A discussion on Units and Standards

21 March 1946

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INTRODUCTION

BY SIR CHARLES DARWIN, F.R.S.

The sequence of papers following, contributed by members of the staff of the National Physical Laboratory, gives an account of most of the units and standards which are studied at the Laboratory. Each paper explains briefly the principal technical points involved, together with a short history. Of necessity a standard is to some extent a convention, and it may therefore be well to introduce the subject by a short review of the legal and conventional authorities on which they depend.

For the metric system the main international authority is the Conférence Générale des Poids et Mesures, at which duly accredited delegates of each of the thirty-two nations subscribing to the 1875 Treaty, the Convention du Mètre, are present, and which, in normal circumstances, meets every six years. The Comité International des Poids et Mesures is the Executive body appointed by the Conférence Générale to manage the affairs of the Bureau International des Poids et Mesures between meetings of the Conférence Générale. It consists of eighteen members, elected by the Conference, each of whom must belong to a different State, but who, as members of the Committee, are no longer representatives only of their individual countries, but of all subscribing countries. The International Committee meets every two years, makes recommendations to the General Conference, or acts on authority delegated to it by the Conference.

There are also three Advisory Committees which deal respectively with electricity, photometry and temperature. These Committees meet whenever necessary and make their recommendations to the International Committee which, if it approves them, offers them to the General Conference for ratification. The original Convention covered length and mass, and matters incidental thereto, such as thermometry and barometry; it was amended in 1921 to bring in the electrical and
photometric units. The headquarters of the organization are at the Bureau International at Sèvres.

Although the countries subscribing to the Convention du Mètre look to the Bureau International as the central authority for all measures relating to the metric system, this body has no direct legal authority in any individual country. It may be well, therefore, to touch on the legal situation in Britain. Our standards go back a long way—there are still in existence standards authorized by Henry VII, Henry VIII and Elizabeth—and due regard was paid by successive administrations to the need of properly legalized standards for purposes of trade and industry. From the scientific angle, however, it must be admitted that the legislation has not been at all systematic. The present legal standards of length and mass, the yard and pound (avoirdupois) were remade after the destruction of the old standards, when the Houses of Parliament were burnt down in 1834, and they were equated as nearly as could be ascertained to the old standards. The provisions for their use were embodied in the Act of 1856, but the specifications were incomplete in some respects, and in others they prescribed methods of measurement which were subsequently shown to be capable of improvement. Similar defects are to be found in the legal definition of the unit of cubic capacity.

For the electrical standards, on the other hand, a better policy was adopted, and the relevant Act made no quantitative specifications, but provided that they should be determined by Order in Council. The legal standard of the volt is at present embodied in an instrument at Teddington, which later experience has shown to be somewhat defective; however, when a certification is demanded it is always for the international and not for the legal volt, so that there have been no serious consequences of the defects.

Though many Acts of Parliament refer to temperature 'on Fahrenheit's scale', there is no enactment that prescribes what this scale is; it is now conventionally taken to be the absolute thermodynamic scale with 32° and 212° F exactly equal to 0° and 100° C. This certainly differs appreciably from the Fahrenheit scale as realized by the mercury-in-glass thermometers which were actually employed at the time the standards were laid down. Similarly, in calorimetry various enactments refer to the British thermal unit, but never specify it, and there are no less than three equally good authorities who might be quoted, all giving slightly different values for it. In photometry, candle-power was defined in 1860 in terms of a sperm candle, but it has been changed since then without formal authority, first to a flame lamp and later to electric lamps.

The general outcome of the papers following is summarized in table 1, which calls for some explanation. Apart from the need that it should keep constant in time, the important character of a fundamental standard is its accuracy of comparison, that is to say the degree of precision with which copies can be compared with their prototype. For derived standards there are two differing qualities of this type, accuracy of comparison and accuracy of realization. The first is the relative accuracy with which two examples of the same unit can be compared,
the second the evaluation of an example of the unit in terms of the fundamental standards of length, mass and time, and it will be noted that in many cases the second is distinctly inferior in accuracy to the first.

**Table 1. Table of Accuracies**

<table>
<thead>
<tr>
<th>Unit</th>
<th>Physical Dimensions</th>
<th>Standard or Reference</th>
<th>Nature</th>
<th>Accuracy of Realization parts per million</th>
<th>Accuracy of Comparison parts per million</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>$L$</td>
<td>international metre</td>
<td>material</td>
<td>---</td>
<td>0.2</td>
</tr>
<tr>
<td>mass</td>
<td>$M$</td>
<td>wave-length of light</td>
<td>natural</td>
<td>0.02</td>
<td>0.01 (1)</td>
</tr>
<tr>
<td>time</td>
<td>$T$</td>
<td>international kilogram</td>
<td>material</td>
<td>---</td>
<td>0.002 (2)</td>
</tr>
<tr>
<td>temperature</td>
<td></td>
<td>mean solar second</td>
<td>natural</td>
<td>---</td>
<td>0.01</td>
</tr>
<tr>
<td>volume</td>
<td>$L^3$</td>
<td>thermodynamic scale</td>
<td>theoretical</td>
<td>--- (3)</td>
<td>1 (4)</td>
</tr>
<tr>
<td>gravity</td>
<td>$LT^{-2}$</td>
<td>earth's attraction</td>
<td>derived (6)</td>
<td>1</td>
<td>1 (6)</td>
</tr>
<tr>
<td>pressure</td>
<td>$ML^{-1}T^{-2}$</td>
<td>standard barometer</td>
<td>natural</td>
<td>2</td>
<td>1 (6)</td>
</tr>
<tr>
<td>electric resistance</td>
<td>$LT^{-1}$</td>
<td>ohm</td>
<td>---</td>
<td>10</td>
<td>---</td>
</tr>
<tr>
<td>electric current</td>
<td>$MLT^{-1}$</td>
<td>ampere</td>
<td>theoretical</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>quantity of heat</td>
<td>$ML^2T^{-2}$</td>
<td>calorie</td>
<td>theoretical</td>
<td>10</td>
<td>1 (7)</td>
</tr>
<tr>
<td>luminous intensity</td>
<td>$ML^2T^{-2}$</td>
<td>candle (9)</td>
<td>material</td>
<td>100 (8)</td>
<td>---</td>
</tr>
<tr>
<td>sound pressure</td>
<td>$ML^{-1}T^{-2}$</td>
<td></td>
<td>derived</td>
<td>2,000</td>
<td>1,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10,000</td>
<td>--- (10)</td>
</tr>
</tbody>
</table>

(1) Comparison of derived end-standards (metre or yard).
(2) Comparison of kilograms.
(3) The absolute zero is accepted as $-273.15 \pm 0.02^\circ$ C.
(4) Reproducibility of the ice-point on the absolute scale.
(5) Derived from the kilogram of water; $1000 \cdot 027 \text{ cm.}^3 = 11$.
(6) Accuracy of relative comparisons between different stations.
(7) The relative comparison is not of amperes but of volts.
(8) Electrical determination of the mechanical equivalent for the $15^\circ$ C cal.
(9) The dimensions are those of power, but there is an additional factor representing the response of the eye to energy at any wave-length.
(10) The acoustic unit selected is the most generally useful; the accuracy of comparison varies with the experimental conditions but would generally be better than the accuracy of realization.

The succeeding papers deal with all the items in the table with two exceptions. One of these is time, and though much work is done on the subject at the laboratory, it is primarily the responsibility of the Astronomer Royal. We owe to him the information entered in the table, which is roughly speaking the accuracy of behaviour of the best crystal clocks; already these clocks are so good that they can show up small irregularities in the earth’s rotation. Time, of course, plays a part in many of the derived standards, but the accuracy of realization of these is for the most part much lower, so that time-keeping is not usually the most serious difficulty in their attainment.

