negative and therefore the fluctuation of the readings reduced to normal pressure is only statistical. There is no indication of a systematic variation due to causes other than variation of pressure.

Since the statistical fluctuation is large it cannot be excluded that systematic variations with amplitudes much smaller than the barometric fluctuation occur but are masked by the statistical fluctuation. Variations with an amplitude comparable to the fluctuations due to pressure changes would have been detected.

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References

Some aspects of the production of mesons and the barometer effect of penetrating showers

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It is shown that the large barometer effect of penetrating showers can be understood in terms of an exponential absorption of the primaries giving rise to the showers. A number of other experimental results favour such an assumption. It is shown further that the exponential absorption of the primaries is compatible with the theory of Hamilton, Heitler & Peng when account is taken of fluctuations.

1. Introduction

From a large number of observations of penetrating showers Jánossy & Rochester (1944) have found the barometer coefficient of penetrating showers to be of the order of 10 % per cm. Hg. This barometer effect corresponds to an absorption of about 1 % per g. of the atmosphere. Jánossy & Rochester (1943 a) have found, however, that the absorption of penetrating showers is only 0·1 % per g. of lead. Thus the absorption in air as deduced from the barometer effect appears to be ten times larger than that in lead.

As penetrating showers probably consist mainly of mesons, one would expect an absorption according to mass, and it would be very difficult to understand a much stronger absorption in air than in lead. The instability of the meson could hardly
account for this behaviour in terms of decaying mesons, for the decay of mesons would give rise to a temperature rather than to a barometer effect.

The difficulty can be overcome by assuming that the barometer effect is not so much due to the absorption of penetrating showers in air but to the absorption of the primaries producing them. The large barometer effect and the small absorption of penetrating showers in lead can be understood in the following way.

Consider primaries which have just sufficient energy to penetrate the atmosphere at the normal pressure $P$. A fraction $\delta N$ of the primaries will be absorbed in $T$ (figure 1) giving rise to penetrating showers and thus will be recorded. If an extra absorber $T_1$ is placed on top of $T$, the fraction $\delta N$ will give rise to penetrating showers in $T_1$ and not in $T$, but as the showers are only slightly absorbed in the absorber $T$ they are almost as likely to be recorded when produced in $T_1$ as when produced in $T$; thus the absorber $T_1$ will have little effect on the observed intensity.

Compare the effect of the extra absorber $T_1$ with the effect of an equivalent increase $P_1$ of the barometric pressure. $P_1$ should be so chosen that the extra air above the arrangement due to the pressure $P_1$ should have the same mass per unit area as the absorber $T_1$.

The effect of the increase of air pressure on the coincidence rate is very different from the effect of the absorber $T_1$. The fraction $\delta N$ which could just reach the top of the apparatus when the air pressure was $P$ will, when the pressure increases to $P + P_1$, be stopped in that layer above the surface at which, after the change, the pressure has now become $P$. Because of the low density of air this layer will have a height of the order of 100 m. for average changes of $P$. Showers produced at these heights will be very diffuse when they reach the apparatus and will therefore usually not be recorded (figure 1).
It follows that the dense absorber $T_1$ placed close above the apparatus produces a decrease according to the absorption of the showers in $T_1$, while an extra mass of air above the apparatus produces a decrease according to the absorption of the primaries in the air.

The large barometer coefficient seems therefore to indicate that the primaries giving rise to penetrating showers are strongly absorbed in air. The absorption coefficient of the primaries which is required to account for the observed barometer effect is of the order

$$1/\mu_P = 100 \text{ g. per cm.}^2.$$  \(1\)

2. Absorption of the Primaries (Experimental)

There is some experimental evidence to show that the primaries giving rise to penetrating showers are absorbed with an absorption coefficient $\mu_P$ of the order given in (1).

(a) The transition effect of penetrating showers shows saturation at approximately 5 cm. of lead. This saturation can be accounted for either by assuming that a large fraction of the incident primaries are absorbed in 5 cm. of lead or by assuming that the primaries are producing a number of penetrating secondaries in 5 cm. of lead sufficient to discharge the shower recording arrangement.

