S_{pq}, although S_{pq} disappears altogether from the final result. Here, then, *the spinor-calculus enables us to obtain by direct calculation a vector which*

† It is true that the tensor S_{pq} might be taken to be the same as R_{pq}, but even in this case D_p depends not solely on R_{pq} but on R_{pq} together with its complex-conjugate R^*_{pq}, which is a distinct tensor, so that the argument in the text still stands.

cannot be obtained by tensor-analysis except by the "catalytic" use of a tensor not related to the problem.

The general problem of the construction of new tensors from known tensors has hitherto been studied chiefly in connexion with direct methods of tensor-formation, such as addition, multiplication, and contraction. There is a large field of possible research regarding tensors which can be defined only by the solution of tensor-equations or by the use of "catalytic" tensors. It would seem that in this field the spinor-calculus may be of great assistance.

4—Dirac's Relativistic Equation of the Electron

From what precedes it will be evident that the connexion between tensor-calculus and spinor-calculus may be used in order to solve difficult problems in tensor-calculus by re-stating them as problems in spinor-calculus. Conversely, the connexion may be used in order to solve difficult problems regarding spinors by re-stating them in terms of tensors; this is notably the case when the investigation of a problem in the space-time of special relativity has led to an equation expressed in terms of spinors, and it is desired to find the equation which is the extension of this to the space-time of general relativity; for tensors can be carried over at once from special relativity to general relativity, whereas spinors (at any rate, as defined originally) belong properly to special relativity, and spinor-equations can be transferred to general relativity only with difficulty. These remarks may be illustrated by reference to Dirac's relativistic equation of the electron.

We shall take Dirac's equations (omitting the terms depending on the external electromagnetic field, which complicate the equations without affecting the principles here involved) in the form†

$$\left.\begin{aligned}
\frac{\partial\psi_1}{\partial x^0}+\frac{\partial\psi_2}{\partial x^1}-i\frac{\partial\psi_2}{\partial x^2}+\frac{\partial\psi_1}{\partial x^3}+\frac{mci}{\hbar}\psi_3=0\\
\frac{\partial\psi_2}{\partial x^0}+\frac{\partial\psi_1}{\partial x^1}+i\frac{\partial\psi_1}{\partial x^2}-\frac{\partial\psi_2}{\partial x^3}+\frac{mci}{\hbar}\psi_4=0\\
\frac{\partial\psi_3}{\partial x^0}-\frac{\partial\psi_4}{\partial x^1}+i\frac{\partial\psi_4}{\partial x^2}-\frac{\partial\psi_3}{\partial x^3}+\frac{mci}{\hbar}\psi_1=0\\
\frac{\partial\psi_4}{\partial x^0}-\frac{\partial\psi_3}{\partial x^1}-i\frac{\partial\psi_3}{\partial x^2}+\frac{\partial\psi_4}{\partial x^3}+\frac{mci}{\hbar}\psi_2=0
\end{aligned}\right\} . \quad (16)$$

† This is the form adopted by H. Weyl, "Gruppentheorie und Quantenmechanik" (1928), and by B. C. van der Waerden, 'Nachr. Ges. Wiss Gött.,' p. 100 (1929); the ψ's are connected with Dirac's original ψ's by the relations

$$(\psi_1)_{\text{Dir.}}=\frac{\psi_1+\psi_3}{\sqrt{2}},\quad(\psi_2)_{\text{Dir.}}=\frac{\psi_2+\psi_4}{\sqrt{2}},\quad(\psi_3)_{\text{Dir.}}=\frac{\psi_1-\psi_3}{\sqrt{2}},\quad(\psi_4)_{\text{Dir.}}=\frac{\psi_2-\psi_4}{\sqrt{2}}.$$

With this form of the equations, (ψ_1^*, ψ_2^*) and $(\psi_4, -\psi_3)$ are spinors, that is, are transformed according to equation (5) when the coordinates are subjected to the Lorentz transformation specified by (3) and (6). By § 2, to each of these spinors there corresponds a self-dual six-vector; calling these six-vectors M_{pq}^* and R_{pq} respectively, we have by (10)

$$M_{01}^* = \psi_1^{*2} - \psi_2^{*2}, \quad R_{01} = \psi_4^2 - \psi_3^2, \quad \text{etc.} \tag{17}$$