There is also no paper on calorimetry. In modern times precise calorimetry is always done electrically, since the measurement of electrical energy is much easier than that of any other form. Indeed, it has been proposed that when the absolute system of electrical units is adopted in place of the present International System, the calorie shall be redefined as a definite quantity of electric energy by the
equation 1 cal. = \( \frac{3600}{860} \) joules or \( \frac{1}{860} \) watt-hour. The effect of this reform will be to remove the question of the specific heat of water from the place of importance which it now occupies, and which it does not deserve.

One entry in the table is the value of gravity, which is not strictly speaking a standard at all. Its presence here is justified because it is an essential intermediary in the evaluation of several of the other standards, in particular those of pressure and of electric current.

The very different natures of all these physical quantities impose some strain in forcing them into a single table. In each case there is some range of values where the measurement is easiest and most precise: for example, absolute temperature is very much more precisely known at ordinary temperatures than at a red-heat. In the table the most favourable case has been taken, and the choice is partly explained by footnotes, but for full appreciation the succeeding papers must be studied.

THE STANDARDS OF LENGTH

BY J. E. SEARS

The fundamental standards

The units of length, on both the British and metric systems, are at present defined by material standards known respectively as the Imperial Standard Yard, and the International Prototype Metre.

The Imperial Standard Yard is a bar of Baily's metal (16 parts copper, \( 2\frac{1}{2} \) parts tin and 1 part zinc) 1 in. square in section and 38 in. long. It is one of a number of similar bars, cast in 1845, and was given legal status as the Imperial Standard by the Weights and Measures Act of 1856. It replaced the former standard—the 'Bird' Yard—which had been destroyed in the fire at the Houses of Parliament in 1834. At 1 in. from either end there is a hole \( \frac{1}{8} \) in. diameter and \( \frac{1}{2} \) in. deep, the bottom of which therefore lies in the neutral plane of the bar. At the centre of the bottom surface of each hole a polished gold stud is inserted on which three lines are ruled, at right angles to the length of the bar. The length of the yard is defined, in the Weights and Measures Act, 1878, as the distance between the central lines at the two ends when the bar is at the temperature of 62° 'of Fahrenheit's thermometer'.

The International Prototype Metre is a bar of platinum-iridium alloy (90 % platinum, 10 % iridium) of a special winged X-form section devised by G. Tresca to give maximum rigidity in relation to the weight of metal used; the neutral plane of the section is exposed throughout the length of the bar, and the metre is defined as the distance between two transverse graduations on the neutral plane, near the ends of the bar, when the latter is at the temperature of 0° C. This bar is one of a number prepared in pursuance of the International Treaty of 1875, known as the Convention du Mètre, and received sanction as the International Prototype,
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replacing the old Mètre des Archives, at the first General Conference of Weights and Measures held in pursuance of the Treaty, in 1889.*

The reason for putting the graduations on the neutral plane is that the material of the bar in this plane is neither extended nor compressed by the elastic distortion of the bar due to its own weight when resting on its supports. It is still necessary, however, to specify the manner in which the bar shall be supported. If supported at its two ends, for example, it would droop in the middle, and the distance between the graduation marks would be slightly shorter than the ‘free’ distance between them. The Weights and Measures Act, 1878, specifies that the Imperial Standard Yard shall be supported on a system of eight rollers connected by pivoted levers in such a manner that the weight of the bar is equally distributed between them, thus minimizing the flexure of the bar, and providing freedom for expansion or contraction according to temperature conditions. The International Prototype Metre is supported for measurement on two rollers symmetrically arranged at a separation of 571 mm., this distance being calculated so that the separation of the graduations is a maximum and therefore, to the first order, independent of slight errors in positioning of the rollers. Actually the difference in the length of the yard as supported on eight rollers, or alternatively as supported on two rollers suitably placed to give maximum length, is quite negligible in relation to the accuracy of measurement, and it is found preferable in practice to make comparisons on two supports only, since there is less risk in this way of accidental movements of the bar during measurement.

Comparisons of line-standards

It will be noted that both the Imperial Standard Yard and the International Prototype Metre bars are ‘line’ standards, adapted to be compared with other measures by means of micrometer microscopes on a comparator. On the comparator two microscopes are used, set at a fixed distance apart, to read the lines at the two ends of the bar, and the two bars to be compared are carried on two parallel girders on the carriage of the comparator, with independent adjustments for aligning the bars and focusing the graduations in the microscopes. The carriage can be moved so as to bring either bar at will into view in the microscopes. In the eyepiece of each microscope a pair of parallel cross-wires is adjusted by means of the micrometer so as to appear symmetrically disposed in relation to the image of the graduation on the bar, and the reading recorded.

In this way the lengths of two bars can be compared. But owing to the effects of possible asymmetry in the graduations and in the cross-wires of the microscopes, and of the ‘personal equations’ of setting of the observers, it is necessary to repeat the readings with the bars reversed in direction, and for each observer to take an equal number of readings with each microscope under each condition. Finally, in order to achieve the greatest possible accuracy it is usual to compare not merely

two bars, but a series of bars, each with each, and to evaluate from the totality of
the measurements, by the method of least squares, the most probable values of
the differences between the individual bars.

It should be remarked that the graduation marks on a line-standard are, in the
last resort, in the nature of furrows, with more or less ragged edges, whose appear-
ance will differ according to the magnifying power of the microscopes employed,
and according to the manner in which they are illuminated. Both these factors
will affect the observer's estimate of the mean position of the line. The complete
specification of a line-standard should therefore, strictly speaking, include instruc-
tions on these two points. Generally speaking it is undesirable to employ a mag-
nification exceeding \( \times 100 \); \( \times 50 \) is more usual. The exact reproduction of illumina-
tion conditions is in any case a matter of some difficulty. The actual nature and
quality of the graduations has, of course, a marked bearing on this question, and
the metric standards have the advantage over the older British standards in this
respect.

Temperature effects

It is necessary, of course, to maintain a close control and make accurate measure-
ments of the temperature of the bars during comparison. For this reason the
comparator is arranged so that the bars can be immersed, during measurement,
in the inner compartment of a double water-bath, provided with stirring devices
to maintain a uniform temperature. In the case of the metric standards it is per-
missible to immerse the bars in distilled water, together with suitable thermo-
meters, during measurement, but the construction of the British standards makes
this procedure undesirable in their case, and the comparisons have to be made in
air, so that the control of temperature presents greater difficulty, though the outer
water-bath can still be used. In any case measurements cannot readily be made
exactly at 62° F, and still less can they conveniently be made at 0° C. Since dif-
ferent bars will have different coefficients of thermal expansion, it is necessary to
determine these and make allowances for them in computing the results of com-
parisons actually made at other temperatures.

The process of determining coefficients of expansions is similar to that of
comparing two bars, except that the bars being compared are kept in separate
compartments on the comparator (or dilatometer as this variant of the apparatus
is named), one of which is kept at a constant temperature while the temperature
of the other is varied by known amounts between successive comparisons.

It is important to note that the units are absolute measures of spatial extension,
and entirely independent of any temperature considerations, once their values
have been ascertained in terms of the defining standards under the prescribed
conditions. There has in the past been a certain confusion of ideas on this point,
with the result, for example, that it was at one time considered that a metre of
any material must have the length of a metre at the temperature of definition,
0° C, and that, in consequence, metres of varying materials would be of varying
lengths at ordinary temperatures. It has now been agreed internationally that industrial standards of length, whether British or metric, shall be adjusted to be correct—i.e. to agree in value with the units—at 20°C (68°F). So long as such standards are made of materials having a similar coefficient of expansion to the articles they are used to measure, and the standards and the articles are at the same temperature at the time of measurement, the comparison will then give directly the true size of the article at 20°C, whatever the actual temperature of measurement.