The second alternative was put forward by Hamilton, Heitler & Peng (1943) [quoted in the following as H.H.P.] and by myself (1943), but the barometer effect seems to favour the first interpretation. It will be shown in the following that the two points of view can be reconciled.

(b) It was shown by Jánossy & Rochester (1943) that part of the penetrating showers is produced by neutral particles. Further, Rossi & Regener (1940) and Jánossy & Rochester (1943b) have observed a penetrating non-ionizing radiation with a mean free path of about 100 g. per cm.$^2$. It is very probable that both of these neutral radiations consist of neutrons. The more energetic neutrons may be able to give rise to penetrating showers, while the bulk of the neutrons having too little energy for producing penetrating showers may give rise only to ionizing secondaries.

The intensity found by Rossi & Regener at Mt Evans (4500 m. above sea-level) was estimated by us to be 30–60 times larger than the intensity of neutral particles in Manchester. The ratio of the intensities suggests an absorption coefficient of the order of (1).

3. Absorption of Primaries (Theoretical)

(a) Two main types of absorption phenomena have to be considered:

(i) Catastrophic absorption. The primary loses all its energy in its first collision. The absorption law in this case is essentially exponential and does not depend on the incident spectrum, provided the collision cross-section does not vary much with energy.
(ii) **Range absorption.** The primaries lose energy gradually. The more energetic primaries penetrate deeper and the absorption law is a function of the energy spectrum.

Intermediate processes may also occur.

Consider the case in which the primaries lose a constant amount $\beta$ of energy per gram of absorber and assume $\beta$ to be independent of energy. Assume an incident spectrum of the type

$$S(E) = (E_0/E)\gamma \quad (E > E_0).$$

(2)

The incident spectrum of the primaries is likely to be of this type, since it is known that the meson spectrum is between wide limits of this type, and it is not likely that the spectrum of the primaries producing the mesons is very different from the secondary meson spectrum.

The energy spectrum at the depth $x$ below the top of the atmosphere will then be

$$S(E, x) = E_0\gamma/(E + \beta x)\gamma \quad (E > E_0),$$

(3)

and, neglecting particles of energy less than $E_0$, the absorption function is given by

$$I(x) = E_0\gamma/(E_0 + \beta x)\gamma.$$  

(4a)

(b) The barometer coefficient near sea-level, i.e. at $x = H$, is given by

$$B = (H/I)(dI/dx)_{x=H} 1.33 \% \text{ per cm. Hg;}$$

(5)

if $\beta H \gg E_0$ this reduces to

$$B = -1.33\gamma \% \text{ per cm. Hg (near sea-level).}$$

(5a)

As $\gamma$ is of the order of 2 the barometer coefficient is of the order of 3 % per cm. Hg. Such a value is observed for the bulk of cosmic rays.

Note that the coefficient $B$ in (5a) is independent of the specific loss $\beta$. Therefore the barometer coefficient for the primaries producing the mesons should also be of the order of 3 % if they lose energy gradually.

(c) The barometric pressure at Mt Evans is 0.62 atm. Therefore the ratio of the intensities at Mt Evans and at sea-level is from (4a) with $\gamma = 2, E_0 \ll \beta H$

$$I_{\text{Mt Evans}}/I_{\text{sea-level}} = 0.62^{-2} = 2.6.$$  

(6)

This value is far too small to be compatible with the estimated ratio from 30 to 60.

Thus, assuming continuous absorption of the primaries, one fails to account either for the ratio of neutron intensities on Mt Evans and at sea-level or for the large barometer coefficient found at sea-level.

Both effects would be accounted for immediately by assuming catastrophic loss. Replacing equation (4a) by

$$I(x) = \exp \left[ -10x/H \right],$$

(4b)

the modified equations (5) and (6) give numerical values in good agreement with the experiments.
Using (4b) instead of (4a) it becomes also somewhat easier to account for the transition effect of penetrating showers.

