The electronic charge

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[Plate 6]

The determination of e by a new oil-drop method in which the electric field is horizontal has been described. The expression for e in terms of quantities measured is similar to that which applies to H. A. Wilson's method. The correction for the departure from Stokes's law is obtained from Millikan's relation

$$e_1^{\frac{2}{3}} = e^{\frac{2}{3}} + m/pa,$$

where e_1 is the uncorrected value of e, p cm. of mercury is the pressure and a cm. the radius of the drop. The oil drops used are larger than those used by previous experimenters, and their velocity of fall and the velocity in the direction of the electric field could be estimated with satisfactory accuracy.

Assuming $\eta_{23} = 1830 \times 10^{-7}$ c.g.s. unit, the value of e obtained is $(4{\cdot}8020 \pm 0{\cdot}0013) \times 10^{-10}$ e.s.u. The probable error calculated by the least squares is about one-third of that obtained by Millikan, whose result becomes $(4{\cdot}7992 \pm 0{\cdot}0037) \times 10^{-10}$ e.s.u., when the above value of η_{23} is assumed. The errors in the determination of η_{23} by the rotating cylinder and the capillary tube method are discussed and the final mean

$$\eta_{23} = (1830{\cdot}06 \pm 2{\cdot}5) \times 10^{-7} \text{ c.g.s. unit.}$$

is derived. This contributes an error of 1 in 500 in e and is its major uncertainty by the oil-drop method. Recent determinations of the electronic charge by the X-ray method have been analysed. The mean value of e by this method is deduced to be $(4{\cdot}8044 \pm 0{\cdot}0007) \times 10^{-10}$ e.s.u., and this result differs from the mean of the determinations of Millikan and the authors $(4{\cdot}8007 \pm 0{\cdot}002) \times 10^{-10}$ e.s.u., by $0{\cdot}0037 \times 10^{-10}$ e.s.u., which is less than the error due to the viscosity of air. Assuming the X-ray value of e, the viscosity of air can be deduced and is found to be $(1830{\cdot}9 \pm 0{\cdot}5) \times 10^{-7}$ c.g.s. unit at 23° C.

Introduction

The values of three of the general constants of physics, e, h and e/m, have been the subject of much discussion and experiment. Discrepancies exist such as the disagreement in the value of e as found by the drop and the X-ray methods, and the disagreement between the value of Rydberg's constant calculated from e, h and e/m and its observed value.

It would appear that these discrepancies can only be removed by improvements in the methods which are used in the determination of the constants. Of the many methods which have been used to determine the electronic charge, the oil-drop and the X-ray methods are accepted as the more reliable. It is true that the reliability of the drop method has been questioned, and one of the authors believed, until he had carried out experiments by the X-ray method, that it would be the more accurate of the two methods. The conclusions which we have reached in the course of the experiments described below is that the highest accuracy attainable by the drop method has not yet been reached, that the uncertainty as to the correct value of the viscosity (η) of air is the major error in the value of e by this method, and that when the uncertainty as to the value of η is removed the accuracy of the drop method will equal that of the X-ray method as it has so far been developed.

Assumptions of the drop method

The drop method, introduced by H. A. Wilson and used by him with water, was improved by Millikan by the use of oil drops. Millikan and his co-workers in experiments from 1906 to 1917 established that Stokes's law in a corrected form applied to a drop of liquid falling through a gas, and that Einstein's theory of the Brownian motion applied too. Cunningham in 1910 gave a theoretical deviation for the correction to be applied to Stokes's law for a sphere moving in a gas. These brilliant investigations placed the assumptions made in the drop method on a sound foundation of experimentally verified theory, and are the justification of an attempt to improve the experimental application of those principles in the measurement of e. (This symbol is used as an abbreviation for 'value of the electronic charge'.)

In the method to be described here there is a departure from the principle of H. A. Wilson's experiment in which the electric and gravitational fields are parallel, and the electric field changes the speed of fall without changing its direction.

In the method of the authors the electric field is horizontal and it changes the direction of fall. This, when combined with a photographic method of measuring the velocity of the drop, has the advantage that departures from Stokes's law, the presence of convection currents in the air, a change in the charge of the drop, and any error in the direction of the electric field can be detected.

The accuracy with which e can be measured by the drop method is limited by the accuracy with which the viscosity of air, η, and its variation with

temperature are known, since $e \propto \eta^{\frac{3}{2}}$. Some forty determinations of η have been made, and the value of η to be accepted is discussed later.

Criticisms of drop measurements of e. Millikan (1913–30) made two series of measurements of e, one of which gave fifty-eight values of e and the other twenty-five values. Ishida, Fukushina and Suetsuga (1937) and Bäcklin and Flemberg (1936) have also measured e by the drop method. Table 1 contains the range of values of a cm., the radius, and t sec., the times of fall, of the ten largest drops used in previous measurements of e by the drop method.

TABLE 1

a cm.	t sec.	Observers
$1{\cdot}7 - 4{\cdot}3 \times 10^{-4}$	$3{\cdot}7 - 22$	Ishida, Fukushina and Suetsuga
$2{\cdot}4 - 5{\cdot}8 \times 10^{-4}$	$4{\cdot}4 - 15$	Millikan (1913)
$1{\cdot}65 - 2{\cdot}3 \times 10^{-4}$	$14{\cdot}1 - 26{\cdot}8$	Millikan (1917)
$0{\cdot}83 - 1{\cdot}8 \times 10^{-4}$	—	Bäcklin and Flemberg

In the above measurements the time t has been measured by an 'eye and hand' method closely similar to that used in meridian observations of stars with a fixed wire micrometer and tapping key. The errors to which such observations are subject have been discussed by Spencer Jones (1938).

Table 2 of this reference (Spencer Jones 1938) shows that the personal equation of two observers may differ by as much as 0·47 sec., and this is evidence that the personal error must be considered in the timing of drops.

We conclude from data given in Millikan's paper that:

(1) his Hipp chronoscope had a rate of $+0{\cdot}2\,\%$;

(2) the combined personal and chronoscope error

$$\text{start} - \text{stop} = +0{\cdot}023 \text{ sec.};$$

(3) the mean departure from the mean $= \pm 0{\cdot}05$ sec.

Millikan corrects for the systematic error (1), and possibly for the systematic error (2). The random error (3) necessarily enters into the value of e derived from the experiments. Insufficient data are available to evaluate the effect of this timing error on Millikan's value of e, but the following indicates its magnitude. If the time of free fall is $30 \pm 0{\cdot}05$ sec., and of ascent in the electric field is $80 \pm 0{\cdot}05$ sec., the error in $(1/t_g + 1/t_F)(1/t_g)^{\frac{1}{2}}$ is $\pm 0{\cdot}20\,\%$, and this is the error in e. If $t_g = 4{\cdot}4 \pm 0{\cdot}05$ sec., the minimum resulting error in e is over $1\,\%$. Thus the eye-and-hand method of timing cannot be used with large drops. A large number of observations will of course reduce the random error of their mean, but the systematic personal error ((2) in above list) cannot be so eliminated.

We have timed drops by photographing them with an exposure of 0·00067 sec. at intervals of 0·04 sec. In the case of the drop of maximum velocity of 1 cm./sec. the image velocity on the photographic plate is 12 cm./sec., and the position of the image could be measured in some cases to better than 0·0007 cm., which is equivalent in time to 0·00006 sec. This estimate gives the error in timing due to the error in measuring the photographic plates as 1 in 10,000. The error in the speed of the timing disk is discussed later.

Optimum size of drop. Errors of timing are least for slowly moving and therefore for small drops, but the errors due to Brownian motion and to Stokes's law correction both increase as the drop radius, a, decreases. In the previous determinations it will be seen from the above table that the 10 drops which are most important to Millikan's 1913 value of e were timed for a period of from 4·4 to 15 sec., and the error of timing for these drops is larger than the Brownian or Stokes errors. Millikan in his 1917 measurements used smaller drops than in 1913, and while this decreases the timing error it increases that due to Brownian motion, and the error of extrapolation required in the Stokes correction.

With photographic timing the use of drops larger than the largest we used (radius $9{\cdot}6\mu$) would have reduced the already small Stokes correction of our experiments. An unexpected difficulty arises in the use of larger drops. Oil drops when formed have a charge $\pm ne$, where n is roughly proportional to a^3 for a given oil. n was so large for castor oil that only a few measurements with that oil could be used. Even for apiezon oil one drop gave

$$n = 276\,(a = 7{\cdot}4\mu),$$

which was as large as was desirable in these experiments. The precision of the measurement of e limits the maximum value of n which can be used if ambiguity in the choice of n is to be avoided.