Materials of construction for length standards

This brings us to a consideration of the kinds of materials used for the construction of standards for various purposes, other than the primary standards. The principal materials so employed are steel, brass and invar, together with fused silica and crystal quartz. Steel and brass are used for industrial standards for the measurement of articles made of materials with corresponding coefficients of expansion. In some cases brass, or gun-metal, gauges are used in preference to steel in explosives factories simply for safety reasons.

Invar, an alloy steel containing 36% nickel, has the remarkable property of having a very low coefficient of thermal expansion, varying according to the size and shape of the piece and the thermal and mechanical treatment to which it has been subjected, from about 1.5 × 10⁻⁶ to −0.5 × 10⁻⁶/¹°C. It is, however, subject to a marked secular growth which renders it unsuitable for use as a reference standard, and its low coefficient makes it also unsuitable as a standard of comparison for use in most industrial measurements. Apart from its special applications in the construction of such articles as thermostats, pendulum rods for clocks and the like, it serves two very useful purposes in metrology, namely, it provides a convenient comparison standard for the determination of coefficients of expansion, and is very valuable as a material of construction for the tapes and wires which are used for the measurement of bases in geodetic surveys where errors due to uncertainty of temperature measurements in the field, with materials of higher coefficients, might be of serious consequence.

Fused silica, with a coefficient of expansion of about 0.4 × 10⁻⁶/¹°C, was considered likely to be a stable material, and therefore as possibly helping to provide a control on the constancy of units of length defined by material standards made of alloy metals. Two standards of fused silica have been made, one of which is at the N.P.L. (Kaye 1911) and one with the Indian Ordnance Survey. Comparisons of the silica metre at the N.P.L. since 1911 with other copies of the metre linked back to the platinum-iridium standard have exhibited no systematic change.

Crystal quartz is another material which, by virtue both of its structure and its antiquity, might be expected to have a high degree of stability. Standards of crystal quartz are necessarily limited to a fraction of the length of the normal unit. End-standards of quartz of 1 dm. length have, however, been made and measured
with high precision by optical interference methods. This work has not yet been repeated systematically over a long enough period to draw any definite conclusions from it.

**Constancy of units**

Control on the reproducibility of the units in the event of loss or damage is maintained by the provision of copies, in the case of the Imperial Standards the Parliamentary Copies, which are held in the custody of the Board of Trade, the Royal Observatory, the Royal Mint and the Royal Society, and in the case of the Metric Prototype Standards the ‘Témoins’, which are held at the Bureau International des Poids et Mesures, and the various national copies distributed to all the countries subscribing to the Convention du Mètre. Periodical intercomparisons between these copies suffice to show whether any one of them has changed relative to the others, and very occasional checks between the principal copies and the fundamental standards maintain the connexion with the latter. Should one of the fundamental standards be found at any time to have changed relative to the copies by an amount appreciably exceeding the experimental error of the comparisons, steps would need to be taken to replace it, or to substitute for it one of the copies presumed not to have changed. It must be remarked, however, that continued agreement between a number of standards all of the same material does not, by itself, suffice to establish the absolute constancy of any of them.

With regard to the continuity of the units and the stability of the standards the following may be stated. There are still in existence, at the South Kensington Museum, the old standard yards of Henry VII and Elizabeth. Both these are endstandards, with roughly incised subdivisions representing fractions of the yard. The former has been found to agree with our present yard within $\frac{1}{37}$ in., and the latter, though broken and roughly mended, within 0·01 in. This bar, which has since been more carefully repaired, remained the standard of the country until 1824, when it was superseded by a yard known as the Bird Yard, whereon the unit length was defined as the distance between two dots on inserted gold plugs, which had been placed in the custody of the Clerk of the House of Commons in 1760, following the Reports of Lord Carysfort’s Committee, but which did not receive legal sanction till 64 years later (Weights and Measures Act, 1824). Unfortunately, its legal existence was all too short, for it was destroyed, as mentioned above, in 1834.

The Weights and Measures Act, 1824, prescribed that if the Imperial Standard Yard were ever lost or destroyed it should be restored by reference to the length of the pendulum beating seconds in London. In a certain sense, then, the unit of that day was theoretically a ‘natural’ unit, of which the standard was no more than the material representation. The Committee entrusted with the duty of


† 1738–65, *Reports from Committees of the House of Commons, Miscellaneous Subjects, 2, 434.*
producing a new standard soon found, however, that the difficulty of reproducing with accuracy the length of the seconds pendulum was greater than that of reproducing the length of the old bar through the agency of other standards which were known to have been compared with it, and this procedure was that which, in fact, was employed for the establishment of the new (and present) standard. It is estimated that the new standard represented the current values of the best scientific standards of the time (which were susceptible of intercomparison with higher accuracy than the Imperial Standard itself) within 0·0002 in. and was in agreement with the standard it replaced within the limit of accuracy to which the latter could be measured.

Evidence for the constancy of the yard in recent times is fairly satisfactory. The results of the comparisons of the Imperial Standard Yard with its three contemporary Parliamentary Copies, Nos. 2, 3 and 5, in 1912, 1922 and 1932,* show that each of the four bars has remained in the same relationship to the mean of the group as when first determined in 1852, within 0·00005 in. In particular P.C. 2 has remained constant in relation to the mean within 0·000004 in., and P.C. 5 within 0·000016 in. for the last three comparisons. It should be mentioned, however, that a new Parliamentary Copy authorized to be made by the Weights and Measures Act, 1878, was constructed as nearly similar as possible to the original series in 1879. Since its first determination, in 1886, this bar has shown a systematic shortening in relation to the rest of the group, successive comparisons, when plotted, falling very closely on a smooth curve, with an asymptotic value of $-228 \times 10^{-6}$ in. which had very nearly been reached 40 years after its construction. If it is assumed that the original bars behaved similarly during the first 40 years of their existence dating from 1845, then at the date of the first comparison of P.C. VI, in 1886, they would have reached an effectively steady condition, but they would, in the meantime, have shortened by just over 0·0002 in. since their own original determinations in 1852.

With regard to the metre, the present international standard, when given legal status in 1889, was declared equal, within the limits of accuracy of measurement estimated at the order of 0·001 mm., with the old Mètre des Archives which it superseded. Evidence of subsequent consistency is available from the results of re-comparisons of twenty national copies since 1919 with the working standards of the Bureau International.† After making due allowance for changes known to have occurred in the values of the working standards, and for certain adjustments in the accepted values of the coefficients of expansion of the various bars, the results of all these individual comparisons were in agreement within $\pm 0·001$ mm. with the original determinations made prior to 1889, the average change being $\pm 0·003$ mm., and the mean of all the results was in agreement with the original

* 1936, Report by the Board of Trade on the comparison of the Parliamentary Copies of the Imperial Standards.
mean within 0·0001 mm. As regards the secular constancy of the metre, there have been a series of determinations of its length in terms of the wave-length of the red radiation of cadmium, commencing in 1893 with that of Michelson & Benoît (1895), which, while they exhibit variations amounting to ±0·0003 mm. from the mean, give no indication of any progressive change. There is therefore good reason for confidence in the constancy of this unit.

The current accuracy of measurement of the yard is estimated at about 0·00002 in., or 0·5 part in 10^8, that of the metre, which has more perfect graduations, at about half this amount. Recently a number of the copies of the metre at the Bureau International have been regraduated, with still better lines, and an accuracy of the order of 1 part in 10^7 is now claimed. Fortunately, the prototype metre itself has particularly good lines.