It seems, therefore, that the experimental facts are better accounted for in terms of catastrophic absorption than in terms of range absorption.

4. COMPARISON WITH THE THEORY OF HAMILTON, HEITLER AND PENG

In this theory it is assumed that the incident primaries are protons and possibly neutrons. It is shown that these fast heavy particles lose energy in inelastic collisions with protons and neutrons resulting in the emission of mesons. The rate of energy loss for primaries of energy $E$ is given by

$$\beta_p = 0.9 \log_e (0.3E).$$

(All energies are given in units of $\mu c^2$, $\mu$ rest mass of the meson.) At first sight one should expect absorption according to range as a consequence of equation (7). In the following it will be shown, however, that owing to the large fluctuations of energy loss connected with the mechanism proposed by H.H.P. the absorption function is of the range type only for very large thicknesses, but for smaller thicknesses the absorption curve is more nearly represented by an exponential function.

Though the number of single encounters of a primary on its way through the atmosphere is large, these encounters are not distributed at random for, as I have shown, a primary particle passing through a single nucleus is likely to collide with several nuclear particles (Jánossy 1943). As a result of this crowding of the single collisions a primary is likely to lose large amounts of energy in a few collisions.

The energy required for a primary to penetrate the atmosphere is found with the help of equation (7) to be of the order of 7000, and it is estimated that this energy is lost in about 10 energetic collisions.

Primitives with energies below $7000/10 = 700$ are therefore expected to be stopped in the first collision. A primary with energy much less than 700 is thus only likely to reach sea-level if it happens not to collide with any nucleus on its way. The probability of a primary not colliding once along $H$ when its mean free path is $H/10$ is $e^{-10} \approx 1/20,000$.

Thus one out of 20,000 low-energy primaries will reach sea-level. The fraction of the primitives having sufficient energy to penetrate the atmosphere without fluctuation is $(E_0/7000)^\gamma$, where $E_0$ is the smallest incident energy sufficient for producing penetrating showers at sea-level. Assuming $\gamma = 2$ and $E_0 = 50$ this fraction is also $1/20,000$. Thus the number of low-energy primaries having reached sea-level after having suffered average loss is of the same order as the number of high-energy primaries reaching sea-level after having suffered average loss.

Roughly speaking the primaries behave as if they consisted of two independent components, one component being absorbed exponentially, the other having a range absorption curve. Down to a certain depth the exponential component is
preponderant, while below this depth the fluctuations become unimportant and the absorption curve corresponds to the average loss.

Assuming that the exponential part of the absorption curve persists to a depth somewhat below sea-level, all experiments described in the last section can be accounted for in terms of the theory of H.H.P.

As the absorption with fluctuation is a rather complex process, it seems worth while to confirm these qualitative considerations by a more detailed calculation.

5. Mathematical treatment of the fluctuation of energy loss

Diffusion equation

In the following the total energy loss will be subdivided into two parts: (1) large losses: write for the probability of an energy loss in a single nuclear collision exceeding $y$ along the path $dx$, $L(y)dx$; (2) small losses: it will be assumed that the loss along a path $dx$ is equal to $\beta dx$, $\beta$ being independent of the energy. The diffusion equation giving the change of the differential energy spectrum $D(E,x)$ can be written as

$$\frac{\partial D(E,x)}{\partial x} = -L(0)D(E,x) + \beta \frac{\partial D(E,x)}{\partial E} - \int_0^\infty D(E+y,x) \frac{dL(y)}{dy} dy. \quad (8)$$

Integrating by parts equation (8) can be transformed into

$$\frac{\partial D(E,x)}{\partial x} = \beta \frac{\partial D(E,x)}{\partial E} + \int_0^\infty \frac{\partial D(E+y,x)}{\partial E} L(y)dy; \quad (9)$$

a particular solution of this equation is

$$D_e(E,x) = \exp \left[ -\epsilon E - \lambda(\epsilon)x \right] \quad (10)$$

with

$$\lambda(\epsilon) = \beta \epsilon + \epsilon \int_0^\infty e^{-y\epsilon} L(y)dy. \quad (11)$$

