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The photodisintegration of the deuteron in the meson theory

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1. Introduction

Bethe and Peierls (1935) have given a theory of the photoelectric effect of the deuteron which was based on the assumption that between a neutron and a proton forces of a very short range exist. This made it possible to calculate the cross-section and the angular distribution in a way which seemed to a great extent to be independent of the nature of the nuclear forces. Massey and Mohr (1935) have extended this theory. Later on Breit and Condon (1936) have taken into account the influence of exchange forces of the Majorana type.

In recent years a theory of nuclear forces has been developed, the fundamental ideas of which are due to Yukawa (1935). In this theory, the existence of free mesons (cosmic rays) is connected with the nuclear forces in a similar way as the electromagnetic forces between electrically charged

particles are correlated with the existence of light quanta (cf. Yukawa, Sakata and Taketani 1938; Bhabha 1938; Fröhlich, Heitler and Kemmer 1938).

According to this meson theory a proton (neutron) can be regarded as surrounded by a *meson field* which, owing to the exchange nature of the nuclear forces, is *electrically charged*, the charge being positive (negative) for the proton (neutron).

The charged nature of the nuclear field has the following important consequence: In treating the interaction of a nucleus with a light quantum one has to take into account not only the direct interaction of the light quantum with the protons but also its *interaction with the charged nuclear field*. The latter consists of two parts: (i) the spin independent part (g-interaction) and (ii) the spin dependent part (f-interaction). The first part gives rise to ordinary exchange forces whose influence on the photoelectric effect has in principle been considered by Breit and Condon. The second part is characteristic for the meson theory in its present form and leads, as we shall see in this paper, to results differing essentially from those of previous theories.

One can thus divide the interaction of a light quantum with a deuteron into two parts: (i) the part in which the light quantum acts directly on the proton (Bethe-Peierls), (ii) the part in which the light quantum interacts with the meson field. It will be shown in this paper that for not too high energies both are of the same order of magnitude. The dependence on the frequency ν is, however, quite different for the two parts. The contribution to the cross-section of (i) decreases, for high energies, like $1/\nu^{\frac{3}{2}}$, whereas the contribution of (ii) decreases only like $1/\nu^{\frac{1}{2}}$. For high energies the new effect is therefore preponderant. For the 17 MeV γ-rays of Li for instance the cross-section turns out to be seven times larger in the new theory than in the old one. Also the angular distribution will be seen to be very different in the new theory.

These effects are characteristic for the spin dependent exchange forces in the present form of the meson theory. We want to emphasize that the effect is mainly due to not too fast mesons and that none of the difficulties of the meson theory connected with fast mesons plays any role in these calculations. The results should have the same degree of reliability as those for the nuclear forces.

It should be possible to check the effect experimentally. We believe that such experiments on the photoelectric effect of the deuteron with hard γ-rays should be considered as a crucial test for or against the present form of the meson theory.

2. Calculation of the cross-section

Our aim is to calculate the cross-section for the photo-disintegration of the deuteron. We shall do this, using Born's approximation. This is justified because it will turn out that the wave function of the ejected proton is not an S-wave function and therefore very little influenced by the proton-neutron interaction (this is not true for the photomagnetic effect which we shall not consider in this paper, cf. § 3).

(i) *The wave functions of the deuteron*

The wave function of a heavy particle depends on the following co-ordinates: The space co-ordinates, the spin co-ordinates and the co-ordinates describing whether the heavy particle is a proton or a neutron. The ground state of the deuteron has an angular momentum 1. It was usually assumed that it is a ^{3}S-state. From the proton-neutron interaction derived from the meson theory it follows, however, that there is a large spin-orbit coupling, and that therefore the ground state is a mixture of ^{3}S and $^3D_1$ (cf. Yukawa, Sakata, Kobayasi and Taketani 1938; Fröhlich *et al.** 1938). The presence of a $^3D_1$ admixture to the wave function finds its expression in the existence of a quadripole moment of the deuteron, discovered recently by Kellogg, Rabi, Ramsey and Zacharias (1939).

It is, however, likely that the contribution of the $^3D_1$ wave function is not very large, owing to the centrifugal forces. In this paper we are mainly interested in the effects arising from the meson character of the nuclear field. These effects are clearly shown even if we assume a ^{3}S-wave function for the ground state of the deuteron as we shall do in this paper for reasons of simplicity.

The wave function for the ground state of the deuteron can thus be written in the form

$$\Psi = \psi(1,2)\, T(1,2)\, U(1,2), \tag{1}$$

where ψ depends on the space co-ordinates $\mathbf{r}_1$, $\mathbf{r}_2$, T on the spin variables, and U on the charge variables of the two particles. For the ^{3}S-state these wave functions are

$$\left.\begin{aligned} & \alpha(1)\,\alpha(2) \\ T &= \frac{1}{\sqrt{2}}\{\alpha(1)\,\beta(2)+\alpha(2)\,\beta(1)\}, \\ & \beta(1)\,\beta(2) \\ U &= \frac{1}{\sqrt{2}}\{\xi(1)\,\eta(2)-\xi(2)\,\eta(1)\}, \end{aligned}\right\} \tag{2}$$

* In the following quoted as F.H.K.

where α, β are the two spin functions and $\xi(\eta)$ is equal to 1, (0) when the particle is a proton and equal to 0, (1) when it is a neutron. For ψ we shall assume the wave function used by Bethe and Peierls (1935):

$$\psi = \sqrt{\left(\frac{\alpha}{2\pi}\right)}\frac{e^{-\alpha r}}{r}, \quad r = |\mathbf{r}_1 - \mathbf{r}_2|, \tag{3}$$

where α is connected with the binding energy E_0 of the deuteron by

$$E_0 = \frac{\hbar^2\alpha^2}{M}. \tag{4}$$

M is the mass of the proton. The wave function (3) corresponds to a neutron-proton potential of the form of a δ-function. For the final state into which the deuteron disintegrates we assume plane waves according to the use of Born's approximation. For parallel spins this wave function is

$$\Psi' = e^{i(\mathbf{K}_1\mathbf{r}_1)+i(\mathbf{K}_2\mathbf{r}_2)}\,\xi(1)\,\eta(2)\begin{Bmatrix} \alpha(1)\,\alpha(2) \\ \frac{1}{\sqrt{2}}\{\alpha(1)\,\beta(2)+\alpha(2)\,\beta(1)\} \\ \beta(1)\,\beta(2) \end{Bmatrix}; \tag{5}$$

$\mathbf{K}_1$ is the wave number of the ejected proton, $\mathbf{K}_2$ of the neutron. Transitions into states with antiparallel spins are only due to the photomagnetic effect, which we do not consider here.