Types of standard

It will be noticed that the standard yard, originally an end-bar, passed through the phases of the 'dot' standard, and the 'natural' standard defined by reference to the 'seconds' pendulum, and finished as a line-standard. The history of the metre is remarkably similar, though the order is somewhat different. Originally a 'natural' standard, defined as the ten-millionth part of the earth's polar quadrant passing through Paris, and represented for practical purposes by the Mètre des Archives, a platinum end-standard 25 × 4 mm. in section, it was fairly soon found that the difficulty of repeating the determination of the value of the standard by reference to the definition was so great that it was necessary to legalize the length of the Mètre des Archives itself as defining the unit. Later, when the new international standard of the metre was established, the principle of defining the unit by means of a material standard was maintained, but preference was given to a line-standard, graduated on the neutral plane, as had already been adopted by British scientists for the yard.

The parallelism between the histories of the British and metric standards of length is worthy of some comment. It is evident that British scientists have not been behind their Continental colleagues in the initiation of improvements. The common change from end- to line-standards, and the common abandonment of natural standards, is clearly indicative of the stage of technical development reached at the epochs concerned. When the earliest standards were constructed end-standards offered greater advantage in accuracy than line-standards. Later, with the application of the ruling engine and microscope, this position was reversed. But the story is not yet complete. There are now very good prospects of being able in the near future to propound a natural standard, in the form of a selected wave-length of monochromatic light, which is capable of definition and reproduction with an accuracy of the order of 1 or 2 parts in 10^8—appreciably higher than can be attained even with the best line-standards. Further, there are now technical means available to produce end-standards (which are those most readily and
directly measurable in terms of wave-lengths) of sufficient perfection to take advantage of this accuracy.

It may be of interest to remark here that the pendulum experiment, regarded as a means of ascertaining the value of gravity in terms of the units of length and time, still remains one of the essential operations of metrology, since a knowledge of the local value of gravity is necessary, for example, to the establishment of the electrical unit of the ampere by weighings on a current balance, and of the standard atmospheric pressure in determining the boiling-point of water for fixing the temperature scale. Mr Clark's note in the present series of papers summarizes the position with regard to the accuracy with which the value of \( g \) has been determined.

**Ratio of yard to metre**

It might be supposed that further evidence as to the constancy of the yard would be available from successive determinations of its relationship to the metre. Unfortunately, however, early determinations of this ratio, by Kater (1818) and Clarke (1867 & 1873) were made with standards about which adequate knowledge is now lacking, so they are not very reliable for this purpose. Three determinations have, however, been made of the ratio of the present Imperial Standard Yard and International Prototype Metre, the results being as follows:

\[
1 \text{ m. } = 39.370113 \text{ in. (Benoît & Chaney 1895)},
\]
\[
1 \text{ m. } = 39.370147 \text{ in. (Sears, Johnson & Jolly 1928)},
\]
\[
1 \text{ m. } = 39.370138 \text{ in. (Sears & Barrell 1934)},
\]

the last of these being an indirect value obtained from independent determinations of the yard and metre in terms of the wave-length of light. With regard to these it can be said that although they suggest a possibility that the yard has shortened slightly in relation to the metre during the past 40 years, the whole range of the variations exhibited does not exceed the possible range of experimental error in the comparisons, so that the previous conclusion as to the present stability of the yard may still be regarded as justified.

While on the subject of the relationship between the yard and the metre it may be of interest to mention that, by an Act of Congress of 1866, legal sanction was given in America for the use of the relationship

\[
1 \text{ m. } = 39.370000 \text{ in.},
\]

and that by a subsequent order of the United States Treasury, known as the Mendenhall Order, this ratio is now accepted as defining the American inch in terms of the International Metre, instead of vice versa. It follows that there is at present a small difference between the U.S. and U.K. inches, the former being equivalent to 1 in. = 25.400051 mm., and the latter (taking the mean of the three determinations above mentioned) to 1 in. = 25.399965 mm. These two values are both so
near to the convenient round value 1 in. = 25·4 mm. that it would be very advantageous if adjustment could be made in both countries to agree on this common ratio for the future by redefining the yard as equal to 0·9144 m. exactly.

It may be remarked that if the conclusion suggested above as to the shortening of the Imperial Standard Yard be correct, then the ratio of the Imperial Yard to the International Metre (had the latter been already in existence at the dates mentioned) would have passed through the value 1 m. = 39·370000 in. about the year 1858 (very soon after the legalization of the British standard) and, curiously enough, through the value 1 m. = 39·370079 in., corresponding to 1 in. = 25·400000 mm., about 1865, just before the former value was adopted in America.

Until comparatively recently the difference has been of no practical significance, but it is now just appreciable in relation to the accuracy guaranteed by the makers of the most accurate end-gauges used in industry. The round figure has the further practical advantage of facilitating mechanical conversion by means, for example, of a 127-tooth gear wheel on a lathe or measuring instrument, between the units of the two systems.

Comparison of line- and end-standards

Whether the primary material representation of the unit of length takes the form of a line-standard or of an end-standard, the operation of transferring from the one to the other is an inevitable necessity, since both types are needed for practical applications in different fields of measurement. Probably the best method of carrying out this comparison which has so far been devised is the following.

Taking, for example, the case of the yard, it is necessary to provide an auxiliary end-bar of say 35 in. in length, with ends highly finished and accurately plane and parallel. In addition, two small end-blocks each 1 in. in length are required, similarly finished as regards their ends, with graduation marks on plugs inserted in holes at the level of their neutral planes, similar to those on the standard yard. The perfection of the surfaces will enable these end-blocks to be adhered to the ends of the 35 in. bar by the process known as ‘wringing’. If both the end-blocks are so attached the combination constitutes a built-up line bar which can be compared with a 36 in. line-standard in the comparator in the usual manner. If only one of the end-blocks at a time is attached to the 35 in. bar the combination can be compared with a 36 in. end-standard on an end-measuring machine. Let the length of the 35 in. bar be $B$, that of the line-standard $L$, and that of the end-standard $E$. And let the distances of the graduation lines from the surfaces of the two end-blocks be $a_1, b_1, a_2, b_2$ respectively. Finally, let the thickness of a ‘wringing film’ be $t$. If the observed differences are $x_1, x_2$ and $y_1, y_2$, the results of the measurements may then be written as follows:

\[
B + a_1 + a_2 + 2t = L + x_1, \quad B + b_1 + b_2 + 2t = L + x_2,
\]

whence

\[
B + \frac{1}{2}(a_1 + b_1 + a_2 + b_2) + 2t = L + \frac{1}{2}(x_1 + x_2),
\]

and

\[
B + a_1 + b_1 + t = E + y_1, \quad B + a_2 + b_2 + t = E + y_2,
\]
A discussion on units and standards

whence

\[ B + \frac{1}{2}(a_1 + b_1 + a_2 + b_2) + t = E + \frac{1}{2}(y_1 + y_2). \]

Thus, finally,

\[ E + t = L + \frac{1}{2}(x_1 + x_2) - \frac{1}{2}(y_1 + y_2), \]

whence it will be seen that the lengths of both the 35 in. bar and of the end-blocks have been eliminated.

Actually, of course, a number of variations can be made in the wrappings which are not included above. In practice all possible variations would be employed, and the mean result taken. But the principle of the comparison should be clear. The fact that the thickness of one wringing film is associated with the length of the end-standard as the result of this comparison corresponds to the fact that, in practical use, precision end-gauges are habitually combined together, by the process of wringing, so as to make up composite standards of any desired size. The calibrations of each gauge must therefore always include the thickness of one wringing film, which may be regarded as distributed half over each end of the gauge, so that when wrung together in groups the thickness of one wringing film is automatically provided for at each interface. In addition, the composite gauge, just as a single gauge representing the same size, will be associated with one film thickness divided between its terminal faces, and allowance may need to be made for this in some circumstances, e.g. when deriving the ‘mechanical’ length of an end-standard from its ‘optical’ length under certain conditions. The actual thickness of an individual wringing film is, however, very small, amounting only to about 0.0000002 in. (Rolt & Barrell 1927).