The general solution of equation (8) is therefore

$$D(E,x) = \int_0^\infty a(\epsilon) \exp \left[ -\epsilon E - \lambda(\epsilon)x \right] d\epsilon, \quad (12)$$

where $a(\epsilon)$ has to be chosen so that the solution obtained satisfies a boundary condition, that is $a(\epsilon)$ has to satisfy equation (13),

$$D(E,0) = \int_0^\infty a(\epsilon) \exp (-\epsilon E) d\epsilon. \quad (13)$$

The general expression for $a(\epsilon)$ can be obtained from equation (13) by means of the Mellin transform. We are, however, only interested in the special case where the incident spectrum obeys a power law. In this case we have

$$D(E,0) = \gamma E_0^{-\gamma}/E^{\gamma+1} \quad \text{and} \quad a(\epsilon) = (\epsilon E_0)^\gamma/\gamma(\gamma - 1). \quad (14)$$
The solution of equation (8) satisfying the boundary condition (14) is therefore

\[ D(E, x) = \frac{1}{(\gamma - 1)!} \int_0^\infty (\epsilon E_0)^\gamma \exp \left[ -\epsilon E - \lambda(\epsilon) x \right] d\epsilon. \]  

Integrating equation (12) with respect to \( E \) from \( E_0 \) to infinity one obtains the integral spectrum. It is found that

\[ S(E_0, x) = \frac{E_0^\gamma}{\gamma !} \int_0^\infty (\epsilon E_0)^\gamma \exp \left[ -\epsilon E_0 - \lambda(\epsilon) x \right] d\epsilon. \]

The values of \( S(E_0, x) \) as obtained from equation (16) depend on the choice of the function \( L(y) \). Before evaluating equation (16) for the case of energy loss due to meson production two extreme cases may be considered.

(a) \( L(y) = \text{const.} = 1/\mu_P \). This represents a case in which the primaries are stopped in the first collision independently of their energy. It is found with the help of equation (11) that

\[ \lambda(\epsilon) = \beta \epsilon + 1/\mu_P; \]  

introducing this expression into equation (16) one finds

\[ S(E_0, x) = \frac{E_0^\gamma}{(E_0 + \beta x)^2} \exp \left[ -\beta x \right]. \]

Thus the absorption is essentially exponential. The barometer coefficient as defined in equation (5) will be found to be

\[ B = -1.33(\gamma + H/\mu_P). \]

It can be inferred from this equation that the observed barometer effect can be accounted for by assuming sufficiently large but infrequent losses for the primaries.

(b) \( L(y) \) is very large for small values of \( y \) and is zero for larger values, so that the average rate of loss due to \( L(y) \) is \( \beta_p \). In this case one finds

\[ \lambda(\epsilon) = (\beta + \beta_p) \epsilon; \]  

and

\[ S(E_0, x) = \frac{E_0^\gamma}{(E_0 + \beta + \beta_p x)^2}; \]

thus the absorption is essentially a range absorption; this case is incompatible with experiment.

(c) To treat the case of energy loss according to the theory of H.H.P. assume

\[ L(y) = \begin{cases} 0.9/(y - B) & (y \geq y_1), \\ 10 \times 7 \times 10^{-3} & (y < y_1), \end{cases} \]

with \( y_1 = 320 \) and \( B = 240 \). A justification of this expression will be given in the Appendix. Further, assume \( \beta = 0.02 \), this representing the ionization loss suffered by a proton.
Introducing equation (17) into equation (11)

\[ \lambda(e) = \beta e - 0.9 e e^{-B e} Ei(-(y_1 - B)e), \]  

\[ (11c) \]

\( Ei(-x) \) is the exponential integral. No analytic expression for equation (16) can be obtained in this case, but the integral can be evaluated by the saddle-point method.

The integrand in equation (14) vanishes both for small and large values of \( e \) and has therefore maxima between.