(ii) *Interaction energy*

The transition from the ground state to the final state are due to the interaction of the deuteron with the light quantum. In the theory of Bethe and Peierls (1935) the proton was considered as the only particle carrying a charge. In the meson theory we have in addition an interaction of the light quantum with the *charged* meson field.

The Hamiltonian of a meson field in the presence of both heavy particles and an electromagnetic field has been given in its complete form by Bhabha (1938). We write down the interaction part of this Hamiltonian in the notation of F.H.K. (1938), making use of the longitudinal and transverse meson wave functions $\boldsymbol{\psi}$ and $\boldsymbol{\phi}$. $\mathbf{A}$ is the electromagnetic vector potential and V the scalar potential.

The Hamiltonian consists of three parts

$$H = H' + H'' + H''', \tag{6}$$

where H' represents the interaction between the electromagnetic field and the meson field, H'' that between the meson field and each of the heavy

particles, H''' is a "mixed interaction" containing the electromagnetic field, each of the heavy particles and the meson field.† The three contributions are (for one heavy particle):

$$H' = \frac{ie}{4\pi\hbar c}\left\{\operatorname{div}\boldsymbol{\psi}\left(\mathbf{A},\boldsymbol{\psi}^*+\frac{1}{\lambda}\dot{\boldsymbol{\phi}}^*\right)+\left(\operatorname{curl}\boldsymbol{\phi}\left[\mathbf{A},\boldsymbol{\phi}^*-\frac{1}{\lambda}\dot{\boldsymbol{\psi}}^*\right]\right)\right.$$
$$\left.+V(\dot{\boldsymbol{\phi}}\boldsymbol{\phi}^*)+V(\dot{\boldsymbol{\psi}}\boldsymbol{\psi}^*)\right\}$$
$$+\frac{e^2}{4\pi\hbar^2c^2}\left\{|(\mathbf{A}\boldsymbol{\psi})|^2+\frac{1}{\lambda^2}|[\mathbf{A}\dot{\boldsymbol{\psi}}]|^2+\frac{1}{\lambda^2}|(\mathbf{A}\dot{\boldsymbol{\phi}})|^2+|[\mathbf{A}\boldsymbol{\phi}]|^2\right.$$
$$\left.+\frac{1}{\lambda}(\mathbf{A}\boldsymbol{\psi})(\mathbf{A}\dot{\boldsymbol{\phi}}^*)+\frac{1}{\lambda}(\mathbf{A}\boldsymbol{\phi})(\mathbf{A}\dot{\boldsymbol{\psi}}^*)\right\}, \tag{7}$$

$$H'' = \frac{g}{\lambda}M\operatorname{div}\boldsymbol{\psi}+\frac{f}{\lambda}(\mathbf{S}\operatorname{curl}\boldsymbol{\phi})+\frac{4\pi g^2}{\lambda^2}|M|^2, \tag{8}$$

$$H''' = -\frac{ieg}{\hbar c\lambda}M\left(\mathbf{A},\boldsymbol{\psi}+\frac{1}{\lambda}\dot{\boldsymbol{\phi}}\right)-\frac{ief}{\hbar c\lambda}\left(\mathbf{S}\left[\mathbf{A},\boldsymbol{\phi}-\frac{1}{\lambda}\dot{\boldsymbol{\psi}}\right]\right). \tag{9}$$

In these expressions it is to be understood that the complex conjugate terms have to be added to all those terms which are not automatically real. The definition of $\dot{\psi}$ is $\psi = 1/c\,\partial\psi/\partial t$. M and $\mathbf{S}$ depend on the wave functions of the heavy particles Ψ and the charge operator Π which is defined as follows:

$$\Pi\xi = \Pi^*\eta = 0, \quad \Pi\eta = \xi, \quad \Pi^*\xi = \eta, \tag{10}$$

$$M = (\Psi^*\Pi\Psi), \quad \mathbf{S} = (\Psi^*\Pi\boldsymbol{\sigma}\Psi). \tag{11}$$

$\boldsymbol{\sigma}$ is the spin vector of the heavy particle. In the Hamiltonian (7)–(9) all terms have been omitted which depend on the velocity of the heavy particles.

We expand the wave functions of the meson $\boldsymbol{\psi}$ and $\boldsymbol{\phi}$ into plane waves

$$\boldsymbol{\psi} = \sum_k Q_k\frac{\mathbf{k}}{k}e^{i(\mathbf{kr})}\sqrt{(4\pi c^2)},$$

$$\boldsymbol{\phi} = \sum_k q_k\mathbf{j}\,e^{i(\mathbf{kr})}\sqrt{(4\pi c^2)}.$$

q_k and Q_k are quantized according to the Pauli-Weisskopf formalism. $\mathbf{j}$ is the unit vector of polarization for transverse mesons. It has to be noticed

† We are indebted to Dr Bhabha for having drawn our attention to the existence of this term.

that the wave number $\mathbf{k}$ is connected with the momentum $\mathbf{p}$ of the free meson in a different way for positive and negative mesons:

$$\text{for } Y^+\text{:} \quad \mathbf{p} = \hbar c\mathbf{k},$$
$$\text{for } Y^-\text{:} \quad \mathbf{p} = -\hbar c\mathbf{k}.$$

This follows immediately from the Pauli-Weisskopf formalism.

(iii) *Matrix elements*

The interaction H gives rise to a number of transitions for which we give the matrix elements. The following transitions are due to H':

(1) A longitudinal meson with wave number $\mathbf{k}$ absorbs a light quantum with wave number $\mathbf{q}$ ($\hbar\nu = \hbar cq$) and has then a wave number $\mathbf{k}'$. The law of conservation of momentum is then

$$\text{for } Y^+\text{:} \quad \mathbf{k}' = \mathbf{k}+\mathbf{q}, \qquad \text{for } Y^-\text{:} \quad -\mathbf{k}' = -\mathbf{k}+\mathbf{q}. \tag{12}$$

(2) The analogous transition for a transverse meson. The transitions in which a longitudinal meson is transformed into a transverse one through the absorption of a light quantum give no contribution to the photoelectric effect.