Determination of submultiple end-standards

Now follows the process of determining the values of end-standards representing subdivisions of the fundamental unit, which are of great importance in practical applications. This can be done by comparing together, in a suitable end-measuring machine, combinations of gauges of equal nominal size. It is convenient, though not essential, that a duplicate series should be available. As a typical case consider the entries in table 2, which refers to the determination of the values of a series of end-gauges 7, 8, 9, 10 and 11 in., in relation to those of 6 and 12 in. supposed already known. Duplicate gauges are indicated by a dash (‘).

| 12 + 10 | 11 + 11’ | ... | ... | ... | ... |
| 12 + 9 | 11 + 10 | 10 + 11’ | ... | ... | ... |
| 12 + 8 | 11 + 9 | 10 + 10’ | 9 + 11’ | ... | ... |
| 12 + 7 | 11 + 8 | 10 + 9 | 9 + 10’ | 8 + 11’ | ... |
| 12 + 6’ | 11 + 7 | 10 + 8 | 9 + 9’ | 8 + 10’ | 7 + 11’ |
| 6 + 11’ | 11 + 6’ | 10 + 7 | 9 + 8 | 8 + 9’ | 7 + 10’ |
| 6 + 10’ | ... | 10 + 6’ | 9 + 7 | 8 + 8’ | 7 + 9’ |
| 6 + 9’ | ... | ... | 9 + 6’ | 8 + 7 | 7 + 8’ |
| 6 + 8’ | ... | ... | ... | 8 + 6’ | 7 + 7’ |
| 6 + 7’ | ... | ... | ... | ... | 7 + 6’ |
It will be noted that all the combinations in any row of this table have the same nominal length. Suppose each combination in any column is compared with the corresponding combination in column 1, and let the observed differences be \( x_1, x_2, \) etc. Adding all the results relating to any particular column it is seen that the sum of the right-hand members in the combinations compared cancels out, and there remains a series of equations of the form, taking, for example, column 3,

\[
10 \times 6 = 12 \times 4 + 6 \times 2 + \Sigma x \quad \text{or} \quad 10 = 12 \times 4/6 + 6 \times 2/6 + \Sigma x \times 1/6.
\]

This case has been selected as exhibiting in a simple form the general procedure involved. It is easy to follow the modifications involved, say, in deriving a series such as 30, 24, 18, 12 and 6 in. from a yard end-standard, or a series such as 0·100, 0·101, 0·102, ..., 0·110 in. which is found in a normal set of slip gauges. Reference gauges of the smaller sizes are, however, more readily measured, provided the flatness and parallelism of their surfaces is sufficiently high, by interference methods in terms of the wave-length of light, other gauges being then compared directly with reference gauges of similar size in a suitable end-measuring machine.

By one or other of the above processes the lengths of reference end-gauges can be established to an accuracy of 1 part in \( 10^6 \) for sizes above 1 in., or to 0·000001 in. for sizes below. It may be worth while to point out that this accuracy is actually necessary to meet industrial demands at the present time. Grouped combinations which may include as many as five such gauges are required to be correct to the nearest 0·0001 in., that is, to within 0·00005 in., and their users expect to be able to rely on this without making use of corrections for known errors. Each individual gauge must therefore be correct within 0·00001 in., and the makers guarantee this, and in some cases even higher, accuracy. Reference gauges used in verifying that such guarantees have been fulfilled must therefore be known at least to 0·000001 in. It follows that the present difference of nearly 4 parts in \( 10^6 \) between the British and American inches is, as already stated, no longer negligible in relation to the highest grade of current commercial accuracy.

**Subdivision of line-standards**

To complete the present survey reference must be made to the operations of calibrating the subdivisions of a divided scale, and of building up from the yard or metre to the longer standards which are required, for example, in field-survey work.

For the first of these processes the bar to be examined is mounted, under two micrometer microscopes, on the girder of a comparator in which the carriage, instead of moving transversely, is arranged to move in the direction of the length of the bar. The separation of the microscopes can be varied to suit the successive stages of the comparisons. Suppose, for example, one wishes to determine the decimetre subdivisions of a 1 m. bar. The microscopes are set first 1 dm. apart and direct comparisons are made between the 0/1 interval and the 1/2, 2/3, etc., intervals up to 9/10. The results of these measurements give a series of values, \( a_1, a_2, a_3, \ldots, a_9 \) for the differences 0/1–1/2, 1/2–2/3, ..., 8/9–9/10 between the
successive adjacent decimetre intervals on the bar. The microscopes are then placed 2 dm. apart, and similar comparisons made of the intervals 0/2, 1/3, etc., up to 8/10. The results of these measurements, subtracting the parts common to the two adjacent intervals compared, give a series of values \( b_1, b_2, \ldots, b_8 \) for the differences \( 0/1-2/3, 1/2-3/4, \) etc., up to \( 7/8-9/10 \) and so on, with successively increasing distances between the microscopes, up to 9 dm. The results of all these measurements can then be tabulated as follows:

<table>
<thead>
<tr>
<th></th>
<th>0/1</th>
<th>1/2</th>
<th>2/3</th>
<th>3/4</th>
<th>4/5</th>
<th>5/6</th>
<th>6/7</th>
<th>7/8</th>
<th>8/9</th>
<th>9/10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0/1</td>
<td>0</td>
<td>( -a_1 )</td>
<td>( -b_1 )</td>
<td>( -c_1 )</td>
<td>( -d_1 )</td>
<td>( -e_1 )</td>
<td>( -f_1 )</td>
<td>( -g_1 )</td>
<td>( -h_1 )</td>
<td>( -k )</td>
</tr>
<tr>
<td>1/2</td>
<td>( a_1 )</td>
<td>0</td>
<td>( -a_2 )</td>
<td>( -b_2 )</td>
<td>( -c_2 )</td>
<td>( -d_2 )</td>
<td>( -e_2 )</td>
<td>( -f_2 )</td>
<td>( -g_2 )</td>
<td>( -h_2 )</td>
</tr>
<tr>
<td>2/3</td>
<td>( b_1 )</td>
<td>( a_2 )</td>
<td>0</td>
<td>( -a_3 )</td>
<td>( -b_3 )</td>
<td>( -c_3 )</td>
<td>( -d_3 )</td>
<td>( -e_3 )</td>
<td>( -f_3 )</td>
<td>( -g_3 )</td>
</tr>
<tr>
<td>3/4</td>
<td>( c_1 )</td>
<td>( b_2 )</td>
<td>( a_3 )</td>
<td>0</td>
<td>( -a_4 )</td>
<td>( -b_4 )</td>
<td>( -c_4 )</td>
<td>( -d_4 )</td>
<td>( -e_4 )</td>
<td>( -f_4 )</td>
</tr>
<tr>
<td>4/5</td>
<td>( d_1 )</td>
<td>( c_2 )</td>
<td>( b_3 )</td>
<td>( a_4 )</td>
<td>0</td>
<td>( -a_5 )</td>
<td>( -b_5 )</td>
<td>( -c_5 )</td>
<td>( -d_5 )</td>
<td>( -e_5 )</td>
</tr>
<tr>
<td>5/6</td>
<td>( e_1 )</td>
<td>( d_2 )</td>
<td>( c_3 )</td>
<td>( b_4 )</td>
<td>( a_5 )</td>
<td>0</td>
<td>( -a_6 )</td>
<td>( -b_6 )</td>
<td>( -c_6 )</td>
<td>( -d_6 )</td>
</tr>
<tr>
<td>6/7</td>
<td>( f_1 )</td>
<td>( e_2 )</td>
<td>( d_3 )</td>
<td>( c_4 )</td>
<td>( b_5 )</td>
<td>( a_6 )</td>
<td>0</td>
<td>( -a_7 )</td>
<td>( -b_7 )</td>
<td>( -c_7 )</td>
</tr>
<tr>
<td>7/8</td>
<td>( g_1 )</td>
<td>( f_2 )</td>
<td>( e_3 )</td>
<td>( d_4 )</td>
<td>( c_5 )</td>
<td>( b_6 )</td>
<td>( a_7 )</td>
<td>0</td>
<td>( -a_8 )</td>
<td>( -b_8 )</td>
</tr>
<tr>
<td>8/9</td>
<td>( h_1 )</td>
<td>( g_2 )</td>
<td>( f_3 )</td>
<td>( e_4 )</td>
<td>( d_5 )</td>
<td>( c_6 )</td>
<td>( b_7 )</td>
<td>( a_8 )</td>
<td>0</td>
<td>( -a_9 )</td>
</tr>
<tr>
<td>9/10</td>
<td>( k )</td>
<td>( h_2 )</td>
<td>( g_3 )</td>
<td>( f_4 )</td>
<td>( e_5 )</td>
<td>( d_6 )</td>
<td>( c_7 )</td>
<td>( b_8 )</td>
<td>( a_9 )</td>
<td>0</td>
</tr>
</tbody>
</table>