Proceeding in the usual way, write for the integrand on the right-hand side of (16)

\[ \text{integrand} = \exp[-f(e)], \]

\[ (18) \]

\[ f(e) = \epsilon E_0 + \lambda(e)x - \ln(\epsilon E_0); \]

\[ (19) \]

saddle points are defined by

\[ f'(e) = E_0 + \lambda'(e)x - \frac{1}{e} = 0 \quad \text{for} \quad e = e_0. \]

\[ (20) \]

It is convenient to represent all quantities as functions of the independent parameter \( e_0 \). Thus it is

\[ x(e_0) = \frac{1}{\frac{e_0}{\lambda'(e_0)}}; \]

\[ (21) \]

\( x \) is plotted as function of \( e_0 \) in figure 2. This graph is rather remarkable. It is seen that in the region

\[ 320 < x < 1600, \]

\[ (22) \]

\( f(e) \) has three extrema, i.e. two minima and a maximum. The value \( S \) of the integral (16) for values of \( x \) in the interval given by equation (22) can be regarded as the sum \( S_1 + S_2 \) of the contributions of the two saddle points.

Figure 2. Position of saddle points.
Consider first the branch corresponding to the larger values of \( x \) and the small values of \( \epsilon_0 \). From equation (11c) the following asymptotic expression valid for small values of \( \epsilon \) can be derived

\[
\lambda'(\epsilon) \approx \frac{\lambda(\epsilon)}{\epsilon} \approx \ln(140\epsilon).
\] (23)

Introducing this into equation (18) and carrying out the integration in the neighbourhood of the first saddle point, then the contribution of this saddle is

\[
S_1(E_0, x) = \frac{1}{\pi \epsilon_0^2} \exp \left\{ \frac{x}{1/(-0.9\epsilon_0 \ln(140\epsilon_0))} \right\}
\] (24)

Comparing equations (24) and (7) it is noted that \( x \) is roughly equal to the range of the primaries with energy \( 1/\epsilon_0 \), while \( S_1 \) is roughly the fraction of incident particles with energies exceeding \( 1/\epsilon_0 \). Thus the branch of the curve (figure 2) corresponding to the small values of \( \epsilon_0 \) represents the fictitious component showing range absorption.

![Figure 3. Absorption of primaries in the atmosphere.](http://rspa.royalsocietypublishing.org/downloaded from http://rspa.royalsocietypublishing.org/)

The contribution \( S_2 \) due to the branch corresponding to the larger values of \( \epsilon_0 \) has been evaluated numerically, and it is found to be very nearly exponential in \( x \). Both contributions \( S_1 \) and \( S_2 \) as well as \( S = S_1 + S_2 \) are plotted in figure 3. It is seen from figure 3 that our expectations regarding the nature of the absorption curve in §3 are confirmed by the quantitative treatment. The absorption is purely exponential down to a critical depth \( H_c \), while for greater depth the curve is represented by a power function. The critical depth is, according to figure 3, about 800 g. per cm.\(^2\). This value is somewhat short of the total depth of the atmosphere. Considering, however, the large uncertainties in the numerical values for the cross-sections involved one cannot attach any significance to the exact numerical value of this critical depth.
It can thus be concluded that the exponential absorption of the primaries should persist down to a critical depth which is of the same order as the height of the atmosphere.

6. **Comparison with experiment**

(a) The ratio of the primary intensities on Mt Evans and near sea-level as calculated from figure 3 is about thirty, in agreement with the experimental estimate.

(b) Cloud-chamber photographs at high altitudes have been obtained by Powell (1940, 1941), Hughes (1941), Wollan (1941) and Hazen (1944). These photographs show groups of penetrating particles. No evidence is, however, found for the occurrence of more complicated processes such as one would expect due to energetic heavy particles capable of passing through more than one nucleus. These experimental findings are to be expected according to the present picture. The small groups of mesons can be attributed to the exponential component which must be expected to be preponderant at 3000 m. above sea-level. More complicated penetrating showers must be comparatively rare at these heights, as most of the energetic primaries are contained in the range component which is relatively weak there.