(3) The creation or annihilation of a pair of longitudinal mesons with absorption of a light quantum. The momentum law is for the two cases

$$\mathbf{q} = \mathbf{k}_+ - \mathbf{k}_-, \quad \mathbf{q}+\mathbf{k}_+ - \mathbf{k}_- = 0. \tag{13}$$

(4) The same for transverse mesons. The matrix elements of H' for these four transitions are (ϵ, ϵ' are the energies corresponding to $\mathbf{k}$, $\mathbf{k}'$; $\epsilon = \hbar c\sqrt{(k^2+\lambda^2)}$; $\mathbf{e}$ is the unit vector of polarization of the light quantum):

$$\left.\begin{aligned}
H_1' &= -\frac{e\hbar^2c^2}{2}\sqrt{\left(\frac{2\pi}{\epsilon\epsilon'\hbar\nu}\right)}\left\{\frac{k}{k'}(\mathbf{k}'\mathbf{e})+\frac{k'}{k}(\mathbf{k}\mathbf{e})\right\},\\
H_2' &= -\frac{e\hbar^2c^2}{2}\sqrt{\left(\frac{2\pi}{\epsilon\epsilon'\hbar\nu}\right)}\{([\mathbf{j}\mathbf{k}]\,[\mathbf{j}'\mathbf{e}])+([\mathbf{j}'\mathbf{k}']\,[\mathbf{j}\mathbf{e}])\},\\
H_3' &= +\frac{e\hbar^2c^2}{2}\sqrt{\left(\frac{2\pi}{\epsilon_+\epsilon_-\hbar\nu}\right)}\left\{\frac{k_-}{k_+}(\mathbf{k}_+\mathbf{e})+\frac{k_+}{k_-}(\mathbf{k}_-\mathbf{e})\right\},\\
H_4' &= +\frac{e\hbar^2c^2}{2}\sqrt{\left(\frac{2\pi}{\epsilon_+\epsilon_-\hbar\nu}\right)}\{([\mathbf{j}_-\mathbf{k}_-]\,[\mathbf{j}_+\mathbf{e}])+([\mathbf{j}_+\mathbf{k}_+]\,[\mathbf{j}_-\mathbf{e}])\}.
\end{aligned}\right\} \tag{14}$$

H_1' and H_2' are valid both for positive and negative mesons if the momentum law (12) is taken into account. H_3' and H_4' are valid for creation and annihilation.

H'' has matrix elements for emission and absorption of mesons by a heavy particle. They are given in F.H.K. (1938), equation (32).

H''' gives rise to the following transitions:

(5) A proton emits a longitudinal Y^+ and absorbs a light quantum.
(6) A neutron emits a longitudinal Y^- and absorbs a light quantum.
(7) A proton absorbs a longitudinal Y^- and a light quantum.
(8) A neutron absorbs a longitudinal Y^+ and a light quantum.
(9)–(12) The same as (5)–(8) for transverse mesons.

These matrix elements are, for the heavy particle at the position $\mathbf{r}_0$,

$$\left.\begin{aligned}
&H_5''' = \Pi^* \frac{2\pi eg\hbar c}{\lambda}\frac{1}{\sqrt{(\epsilon\hbar\nu)}}\left(\mathbf{e}\frac{\mathbf{k}}{k}\right)e^{-i(\mathbf{k}-\mathbf{q},\mathbf{r}_0)},\\
&H_6''' = -\Pi \frac{2\pi eg\hbar c}{\lambda}\frac{1}{\sqrt{(\epsilon\hbar\nu)}}\left(\mathbf{e}\frac{\mathbf{k}}{k}\right)e^{i(\mathbf{k}+\mathbf{q},\mathbf{r}_0)},\\
&H_7''' = -H_5''', \quad H_8''' = -H_6''',\\
&H_9''',\, H_{10}''' \text{ are the same as } H_5''',\, H_6''' \text{ with } g\left(\mathbf{e}\frac{\mathbf{k}}{k}\right) \text{ replaced by } f(\boldsymbol{\sigma}[\mathbf{ej}]),\\
&H_{11}''' = -H_9''', \quad H_{12}''' = -H_{10}'''.
\end{aligned}\right\} \tag{15}$$

In H_5''' we have omitted the contribution arising from the term $+\frac{ief}{\hbar c\lambda^2}(\mathbf{S}[\mathbf{A}\dot{\boldsymbol{\psi}}])$ of H''' (and similar terms in H_6''', etc.), because these terms do not contribute to the photoelectric effect.

(iv) *Total matrix element*

Let V_{AF} be the matrix element for the transition of the deuteron from a ground state A into a final state F under the action of a light quantum. The differential cross-section of this process is then

$$d\Phi = \frac{2\pi}{\hbar c}|V_{AF}|^2 \rho\, d\Omega, \tag{16}$$

where $\rho\, d\Omega$ is the density of final energy states with the direction of the proton in the solid angle $d\Omega$. The ground state is three-fold owing to the three orientations of the spin. Disregarding the photomagnetic effect, the final state is also a triplet and transitions occur from each of the three initial states to each of the three final states. In (16) the sum has to be taken over the three final states and the average over the three initial states.

The matrix element V_{AF} consists for each of these nine transitions of three parts:

$$V_{AF} = V_a + V_b + V_c \tag{17}$$

with the following significance:

(*a*) V_a is due to a combination of the interactions H' and H'' and is a third-order matrix element. The transition takes place by means of *two* subsequent intermediate states. There exist four possible sequences of those intermediate states:

(α) (i) Emission of Y^+ by proton; (ii) absorption of light quantum by Y^+; (iii) reabsorption of Y^+ by one of the two neutrons.

(β) The same for Y^-, emitted by the neutron.

(γ) (i) Creation of a pair of mesons by the light quantum; (ii) absorption of Y^+ by neutron; (iii) absorption of Y^- by one of the protons. (ii) and (iii) can occur in the reverse order.

(δ) (i) Emission of Y^+ by proton; (ii) emission of Y^- by one of the neutrons; (iii) annihilation of the pair with absorption of the light quantum. (i) and (ii) can occur in the reverse order.

All mesons can be longitudinal or transverse. In the approximations used in our calculations (see below) all four contributions are identical and we shall only give the calculations for (α).