Brownian motion. The mean displacement $\overline{\Delta x}$ of a drop in a time τ in a medium of viscosity η is

$$(\overline{\Delta x})^2 = RT\tau/(3\pi\eta Na), \tag{1}$$

where R, N are the gas and Avogadro's constants, and T the absolute temperature. The velocity of free fall is

$$v = 2\rho g a^2/9\eta. \tag{2}$$

Putting $x = v\tau$, where x is the distance moved by the drop in time τ, then

$$\overline{\Delta x}/x = c/a^{\frac{5}{2}}\,\tau^{\frac{1}{2}}, \tag{3}$$

where c is a constant. Thus the Brownian motion error $\Delta\bar{x}/x$ is proportional to $a^{-\frac{5}{2}}$.

$$\text{For } a = 1{\cdot}7\times 10^{-4}\text{ cm. and } \tau = 29\text{ sec.,}\qquad \overline{\Delta x}/x = 2{\cdot}0\times 10^{-3},$$

$$\text{For } a = 10\times 10^{-4}\text{ cm. and } \tau = 0{\cdot}5\text{ sec.,}\qquad \overline{\Delta x}/x = 1{\cdot}8\times 10^{-4}.$$

The first line applies to a drop used by Millikan, and the second line to a drop used in our experiments.

Correction to Stokes's law. Millikan, to correct for the effect of the discontinuity of the medium, plotted $e_1^{\frac{2}{3}}$ against $1/pa$, where e_1 is the uncorrected value of e obtained from the observations and p is the pressure of the gas in which the drop moves. The points fell on a straight line, given by the equation

$$e_1^{\frac{2}{3}} = m/pa + e^{\frac{2}{3}}. \tag{4}$$

Extrapolating to $1/pa \to 0$, the value of $e^{\frac{2}{3}}$ for $pa \to \infty$ is found. The accuracy of $e^{\frac{2}{3}}$ calculated from the observations will depend on the range of the values of $1/pa$ covered and upon the observations being accurate at both ends of the range. It is to be noticed, however, that when $1/pa$ is small with visual observations the timing error is large, and when $1/pa$ is large Brownian motion displacements are large. These considerations require the use of drops with a sufficiently large range of values of a, and that for the smallest drop used the Brownian motion error should not be large.

Principle of method

The drop first falls under gravity between two vertical plates, is illuminated intermittently, and appears as a series of bright stars on a black background. Then a horizontal electric field is applied and the drop is deflected laterally along a straight path inclined at an angle θ to the vertical. The photographic plate shows a series of black dots on a light background (see figure 1, plate 6).

Let v_g be the vertical velocity of the drop when no electric field is applied. Applying Stokes's law we have

$$mg = 6\pi\eta a v_g, \tag{5}$$

where m is the mass of the drop. Thus

$$\tfrac{4}{3}\pi a^3(\rho-\sigma)\,g = 6\pi\eta a v_g$$

(ρ being the density of the oil and σ that of the air), and

$$a^2 = \tfrac{9}{2}\eta v/(\rho-\sigma)\,g. \tag{6}$$

When an electric field of strength X is applied, the drop quickly reaches a terminal velocity of r at an angle θ to the vertical. If v_x is the velocity of the drop in the direction of the field then, applying Stokes's law,

$$Xne = 6\pi\eta a v_x. \tag{7}$$

Substituting for a we have

$$Xne = 9\sqrt{2}\,\pi\eta^{\frac{3}{2}}(\rho-\sigma)^{-\frac{1}{2}}g^{-\frac{1}{2}}v_g^{\frac{1}{2}}v_x. \tag{8}$$

v_g was measured directly, and v_x can be calculated from a measurement of two of the three quantities r, θ and v_g' (v_g' being the vertical velocity of the drop after deflexion). In the tabulated results v_x is calculated from the expression

$$v_x = \frac{r\sin\theta}{\cos\delta}, \tag{9}$$

where δ is the direction of the field from the horizontal and is calculated from the expression

$$\tan\delta = \frac{v_g - v_g'}{r\sin\theta}. \tag{10}$$

It is not necessary to measure v_g, v_g' and r for other oil drops, as only two of these measurements are necessary, but in general all three were measured to check the value of δ.

Time to reach terminal velocity. The time taken for a drop to reach terminal velocity after deflexion can be derived as follows:

Assuming the electric field to be horizontal and to act in the z direction, the equation of motion of the drop is given by the equations

$$Xne - 6\pi\eta a\dot{z} = m\ddot{z} \tag{11}$$

and

$$mg - 6\pi\eta a\dot{y} = m\ddot{y}. \tag{12}$$

Consider equation (11) and let $k = (6\pi\eta a/m)$ and $(Xne/m) = L$, i.e. the equation becomes

$$L - k\dot{z} = \ddot{z}.$$

The solution of this is $z = c_1 + c_2 e^{-kt} + (L/k)\,t$,

where c_1 and c_2 are constants. Initially

$$t = 0, \quad z = 0, \quad \dot{z} = 0, \quad c_1 + c_2 = 0, \quad \text{and} \quad c_2 = L/k^2,$$

i.e.

$$\dot{z} = -(L/k)(e^{-kt} - 1).$$

Let $\dot{z}_c$ be the terminal velocity (i.e. $\dot{z}$ when $t \to \infty$), then $\dot{z}_c/\dot{z} = (1 - e^{-kt})$.

When $(\dot{z}_c - \dot{z})/\dot{z}_c = 1/10{,}000$ and $a = 5\mu$,

$$t < 0{\cdot}003 \text{ sec.}$$

Thus a drop (radius 5μ) takes less than 0·003 sec. to reach within 1 part in 10,000 of its terminal velocity.

Microphotography of moving drops

Accuracy in the calculation of the electric field requires the condenser plates between which the oil drops move to be several times as wide as their distance apart. The plates are 0·4985 cm. apart, and a microscope objective of focal length 3·8 cm. was chosen to enable the plates to be made sufficiently wide.

The photography of small drops moving with a velocity of about 1 cm./sec. with an exposure of 1/1500 sec. using an objective of small numerical aperture requires some departure from normal photographic method.

As it was desired to improve the definition of the image by the use of a fine-grained plate, a high intensity of illumination on a dark background is necessary. A condenser was designed to give a hollow cone of light, the apex of which was at the drop. The angle of the cone of light was sufficiently large for none of its light to enter the objective. A 20 amp. mirror arc gave ample illumination. A system of filters and shutters reduced the radiation from the arc absorbed in the enclosure in which the drops fell to such a small amount that no convection currents were set up in the air surrounding the drops.

The details of the above-mentioned arrangements are shown in figures 2*a* and 2*b* and are described below under the headings: optical system, illumination, shutters, photography, magnification, etc.

Optical system. Light from the arc (see figure 2) is focused by its mirror on the dark-ground condenser, and is refracted by it to form a hollow cone of angle of about 40°. This gives with the objective used dark-ground illumination. Refracted and scattered light from the drop is focused by the Zeiss 1½ in. objective on the photographic plate, at such a distance as to give a magnification of $\times 12{\cdot}001$. A reflecting prism can be turned to throw the light beam into a focusing microscope which is adjusted to be in focus when the camera is exactly focused. It enables visual observations to be made of the field to be photographed; its prism is turned out just before exposures are made.

The Leitz microscope, from which the apparatus is constructed, is supported on a microcamera stand, which is levelled to make the direction of the electric field horizontal. The microcamera and the copper box containing the

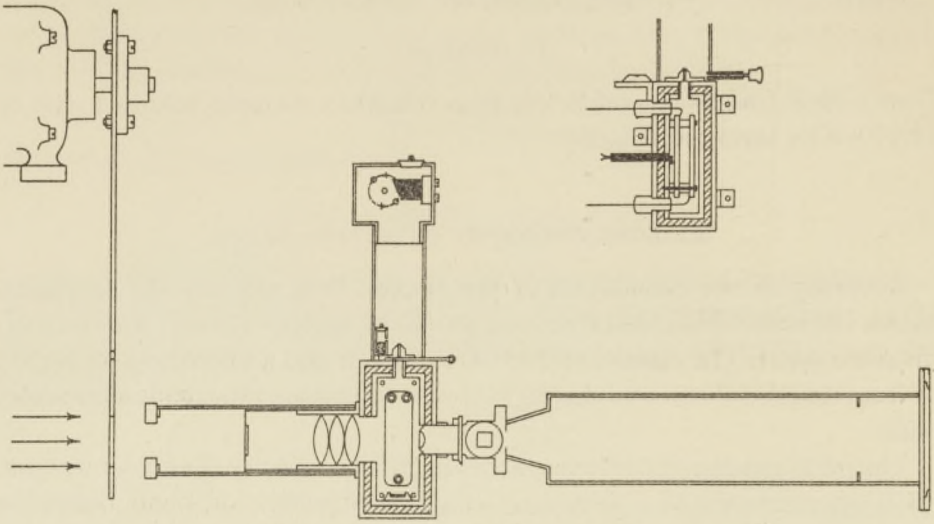

FIGURE 2a. Design of apparatus.