Having obtained these two six-vectors, we can abandon spinor-analysis and proceed by tensor-analysis exclusively. First we determine four ordinary vectors A^p, B^p, C^p, C^{*p}, by the tensor-equations (like equations (12) and (13))

$$\left.\begin{aligned} \sum_k M_{pk}M^{*qk} &= -2A_pA^q, & \sum_k R_{pk}R^{*qk} &= -2B_pB^q, \\ \sum_k M_{pk}R^{qk} &= -2C_pC^q, & \sum_k M_{pk}^*R^{*qk} &= -2C_p^*C^{*q} \end{aligned}\right\}. \tag{18}$$

These four vectors satisfy the relations

$$\left.\begin{aligned} &\sum_p A_p\, A^p = 0, \quad \sum_p B_p\, B^p = 0, \quad \sum_p C_p\, C^p = 0, \quad \sum_p C_p^*\, C^{*p} = 0, \\ &\sum_p A_p\, C^p = 0, \quad \sum_p A_p\, C^{*p} = 0, \quad \sum_p B_p\, C^p = 0, \quad \sum_p B_p\, C^{*p} = 0 \end{aligned}\right\} \tag{19}$$

i.e., each of these is perpendicular to itself and to two of the other three. The vector $-e(A^p + B^p)$ is the "four-vector of electric charge and current" discovered by Darwin,† which determines the probability of finding the electron in unit volume at any place, and the probability that the electron in unit time passes through unit area perpendicular to a given direction.

Next, we obtain two vectors P_p and Q_p by the process which was used in (15), namely, we write

$$\sum_k M_{ks}(A^k)_p = 2A_s\, P_p, \quad \sum_k R_{ks}^*(B^k)_p = 2B_s\, Q_p. \tag{20}$$

The divergences of the six-vectors M_{pq} and R_{pq}^* are, of course, also vectors; let us denote them by S_p and T_p, so

$$S_p = \sum_q (M_{p\cdot}{}^{\cdot q})_q, \qquad T_p = \sum_q (R_{p\cdot}{}^{*\cdot q})_q. \tag{21}$$

From these vectors let us form the vector

$$\Omega_p = \tfrac{1}{2}S_p + \tfrac{1}{2}T_p + P_p + Q_p + \frac{mci}{\hbar}(C_p + C_p^*). \tag{22}$$

† 'Proc. Roy. Soc.,' A, vol. 118, p. 654 (1928).

The actual calculation can be carried out easily by working with spinors; we find at once

$$\tfrac{1}{2}S_0 = \psi_1\left(-\frac{\partial\psi_1}{\partial x^1} - i\frac{\partial\psi_1}{\partial x^2} + \frac{\partial\psi_2}{\partial x^3}\right) + \psi_2\left(\frac{\partial\psi_2}{\partial x^1} - i\frac{\partial\psi_2}{\partial x^2} + \frac{\partial\psi_1}{\partial x^3}\right)$$

$$\tfrac{1}{2}T_0 = \psi_3^*\left(\frac{\partial\psi_3^*}{\partial x^1} - i\frac{\partial\psi_3^*}{\partial x^2} - \frac{\partial\psi_4^*}{\partial x^3}\right) + \psi_4^*\left(-\frac{\partial\psi_4^*}{\partial x^1} - i\frac{\partial\psi_4^*}{\partial x^2} - \frac{\partial\psi_3^*}{\partial x^3}\right)$$

$$P_0 = \psi_2\frac{\partial\psi_1}{\partial x^0} - \psi_1\frac{\partial\psi_2}{\partial x^0}, \qquad Q_0 = \psi_4^*\frac{\partial\psi_3^*}{\partial x^0} - \psi_3^*\frac{\partial\psi_4^*}{\partial x^0}$$

$$C_0 = -\psi_1\psi_4 + \psi_2\psi_3, \qquad C_0^* = -\psi_1^*\psi_4^* + \psi_2^*\psi_3^*,$$