Let us denote by E_n and $E_{n'}$ the energies of the two successive intermediate states (i) and (ii), and E_A, E_F the energies in the initial and final states. Then V_a is given by

$$V_a = 4 \sum_{n,n'} \frac{H_{An} H_{nn'} H_{n'F}}{(E_A - E_n)(E_A - E_{n'})}, \tag{18}$$

where H_{An}, etc. are the matrix elements of $H' + H''$ for the transitions (α). For H_{An} and $H_{n'F}$ only H'' and for $H_{nn'}$ only H' gives a contribution. In the intermediate states n, n' the heavy particles are two free neutrons and have wave numbers $\mathbf{K}_1'$, $\mathbf{K}_2'$, whilst the meson has a wave number $\mathbf{k}$ in n and $\mathbf{k}'$ in n'. The conservation of momentum shows that

$$\mathbf{K}_1' + \mathbf{K}_2' + \mathbf{k} = 0, \quad \mathbf{k}' = \mathbf{k} + \mathbf{q}. \tag{19}$$

In the final state with the wave function (5) the momentum law gives

$$\mathbf{K}_1 = \mathbf{K}_1' + \mathbf{k}', \quad \mathbf{K}_2 = \mathbf{K}_2'. \tag{20}$$

In each of the intermediate states n, n' the spin functions can be either of the four functions $\alpha(1)\alpha(2)$, $\alpha(1)\beta(2)$, $\alpha(2)\beta(1)$, $\beta(1)\beta(2)$. The emitted meson can be longitudinal or transverse; in the latter case one has to take the sum over the two directions of polarization in the intermediate states.

As an example we calculate V_a for the transition $\alpha(1)\,\alpha(2)_A \rightarrow \alpha(1)\,\alpha(2)_F$ for a longitudinal meson. In this case the interaction is independent of the spin and the intermediate states have the same spin functions. According to F.H.K. (equation (32)) we have

$$H_{An} = -\frac{g}{\lambda}\sqrt{\left(\frac{2\pi}{\epsilon}\right)}\hbar ck \iint d\tau_1\, d\tau_2\, e^{-i(\mathbf{K}_1'\mathbf{r})-i(\mathbf{K}_2'\mathbf{r})}\, \eta(1)\,\eta(2)$$

$$\times (\Pi_1^*\, e^{-i(\mathbf{k}\mathbf{r}_1)} + \Pi_2^*\, e^{-i(\mathbf{k}\mathbf{r}_2)})\, \frac{\xi(1)\,\eta(2) - \xi(2)\,\eta(1)}{\sqrt{2}}\, \psi(\mathbf{r}_1 - \mathbf{r}_2)$$

$$= \frac{2ig}{\lambda}\sqrt{\frac{\pi}{\epsilon}}\hbar ck \int d\tau\, e^{-\frac{1}{2}i(\mathbf{r},\, \mathbf{K}_1' - \mathbf{K}_2')} \sin\tfrac{1}{2}(\mathbf{k}\mathbf{r})\, \psi(r), \tag{21}$$

where ψ is the wave function of the deuteron (3). In the second equation (21) we have separated the motion of the centre of gravity and made use of the conservation of momentum.

$H_{nn'} = H_1'$ is given by equation (14). $H_{n'F}$ is again given by F.H.K. (equation (32)):

$$H_{n'F} = -\frac{g}{\lambda}\sqrt{\left(\frac{2\pi}{\epsilon'}\right)}\hbar ck'. \tag{22}$$

The resonance denominators are

$$\left.\begin{aligned} E_A - E_n &= -E_0 - \hbar c\sqrt{(k^2+\lambda^2)} - \frac{\hbar^2}{2M}(K_1'^2 + K_2'^2), \\ E_A - E_{n'} &= -E_0 + \hbar\nu - \hbar c\sqrt{(k'^2+\lambda^2)} - \frac{\hbar^2}{2M}(K_1'^2 + K_2'^2). \end{aligned}\right\} \tag{23}$$

The double summation over n, n' in (18), viz. over $\mathbf{k}$, $\mathbf{k}'$, can be reduced to a simple integral over $\mathbf{k}$ by means of the conservation laws (19) and (20). The contribution to V_a from the transition in question is then

$$V_a = \frac{ig^2 e\hbar^2 c^2}{\pi^{\frac{3}{2}}\lambda^2\sqrt{(\hbar\nu)}} \int d\tau \int d\mathbf{k}\, \frac{kk'}{\sqrt{(k^2+\lambda^2)}\sqrt{(k'^2+\lambda^2)}} \left\{\frac{k}{k'}(\mathbf{k}'\mathbf{e}) + \frac{k'}{k}(\mathbf{k}\mathbf{e})\right\}$$

$$\times \frac{e^{\frac{1}{2}i(\mathbf{r},\, \mathbf{k}+2\mathbf{K}_2)} \sin\tfrac{1}{2}(\mathbf{k}\mathbf{r})\, \psi(r)}{\left(E_0 + \epsilon + \frac{\hbar^2}{2M}(K_1'^2 + K_2'^2)\right)\left(E_0 - \hbar\nu + \epsilon' + \frac{\hbar^2}{2M}(K_1'^2 + K_2'^2)\right)}. \tag{24}$$

In this formula it is to be understood that all wave numbers $\mathbf{k}'$, $\mathbf{K}_1'$, $\mathbf{K}_2'$ are to be expressed by means of (19), (20), by $\mathbf{k}$, $\mathbf{q}$, $\mathbf{K}_1$, $\mathbf{K}_2$.

Before we evaluate this integral we calculate the contributions V_b and V_c to the matrix element.

(*b*) V_b is due to a combination of H'' and H''' and is a *second*-order matrix element. The transition takes place by means of *one* intermediate state. There are again four sequences of those intermediate states:

(α) (i) Emission of Y^+ by proton and absorption of light quantum; (ii) absorption of Y^+.

(β) The same for Y^-.

(γ) (i) Emission of Y^+ by proton; (ii) absorption of Y^+ with absorption of light quantum.

(δ) The same for Y^-.

Again all four sequences give the same contribution resulting in a factor 4. V_b is of the form

$$V_b = 4 \sum_m \frac{H_{Am} H_{mF}}{E_A - E_m}. \tag{25}$$

For (α) H_{Am} is due to H''' and H_{mF} to H''.