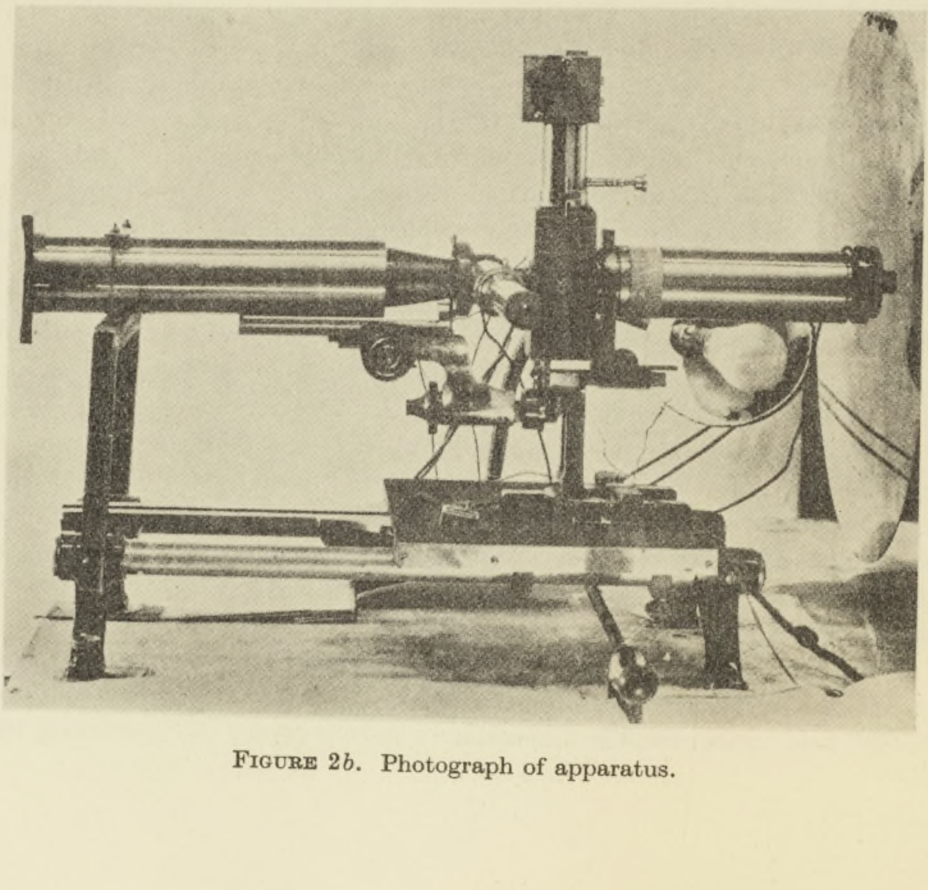

FIGURE 2b. Photograph of apparatus.

condenser were carefully designed to be rigid and retain adjustments once made.

The dark-ground condenser is shown in figure 2. Its design was reached after many other systems had been tested. It consists of three achromatic lenses of 10 cm. focal length, an exit stop of 1 in. diameter placed about 1 in. from the last lens, and a compur shutter with iris diaphragm which covers the entrance to the condenser. This stop and iris diaphragm are carried by telescoping tubing for their adjustment relative to the condenser.

The objective. A Zeiss objective of $1\frac{1}{2}$ in. focal length was chosen for its definition after comparison with a few other lenses.

Illumination. The carbons of the arc are horizontal and in the direction of the axis of the optical system. Complete mechanical adjustments for the carbons and parabolic mirror enabled an image of the arc crater to be formed on the condenser.

Shutters and filters. Three glass boxes filled with water are placed in the beam from the arc to reduce the heat rays; their effectiveness is shown by the water boiling in the first box if the arc remains on for some time.

A large metal shutter protects the apparatus from radiation when observations are not being taken. A rotating disk of aluminium 25 cm. in radius mounted on the axle of a 50 cyc./sec. synchronous motor with a window cut in it near its edge 6° in angular aperture allows the light to pass 25 times per sec. for $\frac{1}{1500}$ sec. It cuts off $\frac{59}{60}$ of the radiation which would enter the condenser. The compur shutter at the entrance to the condenser, normally closed, is open for 2 sec. when exposures are made. Fifty exposures are made of a total duration of $\frac{1}{30}$ sec. All photographic plates used to measure e were examined for evidence of convection currents, but no evidence of convection due to the illumination was found. Good definition in all the images of a drop on a plate is evidence of the absence of convection.

Photography. Kodak ordinary plates developed in Agfa fine-grain developer at 18° C gave a well-defined image free from grain and background fog of even the fastest moving drop. To prevent distortion of the emulsion the plates were kept horizontal while in a wet condition. Using a microscope magnification of about $\times 17$ when measuring the resulting plates, accurate settings of micrometer eyepiece wire could be made. The initial magnification being $\times 12$, the total is 204 in these measurements.

Magnification. The initial magnification is determined by the focal length of the microscope objective and the camera length. Tests showed that when the image on the photographic plate is well defined the magnification is constant to 1 in 20,000. It was measured by photographing a Grayson ruling on glass and measuring the ratio of the distance between two consecutive

lines on the photographic plate to the distance between them on the ruling. These measurements given in the attached table were made by means of the Zeiss travelling microscope used for all plate measurements, the screw of which had been carefully calibrated.

TABLE 2

Plate	No. of obs.	Magnification
1	5	$(42{\cdot}009 \pm 0{\cdot}002)/(3{\cdot}5004 \pm 0{\cdot}0003)$
2	5	$(42{\cdot}005 \pm 0{\cdot}002)/(3{\cdot}5004 \pm 0{\cdot}0003)$
	Mean	$12{\cdot}001 \pm 0{\cdot}001$*

* $\pm 0{\cdot}001$ in the above table is the mean departure from the mean and not the probable error of the mean.

VISCOSITY OF THE AIR

In the expression for e, $e \propto \eta^{\frac{3}{2}}$. This means that the air in which the drops fall must be dry air, that its temperature must be maintained constant and be known to an accuracy of 0·026° C if $\eta^{\frac{3}{2}}$ is to be known to 1 in 10,000. The measures taken for keeping the temperature constant, measuring it, and the value to be assumed for η are discussed below.

Thermostat (Laby and Hopper 1939*a*). To attain uniformity of temperature, the air in which oil drops fall is enclosed in an inner copper box of wall thickness 0·15 cm., surrounded by 0·5 cm. of thin sheets of synthetic cork. The outermost wall is of copper 0·15 cm. thick. To obtain constancy of temperature in this box and the metal and glass parts connected to it, a thermostatic device was devised which proved convenient and very effective. A resistance thermometer of 100 Ω resistance was made by tightly winding no. 45 gauge cotton-covered copper wire round the brass tube of the microscope condenser which is in thermal contact with the copper box. This coil formed one arm of an equal-armed Wheatstone bridge, the other arms being of manganin. With the dissipation of 0·01 W in the copper coil, a change of temperature of 0·001° C caused a galvanometer deflexion of 0·25 cm. The galvanometer spot moved across the window of a gas-filled photoelectric cell when deflected slightly from its equilibrium position. The current from the cell was amplified by a 2A3 triode valve, the plate current of which operated a polarized relay which lit two 20 W 230 V lamps placed on opposite sides of the copper box, each of them being about 45 cm. from it. All sides of the box were heated by the radiant heat of the lamps.

This device kept the box at a constant temperature T, a few degrees above the room temperature, t. T must of course be greater than t. Its effect on the

temperature of the box is shown in figure 3. It proved most convenient and effective.

Temperature measurement. Although it is not in general a good practice to use base metal thermocouples for accurate temperature measurement, a copper-constantan couple was used to measure the air temperature. The constantan wire of the couple was selected for its freedom from inhomogeneities. The cold junction was placed in a mixture of ice and distilled water stirred by a slowly reciprocating vertical plunger, and the other junction was in the air in the copper box adjacent to where the drops fell. A three-dial 'thermokraftfrei' low-resistance potentiometer by Wolff with a galvanometer reading to $0{\cdot}1\,\mu V$ (i.e. $0{\cdot}002°$ C) was used with the thermocouple.