It will be noted that in this table each entry is repeated, above the left to right diagonal, with reversed sign. Taking any particular column of the table, for instance the fourth, and adding up all the entries, it is seen that the total of all the observations relevant to the interval 3/4 leads to the result

\[
10 \times \frac{3}{4} - \sum(0/1+1/2+\ldots+9/10) = -(c_1+b_2+a_3)+(a_4+b_4+\ldots+f_4),
\]
or

\[
\frac{3}{4} = \frac{1}{10} \times 0/10, \quad \frac{1}{10}(c_1+b_2+a_3)+\frac{1}{10}(a_4+b_4+\ldots+f_4),
\]

and similarly for any other interval.

With slight modification the same process can be applied to determining the values of every centimetre of two particular decimetres in terms of the whole length of the other. In this case each space in the square will be occupied by an independent observational result. The millimetres of two particular centimetres can then be determined in the same way.

The accuracy of this process, given good conditions, is remarkably high, since the bar may be assumed to have the same coefficient of thermal expansion throughout its length, and errors due to temperature variations, if kept within reasonable limits, should be negligible. With good lines it is considered that a final accuracy of the order of 0.0002 mm. in the placing of any intermediate subdivision in relation to the terminal graduations can be readily attained.

**Longer standards**

Turning to the converse process, of building up to larger lengths, the first step is the comparison of a long bar, say of 4 m. length, 1 m. at a time with a reference standard metre, in a transverse comparator of suitable capacity. As this bar will
subsequently be used in air, under less favourable conditions for temperature control, it is nowadays usually made of invar. When standardized it is used either as a basis of comparison for 4 m. intervals on a mural base or, alternatively, the bar may be directly compared with 4 m. intervals graduated on the surface of a standard invar tape. These normally have a total length of 24 m.

Tapes intended for use in the field in setting out bases for geodetic surveys are compared either with the appropriate interval on the mural base, or directly with a standard tape of similar length. In the case of English measures the most usual length is 100 ft., built up in eight steps from a bar 12 ft. 6 in. in length. The operation of standardizing this bar in terms of the yard consists (a) of determining each 6 in. length in terms of the whole length by the method of subdivision, and (b) determining the lengths of the 0 to 12 ft. and 6 in. to 12 ft. 6 in. lengths in four steps against a reference standard yard.

It is considered that under the best conditions an accuracy of the order of 1 part in $10^6$ can be attained in the standardization of the field tapes in either the British or the metric systems. Field surveyors find that, even in the adverse conditions of open-air working, they are often able to repeat their measurements of a long base to appreciably higher accuracy than this, and are sometimes inclined to criticize the accuracy offered them as the result of calibration under laboratory conditions. The explanation is, of course, that so many observations are taken in the field that casual errors are averaged out, and the probable error of determination of the whole length of, say, a 20 km. base in terms of the length of a particular tape, becomes very small. In the laboratory, on the other hand, we are concerned, not merely with the repeatability of observations, but with their relationship to the absolute value of the unit. Some countries have already experimented with the possibility of determining the length of a mural base in terms of the wave-length of light, and it is intended to do so also at the National Physical Laboratory when opportunity affords. So far, however, we have no experience of the method in this special application.

It may be of interest, finally, to record that the 50 m. mural base at the National Physical Laboratory has gradually grown, since it was first built in 1908, by something over a centimetre, due presumably to progressive chemical action in the cement with which the bricks are bonded.

THE STANDARDS OF LENGTH IN WAVE-LENGTHS OF LIGHT

By H. Barrell

Historical survey

The first direct measurement of the metre in terms of wave-lengths of the red radiation of cadmium was made by Michelson & Benoit (1895) in 1892–3 at the Bureau International des Poids et Mesures (B.I.P.M.). In 1905–6, Benoit, Fabry & Perot (1913) repeated the determination of the metre, using improved methods and apparatus. The International Solar Union (now the International Astronomical
A discussion on units and standards

Union) adopted, in 1907,* the wave-length of the red line of cadmium as the reference basis for all spectroscopic measurements of wave-lengths of light, and defined its value in terms of the International Ångström. Using the relationship between the wave-length of the red line of cadmium originally quoted by Benoît et al. namely, \( \lambda_R = 6438.4696 \times 10^{-10} \text{ m.} \) in normal† air, the Ångström was defined by assigning the value of 6438.4696 Å to the wave-length of the red line of cadmium in normal air, and hence can differ only to the extent of the error in the above determination from \( 1 \times 10^{-10} \text{ m.} \)

In 1923, the International Committee of Weights and Measures‡ accepted in principle the possibility of the eventual adoption of a definition of the metre in terms of wave-lengths of light, subject to the formulation of satisfactory conditions for practical realization of such a definition, and urged national standards laboratories to undertake investigations for this purpose. Four years later, the International Conference on Weights and Measures§ (1927) gave formal sanction, as an interim measure, to determinations of length being made by reference to the wave-length of the cadmium red radiation, determined by Benoît et al., as an alternative to direct reference to material standards.

Reports of the Board of Trade (1930, 1936)|| have expressed the need for replacing the present Imperial Standard Yard by a standard more in accordance with modern requirements, and also suggest the possibility of defining the yard in terms of wave-lengths.

Following the International Committee's recommendation of 1923, the first result to be announced was that of Watanabe & Imaizumi (1928); it was the outcome of work carried out by means of apparatus, constructed in this country for the Japanese Government, which was almost identical with that of Benoît et al.

Each determination so far mentioned made use of purely line-standard technique by relating the position of a line or lines engraved on an interferometer standard (or étalon) to the graduations on a copy of the metre by microscope observations. With the introduction of modern types of end-standard constructed in the form of steel blocks or bars in lengths up to a metre or more and having flat, parallel, terminal faces with a mirror-like finish of optical perfection, it became possible to apply interferometric methods, as devised by Michelson and others, directly to the measurement of a practical standard of length.