We evaluate V_b for the same transition $\alpha(1)\,\alpha(2)_A \rightarrow \alpha(1)\,\alpha(2)_F$ and for a longitudinal meson. We easily find, according to (15) and F.H.K. (equation (32))

$$H_{Am} = -\frac{2\pi i c g \hbar c}{\lambda} \sqrt{\left(\frac{2}{\epsilon \hbar \nu}\right)} \left(\mathbf{e}\frac{\mathbf{k}}{k}\right) \int d\tau\, e^{-\frac{1}{2}i(\mathbf{K}_1'-\mathbf{K}_2', \mathbf{r})} \sin \tfrac{1}{2}(\mathbf{k}-\mathbf{q}, \mathbf{r})\, \psi(r), \tag{26}$$

with $\mathbf{K}_1'$ and $\mathbf{K}_2'$ given by $\qquad \mathbf{K}_1' + \mathbf{K}_2' + \mathbf{k} = \mathbf{q}$, $\qquad$ (27)

$$H_{mF} = -\frac{g}{\lambda}\sqrt{\left(\frac{2\pi}{\epsilon}\right)}\, \hbar c k, \quad \mathbf{K}_1 = \mathbf{K}_1' + \mathbf{k}, \quad \mathbf{K}_2 = \mathbf{K}_2'. \tag{28}$$

The resonance denominator is

$$E_A - E_m = -E_0 + \hbar\nu - \epsilon - \frac{\hbar^2}{2M}(K_1'^2 + K_2'^2). \tag{29}$$

Hence V_b becomes

$$V_b = \frac{2ieg^2\hbar^2c^2}{\sqrt{(\hbar\nu)}\,\lambda^2\pi^{\frac{3}{2}}} \int d\tau \int d\mathbf{k}(\mathbf{e}\mathbf{k}) \frac{e^{\frac{1}{2}i(\mathbf{k}-\mathbf{q}+2\mathbf{K}_2, \mathbf{r})} \sin\tfrac{1}{2}(\mathbf{k}-\mathbf{q}, \mathbf{r})\, \psi(r)}{\epsilon\left(\hbar\nu - E_0 - \epsilon - \frac{h^2}{2M}(K_1'^2 + K_2'^2)\right)}. \tag{30}$$

Here again $\mathbf{K}_1'$, $\mathbf{K}_2'$ are expressed by $\mathbf{k}$, $\mathbf{q}$ and $\mathbf{k}_1$, $\mathbf{k}_2$.

(*c*) V_c is due to the *direct interaction* of the proton with the light quantum. There is no intermediate state.

In this transition the spin function does not change. V_c is the same for all these three transitions and is given by the well-known formula

$$V_c = \frac{e\hbar^2}{M}\sqrt{\left(\frac{\pi}{\hbar\nu}\right)}\,(\mathbf{K}_2\mathbf{e}) \int d\tau\, e^{-i(\mathbf{K}_2\mathbf{r})}\, \psi(r). \tag{31}$$

(v) *Approximations*

We shall now confine ourselves to energies

$$\hbar\nu \ll \mu c^2 = 80 \times 10^6 \text{ e-volts}. \tag{32}$$

(μ = rest mass of meson.)

In this case we may neglect throughout the recoil of the meson due to the absorption of the light quantum and therefore replace $\mathbf{k}'$ by $\mathbf{k}$. This is justifiable because the main contributions to our integrals arise from values $k \simeq \lambda$. Furthermore, we make use of the conservation of energy in the final state

$$\frac{\hbar^2}{2M}(K_1^2 + K_2^2) = \hbar\nu - E_0 \ll \mu c^2. \tag{33}$$

The term $\dfrac{\hbar^2}{2M}(K_1'^2 + K_2'^2)$ in the denominators of (24) and (30) can then be neglected since $\mathbf{K}_2' = \mathbf{K}_2$ and $\mathbf{K}_1' = \mathbf{K}_1 + \mathbf{k}$ which gives the order of magnitude $\hbar^2 k^2/2M \simeq \hbar^2\lambda^2/2M \simeq \mu c^2 . \mu/2M$. In the resonance denominators of (24) and (30) we can therefore neglect everything except $\epsilon \simeq \epsilon'$.

We want to mention that in making these approximations the transitions to a final *singlet* state all vanish. In these approximations we do not get therefore any photomagnetic effect. For the photoelectric effect these approximations are obviously unimportant for the energies considered.

V_a and V_b (equations (24) and (30)) are then given by

$$V_a = \frac{2ieg^2}{\pi^{\frac{3}{2}}\lambda^2\sqrt{(\hbar\nu)}}\int d\tau \int d\mathbf{k}\, \frac{k^2(\mathbf{ke})}{(k^2+\lambda^2)^2}\, e^{\frac{1}{2}i(\mathbf{k}-2\mathbf{K}_1,\mathbf{r})} \sin \tfrac{1}{2}(\mathbf{kr})\, \psi(r),$$

$$V_b = -\frac{2ieg^2}{\pi^{\frac{3}{2}}\lambda^2\sqrt{(\hbar\nu)}}\int d\tau \int d\mathbf{k}\, \frac{(\mathbf{ek})}{k^2+\lambda^2}\, e^{\frac{1}{2}i(\mathbf{k}-2\mathbf{K}_1,\mathbf{r})} \sin \tfrac{1}{2}(\mathbf{kr})\, \psi(r).$$

Inserting for $\psi(r)$ equation (3), the integration is straightforward and we obtain

$$V_a = +\frac{2^{\frac{3}{2}}g^2e\sqrt{\alpha}\,\pi(\mathbf{eK}_1)}{\lambda^2\sqrt{(\hbar\nu)}\,K_1} \times\left[\frac{\lambda^2}{K_1}\frac{\alpha+\lambda}{(\alpha+\lambda)^2+K_1^2} + \frac{\lambda-\alpha}{K_1} + \left(1+\frac{\alpha^2-2\lambda^2}{K_1^2}\right)\operatorname{arctg}\frac{K_1}{\alpha+\lambda}\right], \tag{34}$$

$$V_b = +\frac{2^{\frac{3}{2}}g^2e\sqrt{\alpha}\,\pi(\mathbf{eK}_1)}{\lambda^2\sqrt{(\hbar\nu)}\,K_1}\left[\frac{\alpha-\lambda}{K_1} - \left(1+\frac{\alpha^2-\lambda^2}{K_1^2}\right)\operatorname{arctg}\frac{K_1}{\alpha+\lambda}\right], \tag{35}$$

$$V_c = -\frac{2^{\frac{3}{2}}\pi e\hbar^2\sqrt{\alpha}\,(\mathbf{eK}_1)}{M\sqrt{(\hbar\nu)}\,(K_1^2+\alpha^2)}. \tag{36}$$