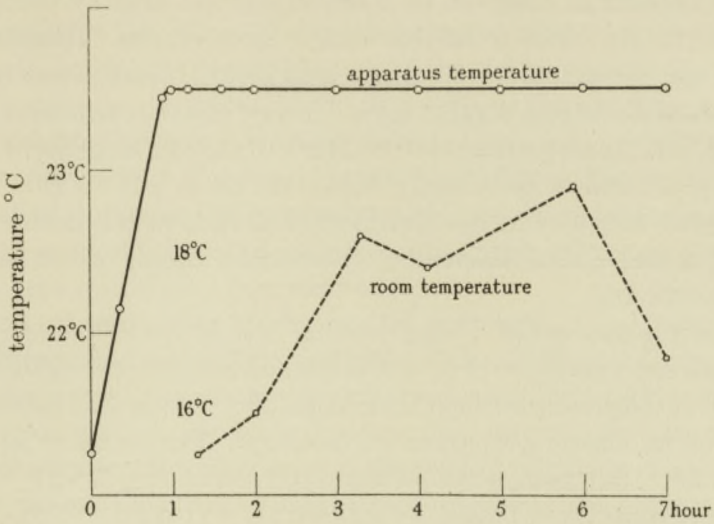

FIGURE 3. Temperature inside apparatus, controlled by radiant heat method.

Viscosity of air. The uncertainty which exists as to the value of the viscosity of air to be accepted sets a limit to the accuracy with which e can be determined by the drop method.

In deriving from the many available values of η for air a 'best' value to be used in calculating e, no procedure is free from arbitrary assumptions. Only two methods of determining the viscosity of a gas will be considered, which are

(1) the steady deflexion of a cylinder, and

(2) flow through a cylindrical tube.

Methods in which the air is accelerated are rejected as not having a valid theory. Fabry and Perot's interference fringe method, in which a plane circular disk is lifted from another plane surface while theoretically sound,

and possibly capable of yielding high accuracy, does not appear to have been used in measurements of the highest precision.

Rotating cylinder method. Determinations of η by the deflexion of a cylinder have been made by Kellström, Houston, Harrington and Gilchrist. The last three of these form a connected investigation which Gilchrist began in Professor Millikan's laboratory, and Harrington continued using the same apparatus. Houston used improved apparatus, and has discussed in detail the theory of the method. Kellström's measurements of the constants of his apparatus are of great precision, but he does not mention having made a correction to the length of the inner cylinder, which is to be regarded as extending into the air gaps at its two ends. Further, the moment of inertia of the inner cylinder as observed by Kellström is too large by the moment of inertia of the air which it carries with it as it rotates. (Harrington and Houston use a vacuum to eliminate this error.) Both these corrections would reduce Kellström's value for η. The air inertia correction cannot be estimated, but the air gap one can be. It is half the two air gaps, i.e. 0·11 mm. on 99·98 mm., and it decreases η from 1834·62 to $183{\cdot}6 \times 10^{7}$. The major experimental difficulty of the deflexion method is in the measurement by means of a suspension filament of the torsion couple which acts on the deflected cylinder.

Bearden's measurement. Since writing the above Bearden (1939) has published a determination of η by this method in which the outer cylinder is deflected. A tungsten wire, heat treated at 1200° C, was used as filament and the law of its elastic properties investigated. The value of η_{23} given is $1834{\cdot}12 \times 10^{-7}$, but using a temperature correction of $4{\cdot}83 \times 10^{-7}$ per ° C at 23° C, for reasons given below, we will use $\eta_{23} = (1833{\cdot}8 \pm 0{\cdot}06) \times 10^{-7}$.

External consistency of viscosity observations. There are nine values of η_{23} in table 3, and the errors of these values can be estimated both from internal and external consistency.

How are the observed values and their probable errors (calculated by internal consistency) to be reconciled? Consider the values $1833{\cdot}8 \pm 0{\cdot}06$, $1830 \pm 0{\cdot}7$, $1822{\cdot}6 \pm 0{\cdot}7$. The chance of an observation with a probable error of $\pm 0{\cdot}06$ having an error of 3·8 or more is vanishingly small, and one with a probable error of $\pm 0{\cdot}7$ having an error of 3·8, that is, 5 times its probable error is $\frac{1}{1340}$, and an error of 11·2 vanishingly small. We conclude that either the random errors of most of the observations of η have been underestimated or that the observations are subject to systematic error. The error of the mean of these values, 1828·7, is $\pm 0{\cdot}67\{\Sigma v^2/(n-1)\}^{\frac{1}{2}} = \pm 3$, that is, the external consistency error includes both their systematic and random errors. As it is larger than all but one of the internal consistency errors of the

observations, we conclude the systematic error is larger than the random error.

In the case of Bearden's value the error $\pm 0 \cdot 06$ may be an underestimate. It is calculated by means of the partial derivative of

$$\eta = F(b^2 - a^2)\,\phi t / 8\pi L b^2 a^2,$$

and the errors of a, b, ... are estimated from their internal consistency. In calculating it the error in $(b^2 - a^2)$ is taken to be $\pm 0 \cdot 45$ in 10,000 for one pair of cylinders, and $\pm 0 \cdot 2$ for another pair. Other figures given by the author show, however, an error of from 4 to 7 in 10,000 in $(b^2 - a^2)$ equivalent to $\pm (0 \cdot 7$ to $1 \cdot 3) \times 10^{-7}$ in η_{23}. This brings the two estimates of error somewhat closer.

In weighting the five values of η obtained by the method account is taken of the interdependence of Harrington and Gilchrist's experiments, and that Kellström's value is not corrected for air inertia in the measurements of I. Using the weights shown in table 3 the weighted mean of η_{23} is $1830 \cdot 6 \times 10^{-7}$.

Capillary tube method. Vogel in 1914 reviewed sixteen determinations of η by this method, and some six have been made since then. In a critical review of the more recent of these experiments it is desirable to know what systematic errors arise in the tube flow method.

Very fine bore glass tubes of less than 0·2 mm. radius have been generally used, but Rapp, Maxwell and Rigden have used somewhat larger bores. The tube is calibrated with a mercury thread, but the methods used to correct the radius are open to experimental and theoretical criticism. The mean velocity of flow may exceed 1000 cm./sec. and approach the velocity for turbulent flow, 8000 cm./sec., as given by Reynolds's criterion. It is evident that the random error in η will be larger the finer the bore of the tube used.

The theory of the flow of gas through a tube assumes the gas follows Boyle's law, that it is at a constant temperature (it expands as it flows), and that there is no loss or gain of kinetic energy at the ends of the tube.

According to Bond's theory these last corrections in η depend on v^2/c^2, where v is the velocity of flow, and c the velocity of sound. Thus these corrections will in general increase as the tube radius decreases. There is evidence that this gives rise to a systematic error in measurements of η by the tube method, and it would account for the higher values found by five earlier observers, who used fine tubes, as compared with the lower values of η found by Rapp, Maxwell and Rigden who used tubes with more than twice the radius of the earlier experimenters. All recent observers have made the same correction for slip at the walls of the tube.

Values of η determined since 1910 are given in table 3. Rigden and Bond

used the same apparatus and method of reduction. Their results differ by $4{\cdot}6 \times 10^{-7}$ which is 6 times the error Bond gives for his experiment. Rapp's value for η_{23}, 1822·7, is low. His experiment was repeated by Maxwell using relatively large-bored tubes. Using a steady oil-pressure head and thermostatic temperature control he made 109 measurements of η.

Using the weights shown the mean for the tube method is $1829{\cdot}7 \times 10^{-7}$.

Weighted mean. The final mean of the nine observations with the weights stated is

$$= (1830{\cdot}0_6 \pm 2{\cdot}5) \times 10^{-7} \text{ unit g.cm.}^{-1} \text{ sec.}^{-1}.$$

TABLE 3. VISCOSITY OF AIR

Observer	Date	Temp. °C	η_{23}	Error	Weight
Rotating cylinder:					
Bearden	1939	20	1833·8	0·06	4
Houston	1937	22	1829·2	2·5	3
Kellström (corr.)	1937	20	1832·6	3·0	2
Harrington	1916	23	1822·6	0·7	1
Gilchrist	1913	20·2	1825·7	1·3	1
		Mean	1830·6		
Capillary tube:					
Rigden	1938	17	1830·0	0·69	2
Bond	1938	15	1834·6	0·8	1
Rapp	1913	26	1822·7	1·8	1
Maxwell	1916	22·5	1827·3	—	1
		Mean	1829·7		

Final weighted mean $= (1830{\cdot}0_6 \pm 2{\cdot}5) \times 10^{-7}$.

Temperature variation. The linear relation

$$\eta_{23} = \eta_t + 4{\cdot}93 \times 10^{-7}(23 - t)$$

has been generally used to reduce observations of η to 23° C, but this relation is not sufficiently accurate. Sutherland's expression

$$\eta_\theta = \eta_{273}(273 + c)/(\theta + c)(\theta/273)^{\frac{3}{2}}$$

is accurate over a wide range of temperatures. We conclude from a critical examination of a number of investigations of the temperature variation of η for air that $C = 117$. This value is used in calculating table 3. The value of the temperature coefficient of viscosity at 23° C is then $4{\cdot}83 \times 10^{-7}$ c.g.s./° C.