Determinations of the metre and the yard made by Sears & Barrell (1932, 1934) at the National Physical Laboratory (N.P.L.) and of the metre by Kösters &

† The modern specification of normal air for spectroscopy and metrology is: dry air at \( 15^\circ \text{ C} \) under a pressure of 760 mm. of mercury at \( 0^\circ \text{ C} \) \( (g=980.665 \text{ cm./sec./sec.}) \) and containing a normal proportion of 0.03 % by volume of carbon dioxide usually found in the open air and in well-ventilated rooms.
‡ 1923, P.V. Com. int. Poids Mes. p. 67.
|| 1930 and 1936, Report of the Board of Trade on the comparisons of the Parliamentary Copies of the Imperial Standards. H.M.S.O.
Lampe* at the Physikalisch-Technische Reichsanstalt (P.T.R.) in 1933 produced evidence of this trend, for the interferometry at both institutions was concerned with the measurement of an end-gauge approximately equal to one of the fundamental units. Correlation of the optically measured end-gauges with the fundamental line standards was effected by the already well-established metrological procedure for transferring from line- to end-measures,† to which reference is made in Mr Sears’s paper in the present series.

Other important changes were introduced in both the new methods. With the N.P.L. apparatus it was possible to measure the end-gauge either in air under known and controlled conditions or in vacuo; with the P.T.R. apparatus the end-gauge was measured in air and a concurrent determination made of refractive index over an optical path equivalent to that of the gauge length. Thus the results obtained from both methods can be expressed, if desired, in terms of wave-lengths in vacuo, rather than in air for which the temperature, pressure and composition must also be specified in order to define the refractive index. Platinum resistance thermometry was used in both determinations for the measurement of temperature of the end-gauge. In the neighbourhood of 20° C it is probable that the temperature scale defined by a platinum resistance thermometer conforming with international specification does not differ by much more than 0·001° C from the thermodynamic Centigrade scale, corresponding to an uncertainty of only about 0·01 μ in the length of a 1 m. end-gauge constructed of steel.

Owing to the disappointingly large discrepancy between the originally announced N.P.L. and P.T.R. results, the determinations were repeated in 1934–5 (Kösters & Sears 1935), but the mean discrepancy remained equivalent to 0·54 μ in the metre. At the same time, however, direct optical comparisons of the N.P.L. and P.T.R. metre end-standards, both independently established in terms of light waves in vacuo, were made at both institutions, and the results of all the optical measurements were found to be mutually consistent to ± 1 part in 36 millions, equivalent to 0·028 μ in the metre.

Results of subsequent P.T.R. work have recently been noticed (Kösters 1938) which indicate that the former P.T.R. results have been modified in a sense which considerably reduces the original divergence, and this change is substantiated by a new determination made in 1937, the result of which differs from the more recent of the two N.P.L. results by an amount equivalent to only 0·14 μ in the metre.

In 1940, a preliminary determination of the metre was made in Russia (Barinov 1941) by methods which appear to be similar to those used in Germany.

Values of the yard have been determined by Tutton (1931) and from the N.P.L. work mentioned above.

It should be mentioned that Williams (1933) has proposed a novel method of measuring an end-gauge by comparison with a reflexion echelon grating constructed of fused silica and of length approximately equal to the metre or the yard.

Wave-length measurements of length made under generally practicable conditions in air effectively involve, for their reduction to comparable terms, concurrent observations of refractive index of the air. Such observations may be made either directly, as in the P.T.R. apparatus, or indirectly from a knowledge of the temperature, barometric pressure, humidity and possibly the CO$_2$ content of the air and of how these variable factors influence the refractive index. For spectroscopic purposes use is made of the experimental data obtained in 1918 by Meggers & Peters (1918–19) on the refraction and dispersion of air. At the 1931 meeting of the International Committee of Weights and Measures it was decided that further precise measurements of the refractive index of air were required. Concordant results for the refraction and dispersion of air in the visible region have since been announced from the B.I.P.M. (Pérard 1934), the P.T.R. (Kösters & Lampe 1934) and the N.P.L. (Barrell & Sears 1939), which appreciably disagree with the hitherto accepted tables of Meggers & Peters for the same spectral region. Suggestions have been made (Sears & Barrell 1934; Kösters 1938) for defining the reference standard of wave-length in the vacuum condition and so eliminating the refractive index entirely from the fundamental definition.

With one exception, all wave-length determinations of the metre have been made directly in terms of the red radiation of cadmium, which was selected by Michelson as the most suitable for the purpose. Certain radiations of krypton have since been proposed as better alternatives, notably the yellow-green line ($\lambda$5651A) by Kösters* which was used during the actual P.T.R. determinations of the metre. The P.T.R. results were finally expressed, for comparison with other work, in terms of the cadmium red radiation by use of an experimentally determined relation between the two wave-lengths.

Discussion of results obtained on the metre and the yard

(a) The metre. The original results of all the determinations mentioned in the previous section are not strictly comparable, as the experimental conditions were not identical, nor in all cases fully specified. But, reducing them as far as possible from the data available to a common basis, the following table has been prepared.

The values originally quoted by the observers are corrected in the fourth column of the table below to take account of subsequent conclusions regarding the values to be attributed to the standards of length employed and adjusted, so far as the information available permits, to uniform conditions of normal air (see footnote, p. 165). It is to be noted that no evidence of any systematic change in the length of the metre is shown, over a period of nearly 50 years, compared with the wave-length of light; also the greatest departure of any individual result from the mean is less than 0·3 in $10^6$, which is of the same order of accuracy as that generally associated by experienced metrologists (Sears 1936; Kösters 1938) with the measurement of length in terms of the existing national copies of the metre.

Furthermore, the mean value of the wave-length is almost identical with that originally quoted by Benoît et al. which has been accepted as the basis of spectroscopic measurements of wave-lengths and involves the definition of the Ångström.

The mean of the two N.P.L. values of the wave-length of the red line in vacuo, determined respectively in 1933 and 1934–5, was $6440.2513 \times 10^{-10}$ m.‡ (Kösters & Sears 1935). If the mean corrected and adjusted P.T.R. value of $6438.4693 \times 10^{-10}$ m. for normal air, abstracted from the table, is reduced to vacuum condition by use of the P.T.R. value of 1000,276,47 (Kösters & Sears 1935) for the refractive index of the red line in normal air, the mean P.T.R. value of the wave-length in vacuo becomes $6440.2493 \times 10^{-10}$ m.

Values of the wave-length of the cadmium red radiation in terms of the International Metre (unit = $1 \times 10^{-10}$ m.)

<table>
<thead>
<tr>
<th>Date of Determination</th>
<th>Observers</th>
<th>Original Values</th>
<th>Corrected and adjusted Values in normal air</th>
<th>Differences from Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1892–3</td>
<td>Michelson &amp; Benoît</td>
<td>6438–4722</td>
<td>6438–4691*</td>
<td>−0·0004 −0·06</td>
</tr>
<tr>
<td>1905–6</td>
<td>Benoît, Fabry &amp; Perot</td>
<td>6438–4696</td>
<td>6438–4703*</td>
<td>+0·0008 +0·12</td>
</tr>
<tr>
<td>1927</td>
<td>Watanabe &amp; Imaizumi</td>
<td>6438–4685</td>
<td>6438–4682</td>
<td>−0·0013 −0·20</td>
</tr>
<tr>
<td>1933</td>
<td>Sears &amp; Barrell</td>
<td>6438–4711</td>
<td>6438–4713</td>
<td>+0·0018 +0·28</td>
</tr>
<tr>
<td>1934–5</td>
<td>Kösters &amp; Lampe</td>
<td>6438–4709</td>
<td>6438–4709</td>
<td>+0·0014 +0·22</td>
</tr>
<tr>
<td>1933</td>
<td>Kösters &amp; Lampe</td>
<td>6438–4672</td>
<td>6438–4689</td>
<td>−0·0006 −0·09</td>
</tr>
<tr>
<td>1934–5</td>
<td>Kösters &amp; Lampe</td>
<td>6438–4685</td>
<td>6438–4690</td>
<td>−0·0005 −0·08</td>
</tr>
<tr>
<td>1937</td>
<td>Kösters &amp; Lampe</td>
<td>6438–4700</td>
<td>6438–4700</td>
<td>+0·0005 +0·08</td>
</tr>
<tr>
<td>1940</td>
<td>Romanova, Varlich, Kartashev &amp; Bartarchukova†</td>
<td>6438–4677</td>
<td>6438–4677</td>
<td>−0·0018 −0·28</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>6438–4695</td>
<td>±0·0010</td>
<td>±0·16</td>
</tr>
</tbody>
</table>

* See Guillaume (1927). † See Barinov (1941).