(c) The barometer coefficient near sea-level taken from the slope of the curve in figure 3 is found to be 7% per cm. Hg. This value is somewhat smaller than the observed value. The disagreement is, however, by no means serious, as the barometer coefficient changes very rapidly with depth in the region of the critical depth. Indeed, at heights above the critical depth where the absorption is nearly exponential the barometer coefficient as calculated from figure 3 is 16% per cm. Hg. Along a comparatively short distance the value of the coefficient drops down to about 3%. Thus a slight change in the estimation of $H_c$ would result in a large change of the estimated value of the barometer coefficient at sea-level.

**Barometer effect of extensive air showers**

It can be shown easily that the exponential component consists mainly of low-energy particles, that is, of particles with energies of about 30–130. Extensive air showers observed by counters separated by a few metres have necessarily much higher energies and therefore the exponentially absorbed component cannot be responsible for the production of the extensive air showers.

The proposed theory gives therefore no explanation for the large barometer coefficients of extensive air showers observed by Auger & Daudin (1942) and Cosyns (1940). The large barometer coefficient of extensive air showers may be due to the absorption of the showers themselves and not to the absorption of the primaries producing them. A different view was put forward by Auger & Daudin (1942).

**Energies of penetrating showers**

To account for the barometer effect of penetrating showers near sea-level in terms of the exponentially absorbed component it is necessary to assume that
about half of the showers at sea-level are due to the exponential component. This 
half of the showers should consist of showers with total energies not exceeding 130 
and having often much lower energies.

Though at present no direct evidence supporting this prediction is available, it 
may be pointed out that this is quite compatible with my original estimation of the 
energies of penetrating showers based on observation (Jánossy 1942, page 369).

**Latitude effect of penetrating showers**

If the present interpretation is correct, penetrating showers should show a 
marked latitude effect. All the energies giving rise to the exponential component 
are forbidden at low latitude, and only the component due to high-energy primaries 
should be observed there. If the exponential component amounts to 50% at high 
lattitudes the latitude effect should also amount to 50%.

Further, because of the absence of the exponential component the barometer 
effect at low latitudes should be of the order of 3% only.

The shape of the transition curve for penetrating showers at low latitudes might 
also prove somewhat different from that at high latitudes.

The only penetrating shower experiments carried out at low latitudes are those 
of Wathaghin, Santos & Pompeia (1940). The coincidence rates observed by Watha-
ghin are of the same order as those recorded in Manchester, but as the arrangements 
used are rather different, no quantitative conclusions can be drawn.

Confirmation of these predictions would support the mechanism put forward in 
this paper. On the other hand, should these predictions prove wrong, one would 
be forced to assume that primaries of sufficient energy to penetrate the earth’s 
magnetic field at low latitudes, i.e. primaries largely exceeding 600, still showed 
exponential absorption curve. Such behaviour might be accounted for by assuming 
that high-energy protons and neutrons suffer energy losses greatly exceeding those 
predicted by H.H.P. At present there is, however, no need for such an assumption.

7. **Appendix**

The differential cross-section for the emission of mesons in close collisions is given 
by H.H.P. as

\[ d\Phi = \frac{\alpha}{E^2} dE \]  
(25)

with \( \alpha = 1.5 \times 10^{-24} \text{cm}^2 \). This cross-section can be interpreted schematically by 
assuming that in a collision with the closest approach \( x \) a meson of energy

\[ E = \frac{\alpha}{\pi x^2} \]  
(26)

is emitted, provided \( x \) is not too large and provided \( E \) is less than the primary energy.

Two kinds of collisions have to be considered: (1) collisions where the fast particle 
crosses the actual area of the nucleus; (2) collisions where the fast particle by-passes 
the nucleus. In the second kind of collision the fast particle suffers distant collisions
with all of the nuclear constituents, and it should give rise to the emission of slow mesons. In this case the assumption that the nuclear particles are independent is certainly not correct and the emission of mesons cannot be estimated in a simple way. It seems, however, reasonable to assume that in the case of the distant collision the nucleus acts more like a single body and the collision will be elastic with little loss of energy. We shall therefore neglect the loss of energy due to the distant collisions.