Electric field

Parallel conducting plates are used to give the horizontal electric field required for the deflexion of the drops. The question arises as to what shape these plates should be given, and what should the ratio of their width, w, to their distance apart, d, be in order that the field may be calculated.

The known solutions, namely, of the field on the axis of parallel circular plates and Helmholtz solution of the semi-infinite planes with straight opposite boundaries are not practically applicable to our problem beyond that Helmholtz's solution is a guide in estimating whether w/d has been chosen sufficiently large.

In deciding the value of w/d a compromise is made between the electrical and optical requirements. The latter require dark-ground illumination of the drop, and a hollow cone of light of vertical angle of 40° is necessary with the microscope objective chosen. Rectangular plates 2·0 cm. wide and 7 cm. long and 0·5 cm. apart have been used.

Helmholtz in 1868 found the field near the edge of two semi-infinite parallel planes, and his solution is discussed by Maxwell, Rayleigh and later writers.

We may apply this solution to find the ratio of the field for plates d apart at a point midway between the plates and distant $2d$ from the edge, to the field at an infinite distance from the edge. It is found that the difference in the fields is less than 1 in 10^6. This is some justification for believing that we may put $X = V/d$ without error important to the accuracy sought in these measurements.

Construction of condenser. The condenser plates are made of glass silver plated over a middle portion, but unsilvered at the ends, which act as part of the electrical insulation between the two conducting surfaces. The glass plates were tested for flatness against a glass optical flat by means of Newton's rings. They were separated by three lengths of pyrex capillary tubing (see figure 2) which were lapped to be of equal length. Ivory bolts and nuts hold the plates together and they are supported by three short lengths of the pyrex tubing on a steel plate 3 mm. thick. This geometrical system of support by pyrex glass from a steel plate gave rigidity and excellent electrical insulation.

The electrical connexion to the silvering required to be carefully made. The insulating bushes for connexion to terminals are of amber.

Adjustment of condenser plates vertical. Before the results with apiezon oil were obtained, the plates were made nearly vertical, and to do this precision in their adjustment was necessary. A small oil-damped plumb bob with a very fine wire filament was placed close to the surface of one of the

condenser plates, the outer copper boxes having been removed. If the glass plate is vertical, the distance (measured by means of a telemicroscope) from the filament to its image in the plate is the same at the top and bottom of the plate. Levelling screws under the platform supporting the microcamera adjusted the condenser plates to be vertical, and the bubble of a circular level attached to the stand of the microcamera is subsequently used to indicate when the plates are vertical.

Distance between plates. The pyrex separating pieces were compared by means of a dial gauge (reading to 1/10,000 in. and by estimation to 1/100,000 in.) with a Pitter 5 mm. steel slip gauge certified by the N.P.L. It was found that

$$\text{Mean length of separating pieces at } 23^\circ\text{ C} = 0{\cdot}49852 \pm 0{\cdot}00006 \text{ cm.}$$

as the mean of 20 measurements. This is taken as the distance between the silvered surfaces of the glass plates.

These condenser plates were not intended for the final measurements of e, and it was intended to construct the equivalent of a Fabry and Perot etalon using, however, three distance pieces, and determine its separation in terms of the wave-length of light. Delay in the construction of the etalon made it necessary to proceed with the condenser measured as described above.

Difference of potential. A difference of potential of 3000 V, which makes X about 6000 V/cm., has been found sufficient to deflect the largest drops we have used. As already explained the large charge found on such drops makes X much less than if the charge on these drops did not increase with their volume.

For the constant potential we have used rectified alternating current from the public supply. We are indebted to Mr T. P. Gill, M.Sc. (1939), for a most effective solution of the design of the stabilizer used. It is a two-stage Evans circuit carefully balanced, and its output potential has the same constancy as has been observed in this laboratory for small storage cells, namely, of about 1 in 10,000. It is very much more convenient than a battery.

The circuit used to connect the stabilizer to the condenser plates and to a potentiometer for measuring, V, the potential applied to the condenser is shown in figure 4.

The potential, V, is given by

$$V = E(R+r)/r,$$

where E is the potential difference across r which is measured on the potentiometer in terms of the e.m.f. of a Weston cell. The potentiometer is a three-

FIGURE 1A. Deflected drops.

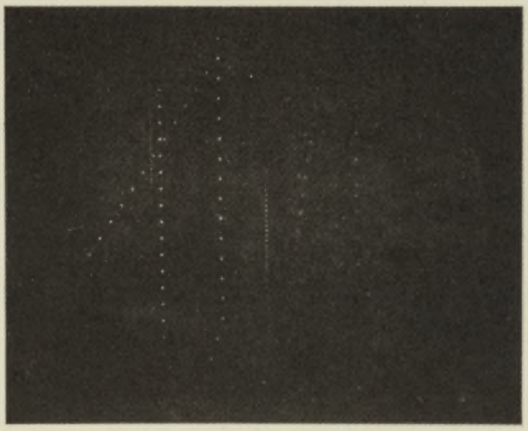

FIGURE 1B. Undeflected drops.

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dial Wolff 'thermokraftfrei' low resistance one in which, as the resistance in the galvanometer circuit is constant, readings of galvanometer deflexions enable the fifth figure in the value of E to be estimated.

V is found in terms of the international volt, but X in the expression for the electronic charge is in electrostatic units. The ratio for these units used is given under calculation of results.

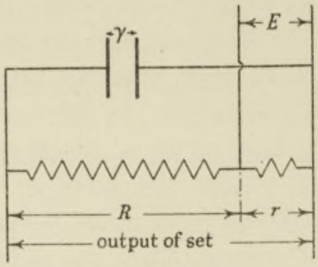

FIGURE 4. Electrical circuit.

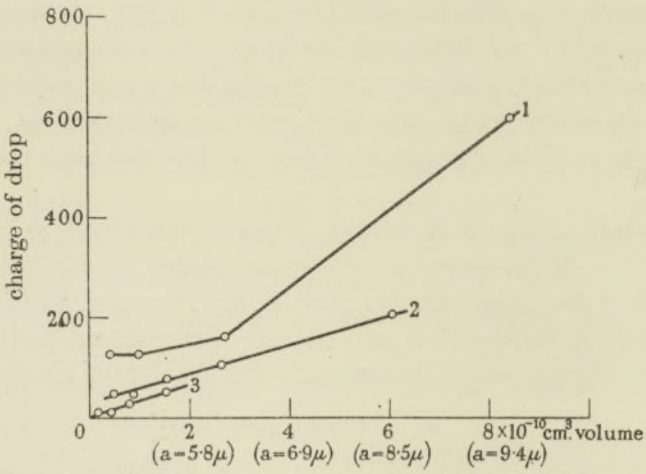

FIGURE 5. Relationship between charge and volume of drops. 1, castor oil; 2, apiezon oil, A; 3, apiezon oil, B.

OILS USED AND THEIR PROPERTIES

We have used in our observations apiezon oil because of its low vapour pressure, and castor oil because it gives drops carrying large charges. We have not observed any evidence of the evaporation of drops as Ishida and his co-workers did. If there is evaporation for apiezon oil it will be less in the following experiments than for the Japanese observations, since the drops used are larger and are under observation for much less time.

In figure 5 the charge $\pm ne$ of a drop on its formation by the method described below is plotted against a^3, where a cm. is the radius of the drop.

The points with some large departures tend to fall on a straight line for each oil, castor oil being more highly charged per unit volume than the other oil.

This property of an oil of forming drops for which n is large is an important one, for it is possible to use proportionately smaller electric fields and potential differences. In one of our experiments $a = 9{\cdot}6 \times 10^{-4}$, $n = 233$, and a potential difference of 2200 V gave a sufficient angle of deflexion. One oil drop was photographed for which n was 1000.

Production of oil drops. An atomizer, and the ejection by air of oil from the fine end of a glass tube, have been tried for producing drops of the radii required of from 3 to 10μ and larger. In the most effective method found after many experiments a drop of oil is placed on the wires of a fine steel brush; a rotating bar first pushes the wires of the brush back and then suddenly releases them with the production of a copious supply of oil drops of about the desired range of sizes. Care was taken to clean the wire brush thoroughly before the oil to be used was dropped on it.

The operation of producing drops is done in a box vertically above the copper box in which the drops are observed. The tube connecting the two boxes is fitted with a tap which serves to separate drops of different sizes, to protect the air in the copper box from external disturbances, and to allow drops to enter so that they are in focus, as they fall past the microscope objective.

Density. This requires to be known, for an accuracy of 1 in 10,000 in e, to 1 in 5000, and to be measured at two temperatures, preferably in the range 15–25° C. A pyrex sinker was weighed in air, in water, and in the oil the density of which is desired. The high viscosity and expansion of the oils make steady and accurately known temperatures necessary for the measurements.