(vi) *Formulae for the cross-section*

(34) and (35) are the contributions from the $\alpha(1)\alpha(2)_A \rightarrow \alpha(1)\alpha(2)_F$ transition and from longitudinal mesons. We give the matrix elements for the other transitions and for transverse mesons without calculation. The matrix elements for transverse mesons depend on the angles between spin and light quantum. We choose the direction of $\mathbf{q}$ as the one to which the spin directions are referred. (α means spin in $+\mathbf{q}$ direction.) Let θ and ϕ be the angles

$$\theta = \sphericalangle(\mathbf{K}_1\mathbf{q}), \quad \phi = \sphericalangle(\mathbf{e};\, \mathbf{K}_1, \mathbf{q}\text{-plane}).$$

Then we have for the total matrix element $V_{AF} = V_a + V_b + V_c$:

$\alpha(1)\alpha(2) \rightarrow \alpha(1)\alpha(2)$ or $\beta(1)\beta(2) \rightarrow \beta(1)\beta(2)$:

$$V = \frac{2^{\frac{3}{2}}\pi e}{\lambda^2}\sqrt{\left(\frac{\alpha}{\hbar\nu}\right)}\cos\phi\{g^2\sin\theta(A+B)+f^2\sin\theta(C+B) \\ +f^2\sin^3\theta(A-\tfrac{5}{4}C)-G^2\sin\theta D\};$$

$\alpha(1)\beta(2)+\alpha(2)\beta(1) \rightarrow \alpha(1)\beta(2)+\alpha(2)\beta(1)$:

$$V = \frac{2^{\frac{3}{2}}\pi e}{\lambda^2}\sqrt{\left(\frac{\alpha}{\hbar\nu}\right)}\cos\phi\{g^2\sin\theta(A+B)+\tfrac{1}{2}f^2\sin\theta C \\ +f^2\sin\theta\cos^2\theta(2A-\tfrac{5}{2}C)-G^2\sin\theta D\};$$

$\alpha(1)\alpha(2) \rightarrow \beta(1)\beta(2)$ or $\beta(1)\beta(2) \rightarrow \alpha(1)\alpha(2)$:

$$V = -\frac{2^{\frac{3}{2}}\pi e}{\lambda^2}\sqrt{\left(\frac{\alpha}{\hbar\nu}\right)}\left\{f^2e^{i\phi}\sin\theta\left(\frac{C}{2}+B\right)+f^2\sin^3\theta\cos\phi(A-\tfrac{5}{4}C)\right\};$$

$\alpha(1)\alpha(2) \rightarrow \alpha(1)\beta(2)+\alpha(2)\beta(1)$ or $\alpha(1)\beta(2)+\beta(1)\alpha(2) \rightarrow \alpha(1)\alpha(2)$ or $\beta(1)\beta(2) \rightarrow \alpha(1)\beta(2)+\alpha(2)\beta(1)$ or $\alpha(1)\beta(2)+\alpha(2)\beta(1) \rightarrow \beta(1)\beta(2)$:

$$V = -\frac{2^{\frac{3}{2}}\pi e}{\lambda^2}\sqrt{\left(\frac{\alpha}{\hbar\nu}\right)}\left\{f^2e^{i\phi}\cos\theta\left(\frac{C}{2^{\frac{3}{2}}}+\frac{B}{\sqrt{2}}\right)+f^2\sin^2\theta\cos\theta\cos\phi\sqrt{2}(A-\tfrac{5}{4}C)\right\};$$

where

$$A = \frac{\lambda^2}{K_1}\frac{\alpha+\lambda}{(\alpha+\lambda)^2+K_1^2}+\frac{\lambda-\alpha}{K_1}+\left(1+\frac{\alpha^2-2\lambda^2}{K_1^2}\right)\operatorname{arctg}\frac{K_1}{\alpha+\lambda},$$

$$B = \frac{\alpha-\lambda}{K_1}-\left(1+\frac{\alpha^2-\lambda^2}{K_1^2}\right)\operatorname{arctg}\frac{K_1}{\alpha+\lambda},$$

$$C = \frac{\alpha+\lambda}{2K_1}+\frac{3}{2}\frac{(\alpha+\lambda)(\alpha-\lambda)^2}{K_1^3}+\left(\frac{1}{2}-\frac{\alpha^2+\lambda^2}{K_1^2}-\frac{3}{2}\frac{(\alpha^2-\lambda^2)^2}{K_1^4}\right)\operatorname{arctg}\frac{K_1}{\alpha+\lambda},$$

$$D = \frac{K_1\lambda}{K_1^2+\alpha^2},$$

$$G^2 = \frac{\mu\hbar c}{M}. \tag{37}$$

Hence we find the differential cross-section according to (16), by taking the sum of $|V|^2$ over all the nine transitions and dividing by 3. We take the average over the directions of polarization of the light quantum and obtain

$$d\Phi = \frac{2\pi}{3}\sin\theta\, d\theta \frac{e^2\alpha K_1 M}{\hbar^3\lambda^4 c\hbar\nu}\Big\{4f^4\left(\frac{C}{2}+B\right)^2 + \sin^2\theta[3\{g^2(A+B)-G^2D\}^2 - 4G^2f^2D(A+B)+4g^2f^2(A+B)^2 + f^4(C^2+2B^2-6BC+4A^2-4AC+8AB)]\Big\}$$

$$\equiv \sin\theta\, d\theta\{\Phi_{\parallel} + \sin^2\theta(\Phi_{\perp}-\Phi_{\parallel})\}. \tag{38}$$

The total cross-section is

$$\Phi = \frac{2\pi}{3}\frac{e^2\alpha K_1 M}{\hbar^3\lambda^4 c\hbar\nu}[4\{g^2(A+B)-G^2D\}^2 - \tfrac{16}{3}G^2f^2D(A+B) + \tfrac{16}{3}g^2f^2(A+B)^2 + \tfrac{2}{3}f^4(5C^2+8A^2-8AC+16B^2+16AB)]. \tag{39}$$

In these formulae K_1 is the wave number of the ejected proton

$$\frac{\hbar^2K_1^2}{M} = \hbar\nu - E_0.$$

3. Results and discussion

We now evaluate the formulae (38) and (39) numerically. The wave function which we have used for the ground state of the deuteron corresponds to a potential with the form of a δ-function. In order to take into account the finite range of the nuclear forces, Bethe and Bacher (1936) have shown that it is sufficient to renormalize the wave function in such a way that it is normalized to

$$1+\alpha/\lambda \tag{40}$$

instead of to unity. We therefore have to multiply the expressions (38) and (39) by this factor. For the numerical evaluation we assume the following figures:

$$E_0 = 2\cdot2\,\text{MeV},$$

$$\text{Meson mass} = 160\text{ m. or } \lambda = 160\frac{mc}{\hbar} = 1\cdot8\alpha.$$