The energy loss in collisions where the primary passes through the nucleus is estimated roughly as follows. The radius $r_A$ of a nucleus of weight $A$ may be assumed as

$$r_A = 0.53(A^{1/3} - 1)(e^2/me^2),$$

(27)
as shown elsewhere (Jánossy 1943). A distance $r_0$ can be so defined that a fast particle traversing a disk with radius $r_0 + r_A$ round the centre of the nucleus suffers in average only one collision having a distance of approach smaller than $r_0$. Ascribing an area $\pi r_0^2$ to each of the $A$ nuclear particles one has

$$A\pi r_0^2 = \pi(r_0 + r_A)^2,$$

(28)

and therefore $r_0 = r_A/3$ for $A = 16$.

Thus a fast particle traversing a nucleus will usually collide with one particle at a distance smaller than $r_0$ and suffer $A - 1$ more distant collisions with the other particles. Averaging over all possible points of intersection of the fast particle with the nucleus I obtained with the help of equation (26) for $A = 16$ for this loss

$$y_1 = 240.$$  

(29)

The energy $y_2$ which is lost in the closest collision has to be added. The probability per nucleus that the distance of this collision lies in the interval $x$, $dx$ is $2\pi A x dx$ with $x < r_0$. Therefore with the help of equation (26)

$$d\Phi_2(y_2) = AAdy_2/y_2^3,$$

(30)
is obtained for the differential cross-section of a loss $y_2$. As the closest collision is assumed to have a distance less than $r_0$ it must be assumed

$$y_2 > a/\pi r_0^2 = 80.$$  

(31)

With the help of equations (28), (30) and (31) one finds for the integral cross-section

$$\Phi_2(y_2) = \pi(r_0 + r_A)^2 80/y_2.$$  

(32)

The total loss suffered is, however, $y = y_1 + y_2$, and therefore the cross-section for the loss of energy exceeding $y$ is

$$\Phi(y) = (r_0 + r_A)^2 80/(y - 240) \quad (y \geq 320).$$  

(33)

Introducing numerical values into equation (33) one obtains finally,

$$L(y) = (N/A)\Phi(y) = 0.9/(y - 240) \quad (y \geq 320).$$  

(34)
Losses smaller than 320 occur according to our picture only when a particle of energy 
$E$ less than 320 hits a nucleus. Such a particle is expected to lose the whole of its 
energy. Thus

$$L(y) = L(320) = 10^{-7} \times 10^{-3} \quad (y < 320).$$  \quad (35)

Equations (34) and (35) are valid for $y \leq E$; they have to be replaced by $L(y) = 0$ for 
$y > E$.

The actual shape of $L(y)$ is not very important for the conclusions given here. It is, however, important for the conclusions that the losses due to the more distant 
collisions shall be neglected.

If one did not neglect the more distant collisions, but assumed that the energy loss 
in distant collisions was still given by the sum of the individual losses with the 
nuclear particles, one would obtain an additional term to $L(y)$. The effect of this 
additional term would be the same as if the ionization loss of the particles was 
increased about ten times. Thus with this extra term no particle with energy less 
than 300 could reach sea-level and therefore the whole exponential component 
would be cut out.

It seems, however, not unreasonable to cut off the small losses as has been done 
above.

I am indebted to Professor W. Heitler for commenting on this paper.

**References**

Hamilton, Heitler & Peng 1943 *Phys. Rev.* 64, 78.
Jánossy 1943 *Phys. Rev.* 64, 345.
Powell 1940 *Phys. Rev.* 58, 474.
Powell 1941 *Phys. Rev.* 60, 413.
Rossi & Regener 1940 *Phys. Rev.* 58, 837.
Wollan 1941 *Phys. Rev.* 60, 532.