Table 4

Oil	Density	Temperature °C	Coef. of density change per °C
Castor	0·96166 g./cm.^{3}	17·73	0·000630
Apiezon	0·88762	25·20	0·000641

The densities and temperatures are graphically interpolated from observations over the range 16·85–27·7° C for the first, and 19–29·8° C for the second oil. We are indebted to Mr P. G. Law for taking most of these rather exacting density observations.

Velocities of drops

The distance between consecutive images of a drop taken at intervals of 0·04 sec. are measured with the Zeiss microscope to test if the velocity is

uniform. If it is not, the reason for the departure is investigated and the drop is not used for measuring e. If it is uniform the velocity of the drop is

$$v = fs/Mn,$$

where s cm. is distance on the plate between the first and last of $(n+1)$ consecutive images, $1/f$ sec. is interval between successive exposures, and M is the magnification of the camera. f is taken to be 25, a correction being made for the small departures from this value, and $M = 12{\cdot}001$. As v is observed for freely falling drops, their path is vertical and s is measured in the direction of the drops.

In the case of drops deflected by the electric field two methods were tried in the reduction of the observations. In one the angle, θ, between the direction of the path of the freely falling drops and that of the deflected ones was measured by means of a device constructed from the azimuth bearing and circle of a theodolite which read to 20″ of angle.

In the other the successive distances between the images of the deflected drops were measured by means of the Zeiss travelling microscope, and the terminal velocity of the drops could then be determined. This latter method proved the more satisfactory, and was employed for the results with the apiezon oil. With the castor oil, both methods were employed to enable the angle of the plates from the vertical to be calculated.

Frequency of illumination. The rotating disk could have been driven with a synchronous motor supplied by current from an elinvar tuning fork. For the power needed such equipment would be expensive. The disk is actually driven by means of a synchronous motor connected to the a.c. supply which is subject only to small departures from its average frequency of 50 cyc./sec. For that frequency the disk makes 25 rev./sec.

To measure the actual speed of the aluminium disk forty holes 1 mm. in diameter and 9° apart were drilled in a circle concentric with the axis of rotation. A neon lamp supplied by 1000 cyc./sec. a.c. is viewed through the holes. Since the lamp is extinguished 2000 times/sec. the rate of revolution of the disk, f, is given by

$$f = 25 \pm z/(80t),$$

where z is the number of black bands which appear to pass in t sec., t being measured by a stop-watch. It is to be explained that sharp black bands appear to cross the lamp, and the number crossing per second can readily be counted. The determination of f takes about 3 sec. The correction $z/80t$ is rarely greater than $\pm 0{\cdot}02$ sec.$^{-1}$, and z/t is observed whenever photographs are taken to measure drop velocities.

The 1000 cyc./sec. a.c. is obtained from an elinvar valve-maintained tuning fork made by Sullivan. The output from the fork is connected to a power amplifier, the output of which is connected to the neon lamp. We are indebted to Mr Frank Kerr, M.Sc., for designing and constructing the amplifier.

Calibrations

Screw of measuring microscope. The Zeiss microscope used to measure photographs of oil drops, etc. consists of a microscope with a 3 × objective and 5·5 × eyepiece, which is moved relative to the plate, being measured by a 50 mm. screw of 1 mm. pitch. A telemicroscope was used in calibrating the screw to read the drum to 0·0001 mm. Each millimetre of the pitch was compared with 1 mm. of a Grayson ruling on glass. Some 600 observations were taken. They showed that from 21 to 50 mm. the pitch was most regular, and only this part of the screw was used. Over this section of it

$$\text{Mean pitch of screw} = 0{\cdot}9982, \ \pm 0{\cdot}00026 \text{ mm. (Grayson).}$$

What part of this small error was due to errors of setting and what to errors of pitch could not be distinguished. It is therefore assumed the pitch is constant to $0{\cdot}3\,\mu$.

The screw was compared with a Nickel metre of H section of the highest quality made by the Société Genèvoise for which there is a certificate of the International Bureau of Weights and Measures. The mean of twelve observations is given in table 5.

Table 5

Temp. °C	Nickel metre mm.	Length calculated from certificate cm.	Length read on Zeiss microscope
20	500–525	2·4999	2·5000 ± 0·0002

Standard of potential. As explained above the potential difference, V, applied to the condenser plates is measured in each experiment in terms of the e.m.f. of a Weston cell. The cell used, supplied by the Cambridge Instrument Co., has an N.P.L. certificate No. 57384, which gives for the e.m.f. of the cell

$$e_{25} = 1{\cdot}01805 \text{ international volts.}$$

This cell was compared on 22 November 1938 with two other cells with N.P.L. certificates. Such a comparison only shows whether the relative

values of the e.m.f.'s of the cells has remained constant, which it has as set out in table 6.

TABLE 6

N.P.L. certificate cell no.	57380	57384	58504
e_{25} international volt	1·01805	1·01805	1·01815
Relative 22 Nov. 1938	1·01805	(1·01805)	1·01815

These cells were again carefully compared by Mr P. G. Law, in November 1939, and the same results were obtained.

Elinvar electrically driven tuning fork. This 1000 cyc./sec. fork made by Sullivan has an N.P.L. certificate of its frequency when used with a certain valve. The valve having been replaced by another of the same type, Mr Gill confirmed the previous calibration. The fork was connected to a General Radio synchronous motor and counter in the form of a clock, and compared with the hourly wireless time signals over a period of 24 hr. The frequency of the fork and associated valve

$$= 1000{\cdot}041 \pm 0{\cdot}002 \text{ cyc./sec. at } 16^\circ \text{ C.}$$

The N.P.L. certificate gives the temperature coefficient as 4 in $10^6/^\circ$ C.

EXAMPLE OF CALCULATION

The calculation of e for plate 4, which is a general case, is as follows:

$$ne = 9\sqrt{2}\,.\,\pi g^{-\frac{1}{2}}(\rho-\sigma)^{-\frac{1}{2}}\,\eta^{\frac{3}{2}}dc \times 10^{-8}\,pq^{-1}\,i^{-1}\,\Omega^{-1}M^{-\frac{3}{2}}f^{\frac{3}{2}}s_g^{\frac{1}{2}}s_x,$$

where ρ = density of the oil (0·89139 g./cm.^{3}),

σ = density of the air (0·0012 g./cm.^{3}),

η = viscosity of air ($1{\cdot}8163 \times 10^{-7}$ c.g.s.),

d = distance between the plates (0·49852 cm.),

c = ratio of e.s.u. to e.m.u. of potential difference ($2{\cdot}99776 \times 10^{10}$),

q = conversion factor from international to absolute units of potential (1·00034),

i = current through resistance (0·58660 mA),

Ω = resistance in parallel with plates (3, 002, 958 ohms),

M = magnification due to camera (12·001),

f = speed of rotation of disk (24·991 cyc./sec.),

s_g = mean distance between images of the drop—no electric field (0·21636 cm.),

s'_g = mean distance between images of the drop in the vertical direction with electric field (0·21720 cm.),

s_x = mean distance between images of the drop in the direction of the electric field (0·10317 cm.),

s_r = mean distance between images of the drop in the direction of deflexion (0·24031 cm.).

$$s_x = s_r \sin\theta/\cos\delta,$$

where
$$\theta = \arccos s'_g/s_r,$$

and δ is the angle between the direction of the electric field and the horizontal. It is given by

$$\tan\delta = (s_g - s'_g)/s_r \sin\theta,$$

s_r being the mean distance between successive images after deflexion, and s'_g being its vertical component. The observed values of s_g, s_r, s'_g for plate 4 are as follows:

s_g: 0·2164, 0·2166, 0·2162, 0·2163, 0·2164, 0·2163, 0·2163, 0·2164; mean 0·21636 cm.

s_r: 0·2403, 0·2399, 0·2408, 0·2401, 0·2403, 0·2410, 0·2398; mean 0·24031 cm.

s'_g: 0·2175, 0·2170, 0·2170, 0·2178, 0·2171, 0·2170, 0·2169, 0·2166, 0·2174, 0·2177, 0·2172, 0·2174; mean 0·21720 cm.

s_x is calculated from the formula above.

Substituting in formula (15), we have

$$ne = 402{\cdot}552 \times 10^{-10} \text{ e.s.u.}$$

For $n=$	81	82	83
$e_1^{\frac{2}{3}}$	62·742	62·231	$61{\cdot}730 \times 10^{-8}$
Residual v^2	261,121	0	$249{,}000 \times 10^{-22}$

The value of n is evidently 82, and for that value $e_1^{\frac{2}{3}} = 62{\cdot}231 \times 10^{-10}$ e.s.u. The values 81 or 82 for n give a v^2 very much larger than the largest value, 841, found in table 8.