and hence

$$\begin{aligned}\Omega_0 = &-\psi_1\left(\frac{\partial\psi_1}{\partial x^1} + i\frac{\partial\psi_1}{\partial x^2} - \frac{\partial\psi_2}{\partial x^3} + \frac{\partial\psi_2}{\partial x^0} + \frac{mci}{\hbar}\psi_4\right)\\ &+ \psi_2\left(\frac{\partial\psi_2}{\partial x^1} - i\frac{\partial\psi_2}{\partial x^2} + \frac{\partial\psi_1}{\partial x^3} + \frac{\partial\psi_1}{\partial x^0} + \frac{mci}{\hbar}\psi_3\right)\\ &-\psi_3^*\left(-\frac{\partial\psi_3^*}{\partial x^1} + i\frac{\partial\psi_3^*}{\partial x^2} + \frac{\partial\psi_4^*}{\partial x^3} + \frac{\partial\psi_4^*}{\partial x^0} - \frac{m}{\hbar}\psi_2^*\right)\\ &+\psi_4^*\left(-\frac{\partial\psi_4^*}{\partial x^1} - i\frac{\partial\psi_4^*}{\partial x^2} - \frac{\partial\psi_3^*}{\partial x^3} + \frac{\partial\psi_3^*}{\partial x^0} - \frac{mci}{\hbar}\psi_1^*\right).\end{aligned}$$

But the right-hand side vanishes in consequence of Dirac's equations (namely the first two of those given in (16) and the complex-conjugates of the other two); and similarly the components Ω_1, Ω_2, Ω_3 all vanish in consequence of Dirac's equations. Thus the vector Ω_p vanishes in consequence of Dirac's equations. Conversely, the vanishing of the four components Ω_0, Ω_1, Ω_2, Ω_3, necessitates Dirac's equations, provided the determinant of the coefficients of the Dirac-expressions in the four equations $\Omega_p = 0$ does not vanish; but this determinant is

$$\begin{vmatrix} -\psi_1 & \psi_2 & \psi_3^* & \psi_4^* \\ \psi_1 & -\psi_2 & \psi_3^* & -\psi_4^* \\ i\psi_1 & i\psi_2 & -i\psi_3^* & -i\psi_4^* \\ -\psi_2 & -\psi_1 & -\psi_4^* & -\psi_3^* \end{vmatrix},$$

which has the value $-i(\psi_1\psi_3^* + \psi_2\psi_4^*)^2$, and therefore cannot vanish unless $\psi_1\psi_3^* + \psi_2\psi_4^*$ vanishes, which physically is exceptional, if not

impossible. Thus finally *Dirac's equations are equivalent to the statement that the vector* Ω_p *is zero*

$$\Omega_p = 0 \qquad (p = 0, 1, 2, 3). \tag{23}$$

Needless to say, the extension of the theory to the space-time of general relativity is much more straightforward when the equations are expressed in this tensorial form than when they are expressed in terms of spinors.

The Auger Effect in Xenon and Krypton

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[Plates 4–8]

Introduction

In an earlier paper* an account was given of a determination of the efficiency of emission of K series radiations from argon atoms ionized in the K shell. This efficiency, usually termed the K yield, was calculated from a statistical count of ordinary K photoelectron tracks and Auger pair tracks observed in a Wilson expansion chamber. These experiments have been extended to xenon and krypton, and the present paper describes the measurement of the K yields of these two gases.

Determinations of the K yields for elements of high atomic number with the cloud expansion chamber should be of especial value, as other methods† which depend on the measurement of X-ray energy by ionization currents encounter considerable difficulties owing to the increasing importance of scattering with short wave-length X-rays. The need for new measurements for the heavier elements has been emphasized recently by the striking divergence between the results of two extensive series of measurements, one due to Berkey‡ and the other to Arends,§ both of

* Martin, Bower, and Laby, ' Proc. Roy. Soc.,' A, vol. 148, p. 40 (1935).

† Compton and Allison, " X-Rays in Theory and Experiment," p. 477.

‡ ' Phys. Rev.,' vol. 45, p. 437 (1934).

§ ' Ann. Physik,' B, vol. 22, p. 281 (1935).