Hence $\qquad 1+\alpha/\lambda = 1\cdot56, \quad G^2/\hbar c = 0\cdot087.$

For g^2 and f^2 we shall use the figures derived by Kemmer (1938) from the nuclear forces, i.e. $g^2/\hbar c = 0\cdot057$, $f^2/\hbar c = 0\cdot143$. These figures are derived from the hypothesis of charge-independent nuclear forces and may not be

the true values. The order of magnitude, however, is certainly correct. The functions A, B, C, D are independent of the choice of g and f and are given in table 1. In this table we give the values of the total cross-section Φ according to equation (39) (including the factor $1+\alpha/\lambda$). $r_0 = e^2/mc^2$ is the electronic radius. For comparison, we also give the cross-section Φ_{old} according to the old theory of Bethe and Peierls (1935). For the discussion of the angular dependence we give the ratio $\Phi_{\parallel}/\Phi_{\perp}$, i.e. the ratio of the cross-sections for $\theta = 0$ and $\theta = \frac{1}{2}\pi$.

In figure 1, Φ (in units r_0^2) and the ratio $\Phi_{\parallel}/\Phi_{\perp}$ are plotted as a function of the frequency ν together with the cross-section Φ_{old} according to the Bethe-Peierls theory. In the Bethe-Peierls theory $\Phi_{\parallel} = 0$.

Table 1

K_1/α	$\hbar\nu$ (MeV)	A	$-B$	C	D	Φ/r_0^2	Φ_{old}/r_0^2	$\Phi_{\parallel}/\Phi_{\perp}$
0·43	2·61	0·126	0·17	0·10	0·65	0·016	0·011	0·11
0·9	3·98	0·26	0·34	0·20	0·90	0·048	0·028	0·18
1·3	5·8	0·37	0·47	0·28	0·87	0·058	0·025	0·27
2·0	11·0	0·55	0·67	0·40	0·72	0·061	0·014	0·46
2·6	17·1	0·69	0·81	0·47	0·60	0·056	0·0084	0·65
4·0	37·4	0·93	1·03	0·59	0·42	0·048	0·0030	0·86
10*	222·2	1·31	1·35	0·74	0·18	0·030	0·00022	1·0

* This value is beyond the validity of our approximations.

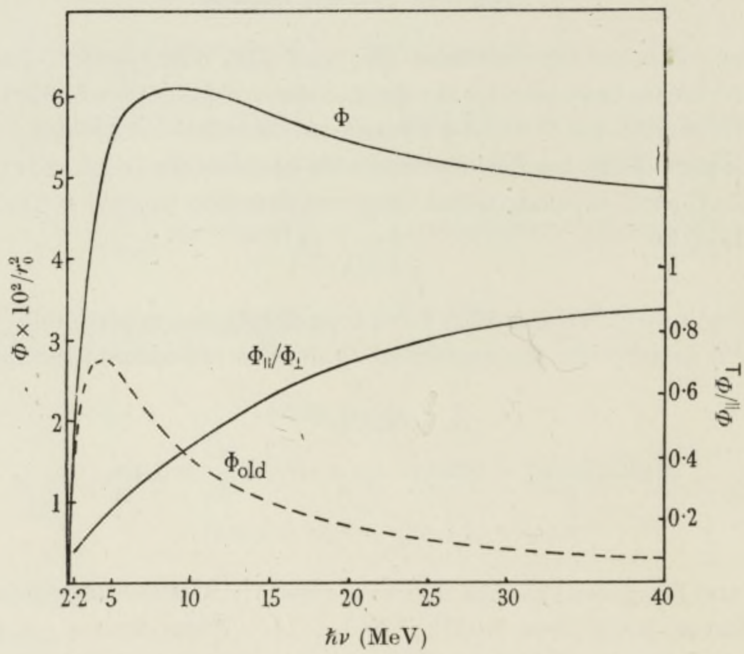

Figure 1

As it is seen from the figure, the total cross-section is for high frequencies very much larger than in the old theory. For the Li γ-rays (17 MeV), for instance, the cross-section is seven times larger in the new theory. The difference between the two theories is most striking in the asymptotic region for high frequencies, i.e. $\hbar\nu \gg E_0$, or $K_1 \gg \lambda, \alpha$. In this case we obtain

$$d\Phi = \frac{\pi^3}{8}\sin\theta\, d\theta \frac{1}{\lambda^2}\frac{e^2}{\hbar c}\left(\frac{f^2}{\hbar c}\right)^2 \times \frac{Mc\alpha}{\hbar\lambda^2}\sqrt{\left(\frac{Mc^2}{\hbar\nu}\right)}(3-\sin^2\theta), \tag{41}$$

$$\Phi = \frac{7\pi^3}{12}\frac{1}{\lambda^2}\frac{e^2}{\hbar c}\left(\frac{f^2}{\hbar c}\right)^2 \frac{Mc\alpha}{\hbar\lambda^2}\sqrt{\left(\frac{Mc^2}{\hbar\nu}\right)}. \tag{42}$$

In the old theory, the cross-section is

$$d\Phi_{\text{old}} = 2\pi\sin\theta\, d\theta \frac{e^2}{Mc^2}\frac{\hbar}{Mc}\frac{\hbar\alpha}{Mc}\left(\frac{Mc^2}{\hbar\nu}\right)^{\frac{3}{2}}\sin^2\theta. \tag{43}$$

Hence
$$\frac{\Phi}{\Phi_{\text{old}}} = \frac{7\pi^2}{32}\left(\frac{f^2}{\hbar c}\right)^2\left(\frac{M}{\mu}\right)^3\frac{\hbar\nu}{\mu c^2}. \tag{44}$$

Thus in the new theory the cross-section decreases like $1/\sqrt{\nu}$ whereas in the old theory it decreases more rapidly, like $1/\nu^{\frac{3}{2}}$. As it is seen from equ. (42), this asymptotic behaviour is solely due to the f-type of interaction. Ordinary exchange forces also give an increase of the cross-section at smaller energies as has already been shown by Breit and Condon (1936). The angular distribution contains a constant term and a term proportional to $\sin^2\theta$. This is, however, not due as it might seem at first sight to a superposition of a S- and a P-distribution, but to a superposition of P- and F-distributions, as can be seen from the derivation in § 2. The process is a three-stage process; in each stage the angular momentum can change by one.