Discussion of results

In deducing the values of ne given in table 8, the provisional value of η_{23} used was $1834{\cdot}0 \times 10^{-7}$ c.g.s. unit, and the value of η at other temperatures was calculated by Sutherland's formula using $c = 115$. The values of e and m in the expression

$$e_1^{\frac{2}{3}} = e_0^{\frac{2}{3}} + m/pa,$$

which gave the best fit with the values of $e_1^{\frac{2}{3}}$ and $1/pa$ contained in table 8, were calculated by the method of least squares. Writing the above

$$y = a_{01} + a_{11}x,$$

then
$$a_{01} = \frac{(\Sigma x)(\Sigma xy) - \Sigma y\,.\,\Sigma x^2}{n(\Sigma x^2) - (\Sigma x)^2},$$

and
$$a_{11} = \frac{\Sigma x\,.\,\Sigma y - n\Sigma xy}{n(\Sigma x^2) - (\Sigma x)^2}.$$

These relations give for

$$e_0^{\frac{2}{3}} = 61{\cdot}456 \pm 0{\cdot}0011 \times 10^{-8} \text{ e.s.u.},$$

$$m = 0{\cdot}03790 \pm 0{\cdot}00044 \times 10^{-8} \ (p \text{ in cm. of mercury, } a \text{ in cm.}).$$

In calculating e_0 from the above value of $e_0^{\frac{2}{3}}$ it has to be corrected to $\eta_{23} = 1830 \times 10^{-7}$, and the value given by our observations for the electronic charge is

$$e_0 = (4{\cdot}8020 \pm 0{\cdot}0013) \times 10^{-10} \text{ absolute e.s.u.}$$

e by oil-drop method. In the following table the value of e found by other observers assuming that $\eta_{23} = 1830 \times 10^{-7}$ is given. Millikan's value is also corrected to absolute electrostatic units and his value of m is given. Where errors are given against an observer they are calculated by the expression for a_{01} and a_{11} given above.

			Errors		
			$\times 10^{10}$	$\times 10^{10}$	$\times 10^{8}$
Observer	$e_0 \times 10^{10}$	$m \times 10^{8}$	e_0	mean e_1	m
Authors	4·8020	0·03777	± 0·0013	± 0·00036	± 0·00044
Millikan	4·7992	0·03784	± 0·0037	± 0·0014	± 0·00013
Bäcklin and Flemberg	4·7811	0·0372	± 0·018	± 0·004	± 0·0014
Ishida and others	4·8350	0·0361			

Our value of e differs from Millikan's by $0{\cdot}0028 \times 10^{-10}$, and this is less than his error. Our and Millikan's value of m only differ by $0{\cdot}00007 \times 10^{-8}$. Bäcklin and Flemberg's e and ours differ by $0{\cdot}021 \times 10^{-10}$, but their error is $\pm 0{\cdot}018 \times 10^{-10}$. Ishida and others value of e is much higher than any of the other three values of e. These investigators assume that the density of their oils changed slowly with time when in the drop form, even before the light from the arc fell on the drops. Their high result is a consequence of this assumption which does not appear to be justified.

X-ray method. In this method of determining the electronic charge, e e.s.u., the wave-length of an X-ray line is measured in cm. and in XU. The

ratio of the XU to the cm. is measured. The XU is conventionally defined as 1/3029·45 of the distance at 18° C between adjacent (100) planes of calcite. The wave-length, λ_g cm., is measured by means of a ruled grating.

Bragg's law, $$n\lambda = 2d_n\{1-4d^2\delta/n^2\lambda^2\}\sin\phi_n,$$

is used to calculate λ in terms of d_n and ϕ_n, the angle of 'reflexion' for the nth order. The second term in the bracket corrects for refraction, where $\mu = 1-\delta$ is the refractive index of the crystal. If $d_\infty = 3029{\cdot}45$ XU, then $d_1 = 3029{\cdot}04$, $d_2 = 3029{\cdot}34$.

The grating space of a crystal, $d_{\infty 18}$, is calculated by means of the following relation, to the mass of the half-molecule which is associated with that cell when it is a rhombohedron:

$$d_{18}^3\phi(\beta)\,\rho_{18} = M/2N = Me/2F,$$

and $$e = 2d_{18}^3\phi(\beta)\,\rho_{18}F/M,$$

where M is the molecular weight, ρ_{18} g.cm.$^{-3}$ the density of the crystal, $d_{18}\,\phi(\beta)$ the volume of the unit rhombohedron cell, N the number of molecules per g.mol., and F e.s.u./g. equivalent to the Faraday, e e.s.u. the electronic charge.

In the reduction of the X-ray determinations of e we use the following values of the above constants:

Symbol	ρ_{18}	$\phi(\beta)$	δ	M	c	$F\times10^{-14}$
Value	2·710467	1·09594	$10{\cdot}2\times10^{-6}$	100·09	$2{\cdot}99776\times10^{10}$	2·89225
Error in 10^4	±0·1	±0·1	—	±0·5	±0·13	±1·1
	Bearden	Birge	Cooksey	1939 at. wt.	Birge	Int. Bur. W. and M.

F is calculated from the definition of the ampere, i.e. 0·001118 g. of silver per coulomb and 1 int. coulomb = $0{\cdot}99986\times10^{-1}$ e.m.u. The third line gives the errors as estimated by Birge and in parts per 10,000. We will calculate, e, the charge for which $d_{\infty 18} = 3{\cdot}02945\times10^{-8}$ cm. It is

$$e_0 = 4{\cdot}77306\times10^{-10} \text{ e.s.u.}$$

In the expression used to calculate e_0, e_0 is proportional to d^3, it follows from this that

$$e = e_0(\lambda_g/\lambda_c)^3 = 4{\cdot}77306\times10^{-10}(\lambda_g/\lambda_c)^3,$$

which is a convenient relation for the reduction of measurements of e by the X-ray method. It is to be noted that the error in e is that contributed by the constants which enter into its calculation. F and M contribute the largest

TABLE 7

Plate	Temp. °C	$\eta \times 10^{-4}$ c.g.s.	ρ_t c.g.s.	p cm. Hg	$a \times 10^{-4}$ cm.	$1/pa$	i mA	Ω ohms resistance	f rev./sec.	M magnification
1	15·27	1·7950	0·96339	75·59	9·570	13·824	0·58183	3,803,608	25·018	11·983
2	22·10	1·8296	0·89161	75·40	7·4124	17·892	0·74110	3,002,799	25·007	12·001
3	16·51	1·8024	0·96242	74·76	7·0678	18·925	0·58045	3,803,552	25·000	11·983
4	19·34	1·8163	0·89139	75·30	6·4925	20·455	0·58660	3,002,958	24·991	12·001
5	12·70	1·7838	0·96482	76·68	6·3267	20·613	0·58653	3,803,724	25·018	11·983
6	19·35	1·8163	0·89138	75·30	6·3264	20·992	0·58900	3,002,957	25·016	12·001
7	18·90	1·8141	0·89167	75·80	6·168	21·390	0·57884	3,002,977	24·991	12·001
8	22·02	1·82925	0·88967	75·25	5·8672	22·650	0·58560	3,002,805	24·992	12·001
9	22·10	1·8296	0·88962	75·40	5·7116	23·220	0·74080	3,002,799	25·022	12·001
10	24·71	1·8422	0·88795	75·50	5·6768	23·332	0·58220	3,002,596	24·985	12·001
11	24·71	1·8422	0·88795	75·50	5·1285	25·826	0·58220	3,002,596	24·985	12·001
12	17·76	1·8086	0·89240	74·80	5·048	26·486	0·60850	3,003,024	25·013	12·001
13	18·70	1·8133	0·89180	75·80	4·3307	30·464	0·57814	3,002,985	25·010	12·001
14	24·71	1·8422	0·88795	75·50	3·973	33·299	0·58220	3,002,596	25·000	12·001
15	15·16	1·7959	0·96327	75·30	3·7873	35·066	0·58163	3,803,613	25·010	11·983
16	17·02	1·8049	0·96210	74·71	3·379	39·607	0·58048	3,803,529	25·003	11·983

TABLE 8

Plate	s_g mm.	s_g' mm.	s_r mm.	$\theta°$	$\times 10^{10}$ e.s.u. ne_1	n	$\times 10^{8}$ e.s.u. $e_1^{\frac{2}{3}}$	$\times 10^{8}$ e.s.u. $e_1^{\frac{2}{3}}$ calc.	$\times 10^{23}$ e.s.u. v^2
1	5·1211	5·0286	5·610	26° 19′	1136·58	233	61·968	61·980	144
2	2·7898	2·8132	4·7113	53° 20′	1351·72	276	62·132	62·134	4
3	2·7808	2·7418	3·4937	38° 18′	735·09	150	62·158	62·173	225
4	2·1636	2·1720	2·4031	25° 20′	402·552	82	62·231	62·231	0
5	2·2554	2·3089	2·492	22° 10′	280·023	57	62·261	62·237	576
6	2·0494	2·0541	2·4592	33° 21⅓′	515·771	105	62·257	62·252	25
7	1·95214	1·9370	2·0058	15° 3′	196·642	40	62·289	62·267	484
8	1·7485	1·7517	1·9725	27° 22′	324·531	66	62·298	62·314	256
9	1·655	1·663	2·5729	49° 44′	541·557	110	62·350	62·336	196
10	1·6196	1·6155	1·7850	64° 52′	265·904	54	62·357	62·340	289
11	1·3551	1·3628	2·562	57° 22′	695·399	141	62·423	62·435	144
12	1·3120	1·3120	1·4382	24° 11′	172·762	35	62·458	62·460	4
13	0·9626	0·9328	1·3860	47° 42′	272·292	55	62·582	62·611	841
14	0·7962	0·7957	1·6046	60° 15⅓′	342·597	69	62·703	62·717	225
15	0·8018	1·0014	3·787	74° 40′	662·083	133	62·812	62·785	729
16	0·63455	0·65759	7·1280	84° 48′	1153·913	231	62·957	62·957	0

errors, and using Birge's estimates (see table above) the error in e is 1·1 in 10,000.