In the old theory the angular distribution is a pure P-distribution ($\sim\sin^2\theta$) as long as the photomagnetic effect is neglected. The latter leads to a pure S-distribution, i.e. to a term independent of θ. In the old theory (cf. Bethe and Bacher 1936), the photomagnetic effect is only appreciable for energies just above E_0 and is for high energies very small. In our theory the photomagnetic effect is zero in the approximations made in § 2. The two essential approximations which we have made were (i) $\hbar\nu \ll \mu c^2$ and (ii) $\hbar\nu$ not too small (Born's approximation). For energies well above E_0 only the first approximation is essential, which means that the recoil of the meson due to the light quantum is neglected. If this approximation is not made we would obtain transitions to singlet terms, i.e. a photomagnetic effect,

as well, but the cross-section for those transitions would be smaller by a factor of the order of at least $\hbar\nu/\mu c^2$.

The angular distribution is also influenced by the fact that the ground state of the deuteron contains a D-function. Even in the old theory one would obtain in addition to the P-, a F-distribution. The admixture of the D-wave function is probably quite small, as can be seen from the size of the quadripole moment which is about fifty times smaller than the cross-section of the deuteron (Kellogg *et al.* 1939). We, therefore, hardly expect any considerable modification of our results.

It should be possible to check this theory experimentally. At present, no experiments in the high-energy region are available. Such experiments would provide a very valuable check for the meson theory. A theory in which the nuclear forces are described by a neutral field would not lead to any such high cross-section and it is just the exchange nature of the nuclear forces which is responsible for our effect. Furthermore, as it is seen from the asymptotic formula (42) the large cross-section and the peculiar angular distribution is due to the f-type of the interaction of the meson and a heavy particle, i.e. to the spin-dependent interaction. This interaction which has no analogy in electrodynamics leads to new types of effects, such as the magnetic moment of the neutron, and to the spin-dependent proton-neutron interaction. In all these effects some kind of divergency occurs which has led to the assumption that the meson theory is only correct for small meson energies. Our effect is mainly due to mesons with kinetic energies not much higher than the rest energy, and for those energies the theory should be essentially correct.

The photoelectric effect of the deuteron provides therefore a test of some of the most interesting features of the meson theory, namely the f-type of the interaction and the charged nature of the field, in a region where it is not seriously affected by high-energy difficulties.

One of us (B. K.) would like to express his thanks to the Cavendish Professor for a grant from the Scott Fund.

Summary

We have calculated the photoelectric effect of the deuteron on grounds of the meson theory of nuclear forces. In this theory the nuclear field is considered as electrically charged and, consequently, the light quantum can act on the nuclear field as well as on the proton. Because of the small mass of the meson, this effect is large. For the calculation we have assumed

for the ground state of the deuteron a ^{3}S-state. The result is that at high frequencies ν the cross-section is much larger than in the old theory, and decreases only like $1/\sqrt{\nu}$ as compared with the $1/\nu^{\frac{3}{2}}$ law in the Bethe-Peierls theory. Also the angular distribution is different. In addition to the $\sin^2\theta$ distribution about the direction of the γ-ray, there is a θ-independent term which becomes increasingly important for increasing energies. For the Li γ-rays (17 MeV), for instance, the cross-section is seven times larger than in the old theory and the ratio of the number of protons emitted parallel to the γ-ray to that emitted perpendicular to it is 0·7.

[*Note added in proof.*

While this paper was in press we noticed that the derivation of the matrix element (37) can be simplified if the approximations made in this paper (dipole radiation) are applied from the very beginning. The energy of the heavy particles does then not occur in the resonance denominators and the rules of ordinary matrix multiplication can therefore be applied for the summation over the intermediate states. The total matrix element then takes the following simple form:

$$V_a + V_b = ie\sqrt{\frac{2\pi}{\hbar\nu}}\int d\tau\, \psi_F^*(\mathbf{er})\, W\psi, \tag{45}$$

whilst V_c was:

$$V_c = -\frac{e\hbar}{M}\sqrt{\frac{2\pi}{\hbar\nu}}\int d\tau\, \psi_F^*(\mathbf{ep})\, \psi. \tag{46}$$

Here ψ_F is the deuteron wave function in the final state, $\mathbf{p}$ is the relative momentum of the deuteron and W the exchange potential of the deuteron (without the exchange operator) as given in F.H.K. equ. (44). Thus the meson effect finds its expression in the fact that the operator $(\mathbf{ep})/M$ which usually describes the interaction with light is replaced by

$$\frac{(\mathbf{ep})}{M} - \frac{i}{\hbar}(\mathbf{er})\, W. \tag{47}$$

This is true for dipole radiation only. Equ. (47) is in agreement with a general theorem derived by Siegert (1937) and confirmed by Lamb and Schiff (1938). These authors have also shown that the sum of the two operators (47) is equal to the operator $(\mathbf{er})$ if exact wave functions are used. We have used approximate wave functions and in this case, as shown in our paper, the consistent procedure is to take the operator (47).

The formula (45) is useful because it allows one to see more clearly in which way the frequency dependence of the cross-section is connected with the expression for the nuclear forces. The cross-section is proportional

to $|V|^2\sqrt{\nu}$. From mere dimensional considerations it can easily be seen that terms of W proportional to $1/r^n$ give a contribution to the cross-section which asymptotically depends on ν like $\nu^{n-\frac{7}{2}}$. Thus our asymptotic formula (42) ($\Phi \sim 1/\sqrt{\nu}$) is due to the $1/r^3$ terms in the proton neutron interaction.]

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Critical and co-operative phenomena
V. Specific heats of solids and liquids

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1. Introduction

In the first two papers in this series (Lennard-Jones and Devonshire 1937–8) we developed a simple method of calculating the free energy of a dense gas or a liquid in terms of interatomic forces. We used this to calculate critical temperatures and also vapour pressures and boiling-points. In later papers (Lennard-Jones and Devonshire 1939) we showed that the model used in the earlier papers was more appropriate to a solid than to a liquid, and that to obtain a satisfactory theory for a liquid we must modify it by introducing the concept of disorder. In this way we were able to account satisfactorily for the phenomenon of melting.