In the measurements of Söderman and Bäcklin the X-ray line used was the Al $K_{\alpha 12}$ line, and in Bearden's the α_1, α_2 and β lines of the K spectra of Cu and Cr.

Söderman used a concave grating with glancing incidence but with the photographic plate normal to one of the diffracted rays. His value of λ_g is measured by comparing the Al $K_{\alpha 12}$ line (8·34 A) in its 8th, 9th and 10th orders relative to a number of 1st order of certain intense and well-defined Al spark lines, the wave-length of these having been compared by Söderqvist with oxygen lines in the ultra-violet optical spectrum. This is an application of Rowland's method of coincidence in different orders, and its accuracy is only limited by the homogeneity of the lines used, and resolving power of the glancing incidence concave grating spectrometer. Söderman mentions $\lambda/\delta\lambda = 1200$ as estimate of the resolving power of his spectrometer. He found Al $K_{\alpha 12}$ $\lambda_g = 8{\cdot}3401 \times 10^{-8}$ cm. Söderman and Bäcklin use for the crystal wave-length, λ_c, of the Al $K_{\alpha 12}$, the value found by Larsson

$$\lambda \,\mathrm{Al}\, K_{\alpha 12} = 8321{\cdot}35\,\mathrm{XU}$$

using a mica crystal, which did not resolve the $\alpha 12$ doublet.

A comparison of Haglund's (1935) measurements of λ_c for the Al $K_{\alpha 12}$ doublet shows that its width and asymmetrical structure set the limit to the precision of the measurement of λ_c for this doublet. He found, using the crystals stated, the effective wave-length to be

$$\text{Mica} \quad \lambda_c = 8322{\cdot}93\,\mathrm{XU};$$

$$\text{Gypsum} \quad \lambda_c = 8322{\cdot}87\,\mathrm{XU}.$$

A quartz crystal resolved the doublet and gave

$$\lambda_{\alpha_1} = 8322{\cdot}18, \quad \lambda_{\alpha_2} = 8324{\cdot}62\,\mathrm{XU}.$$

Larsson's value is less than the smallest of Haglund's values.

Bearden (1931*a*, 1935) has published two determinations of λ_g/λ_c for K lines of Cu and Cr. In his 1935 paper he describes isolating the Cu $K_{\alpha 1}$ line with a double crystal monochromator, and measuring λ_g with a ruled grating. An accurate circle is used to measure angles of incidence and diffraction in the second order. These ranged from 480″ to 1580″, and the error of reading, etc. was about $\pm 0{\cdot}25''$. Twelve observations gave $\lambda_g = 1{\cdot}5406 \times 10^{-8}$ cm.

The following closely agreeing values of the crystal wave-length of λCu $K_{\alpha 1}$ are available:

1537·396 XU, Siegbahn,

1537·395 XU, Wennerlof,

1537·399 XU, Bearden and Shaw.

The mean is 1537·397 and

$$\lambda_g/\lambda_c = 1540{\cdot}6/1537{\cdot}397 = 1002{\cdot}08,$$

i.e.
$$1\ \mathrm{XU} = 1{\cdot}00208 \times 10^{-11}\ \mathrm{cm.}$$

and
$$e = 4{\cdot}8029 \times 10^{-10}\ \mathrm{e.s.u.}$$

The author does not reduce his 1935 observations separately, but combines them in a weighted mean with his 1931 measurements and with those of Söderman and Bäcklin. This mean is = 1·00248, which is the mean of his 1931 measurements as given in his 1935 paper. With much hesitation we calculate separately his 1931 and his 1935 values of e, believing the considerable improvements which the author has made in the 1935 measurements justify their separate reduction. In 1931 he used Cu and Cr K_β lines and the Cu and Cr $K_{\alpha 12}$ unresolved doublet and found λ_g/λ_c as a mean (of over 700 observations) for these lines = 1·00222, and therefore

$$e = 4{\cdot}8049 \times 10^{-10}.$$

Electronic charge X-ray method

Observer	X-ray line	λ_g cm.	λ_c XU	λ_g/λ_c	e e.s.u.
Söderman	Al $K_{\alpha 12}$	8340·1	8321·35	1·00225	$4{\cdot}805_4$
Bäcklin	Al $K_{\alpha 12}$	8339·5	8321·35	1·00218	$4{\cdot}804_4$
Bearden 1935	Cu K_{α_1} etc.	1540·6	1539·397	1·00208	$4{\cdot}802_9$
Bearden 1931	Cu and Cr K	—	—	1·00222	$4{\cdot}804_9$

Mean value $e = (4{\cdot}804_4 \pm 0{\cdot}0007) \times 10^{-10}$.

Conclusion

The value of e by the X-ray method is $(4{\cdot}8044 \pm 0{\cdot}0007) \times 10^{-10}$ e.s.u., whilst that obtained by the oil-drop determinations of Millikan and the authors is $(4{\cdot}8007 \pm 0{\cdot}002) \times 10^{-10}$ e.s.u., if η_{23} is assumed to be 1830×10^{-7} c.g.s. units. The main uncertainty in the value of e as found by the drop method is in the value which is used for viscosity of air. The value of η obtained from the weighted mean of nine separate experiments is

$(1830 \pm 2{\cdot}5) \times 10^{-7}$ c.g.s. units, and as it enters to the $\frac{3}{2}$ power in the expression for e the final error in e is 1 in 500. This error far exceeds the difference between the X-ray and oil-drop values of e given above. It will be noticed that the other errors entering into the determination of e by the oil-drop method can be made very small and have been estimated at less than 1 part in 3000 in the experiment described in this paper. A determination of the viscosity of air is being attempted in this laboratory using a method suggested by Fabry and Perot in which there is lamina flow of air between optically flat disks. It is hoped that improved precision will be obtained by this method.

The oil-drop method can be used to determine the viscosity of air if the X-ray value of e is assumed. Thus if e be taken as

$$(4{\cdot}8044 \pm 0{\cdot}0007) \times 10^{-10} \text{ e.s.u.},$$

the mean result of Millikan's and of the authors' experiments gives

$$\eta_{23} = (1830{\cdot}9 \pm 0{\cdot}5) \times 10^{-7} \text{ c.g.s. units},$$

whilst the authors' results alone give

$$\eta_{23} = (1830{\cdot}6 \pm 0{\cdot}4) \times 10^{-7} \text{ c.g.s. units}.$$

It might be of interest to note that if $e = 4{\cdot}802 \times 10^{-10}$, $h/e = 1{\cdot}3765 \times 10^{-17}$ (a mean of h/e determined by the X-ray method and by Wien's law), and $c = 2{\cdot}99774 \times 10^{10}$, then $1/\alpha = 136{\cdot}76$. If, however, Rydberg's constant $R(109{,}737{\cdot}43)$ and $e/m(1{\cdot}7589 \times 10^{7})$ are substituted in the formula $1/\alpha = [c/4\pi R(e/m)\,e]^{\frac{1}{3}}$, it becomes 137·03 which is nearly integral. Also if the above values of e, e/m and R are substituted in Rydberg's formula, then $h/e = 1{\cdot}3792 \times 10^{-17}$ or 1 in 500 larger than the weighted mean of the recent values of h/e. The recent work of Ohlin (1940) indicates that the previously determined values of h/e by the short wave-length limit of the continuous X-ray spectrum might be low owing to the fact that the X-ray tubes had not been evacuated to a sufficiently high degree. If this is so, then it is possible that both Eddington's relation and Rydberg's formula will be experimentally satisfied.

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