(b) The yard. The number of wave-lengths of the cadmium red radiation contained in the yard under normal air conditions, as determined by Tutton (1931), was 1,420,209.8.

The following N.P.L. results§ (Sears & Barrell 1934) were obtained in 1933, using the red radiation of cadmium:

$$1 \text{ yard} = 1,419,818.24 \text{ wave-lengths in vacuo}$$

$$= 1,420,210.74 \text{ wave-lengths in normal air}.$$  

‡ The corresponding number of wave-lengths in the metre is 1,552,734.44, which may be associated with the value of the yard in terms of wave-lengths in vacuo to determine the relationship between the yard and the metre, as quoted later.

§ These values are slightly lower than those originally published owing to a reduction of 0.07 wave-length in the value previously assigned to the correction applied to the optically measured end-gauge, in the N.P.L. method, to derive the practical mechanical length (Rolt & Barrell 1929). This small change has been made as the result of further work on determining the phase loss in light reflected at the lapped steel surfaces of the gauge.
A discussion on units and standards

The difference between the Tutton and N.P.L. values in normal air accords with the precision of measurement actually attainable with the present Imperial Standard Yard, which is assessed at about 0.5 part in 10^6 (Sears 1936).

The relationship of the yard to the metre, derived from the comparison of the N.P.L. wave-length determinations quoted above, is given by

\[ 1 \text{ m.} = 39.370138 \text{ in. (or 1 in.} = 25.40099962 \text{ mm.}). \]

Future definition of the units of length

From the foregoing, it is clear that it may soon be possible to supersede the present definitions of the fundamental units of length, based on the distances between lines engraved on certain metal bars, by a definition in terms of the natural standard provided by the wave-length of a monochromatic ray of light. The various national laboratories already employ interferometric methods in routine procedures (Kösters 1938; Barrell 1945) for directly calibrating the practical reference standards of measurement—such as, for instance, sets of slip or block gauges containing sizes up to 10 cm. or 4 in.—in terms of this natural, invariable and easily reproducible unit of length to an accuracy of 0.025μ or one-millionth of an inch. Experience in this use of a wave-length standard shows that it is capable of a higher precision and greater convenience in the measurement of such practical standards than the line-standard. The results of the British and German experiments on the calibration of end-gauges 1 m. in length demonstrate that it is possible to establish metre lengths independently in two countries on the basis of a wave-length standard of reference with a precision of ± 0.03μ, which is approximately ten times better than is generally attainable with the present material standards. If the definitions of the units of length were established in terms of wave-lengths the end-standard would obviously become the first material representation of both units, rather than the line-standard.

The red radiation of cadmium, selected 50 years ago by Michelson as the best available, is still the approved standard for all measurements of wave-lengths of light, but it is not now considered to be quite as ideally monochromatic as is desirable for a universal standard of length. Cadmium is known to consist of a mixture of six isotopes, four being of even and two of odd atomic masses; krypton, the suggested alternative source of monochromatic radiation, also consists of six isotopes, five being of even atomic masses and one odd. It is found that the hyperfine structure of certain radiations emitted by cadmium and krypton may be accounted for by associating a nuclear spin of \( I = \frac{1}{2} \) with each of the odd isotopes of cadmium and of \( I \geq \frac{7}{2} \) with the odd isotope of krypton. Some evidence of hyperfine structure, due to the odd isotopes, has, indeed, been observed in the red line of cadmium, but none has so far been observed in the yellow-green line of krypton, possibly because of its rather low intensity. Although the even isotopes of both elements, by reason of their zero nuclear spin, are not expected to produce complexity, there may be some minute mass displacement effect between
the corresponding radiations emitted by the different even isotopes in each element.

The conditions under which excitation of an element takes place in the discharge lamp are also known to affect the monochromatic quality of the emitted radiations, and care must be taken to reduce, as far as possible, perturbative effects due to Doppler motions of the radiating atoms, electric and magnetic fields (Stark and Zeeman effects), collisions, pressure and reversal. As regards the Doppler effect of motions in a radiating gas or vapour, it is known, from kinetic theory, that the ‘width’ of a line is proportional to $\sqrt{T/A}$, where $T$ is the absolute temperature and $A$ the atomic mass, so that radiation tends to become more monochromatic when excited from heavy atoms at low temperature.

A new possibility now exists of finding a more suitable radiation from a single isotope of a heavy element with zero nuclear spin (and therefore with even mass number), owing to the recent advances in the technique of separating isotopes. If a single even-number isotope of mercury, for instance, could be isolated and caused to emit the well-known green radiation 5461 A, either in a Paschen and Schüler hollow-cathode type of discharge lamp at the temperature of liquid air or by very high-frequency excitation of a fine atomic beam of such mercury viewed normally to the direction of motion in the beam, it might provide an ideal source of monochromatic radiation. Information was received from America in 1941 that a suitable isotope of mercury had been obtained from a radioactive isotope of gold.

Although there may be some minor objections in practice to adopting a wavelength in vacuo as the basis of definition, it is considered that the weight of ultimate scientific advantage is greatly in its favour. Whatever radiation may be eventually selected for this purpose, the new definition of the metre should preferably be so adjusted as to yield a value of $6438.4696 \times 10^{-10}$ m. for the wave-length of cadmium red radiation in normal air, which happens to be almost identical with the mean experimentally determined value (see table), thus preserving the definition of the Ångström which from thenceforth could be redefined simply as $10^{-10}$ m. The yard could then with advantage be defined in similar terms to the metre, on a basis which would make the yard exactly equal to

$$0.9144 \text{ m. (or 1 in. = 25.4 mm.).}$$

With the improvements that are now believed to be possible in producing pure monochromatic radiation, combined with new definitions based on wave-lengths in vacuo, it is considered that the units of length could be reproduced through the agency of end-standards with a precision approaching 1 part in $10^8$. The best precision that could be expected from the existing definitions in terms of material line-standards, utilizing all modern resources for their production and observation, is unlikely to be superior to 1 part in $10^7$. 
A discussion on units and standards

The standards of mass

By F. A. Gould

In this review it is proposed to consider briefly the various standards of mass, including primary, secondary and also the more ordinary standards in everyday use in trade, industry and scientific work. Particular reference will be made to the principal standards, their comparisons, and the technique of this class of weighing. Consideration will also be given to air buoyancy in relation to the derivation of secondary and other standards from the primary standards, and to conventions for the adjustment of ordinary standards.

Primary standards

The fundamental standard in the British system of units is the Imperial Standard Pound (Weights and Measures Act, 1878). This was constructed in 1844 in the form of a cylindrical piece of platinum of diameter slightly smaller than its height, and with a shallow circular groove around its flank to take an ivory lifting fork. There are five Parliamentary Copies of this standard, four of which are of platinum and one of platinum-iridium alloy, which serve for the replacement of the Imperial Standard in the event of its being lost or damaged. It is to be noted that this standard is the avoirdupois pound containing 7000 gr. The troy pound, which was the sole standard from 1824 to 1834, is now no longer legal and the troy ounce, defined as 480 gr. is now the basis of the troy system.

In the metric system it is of interest to refer to the original conception underlying the definition of the unit of mass. The kilogram was originally defined by reference to a ‘natural’ standard, i.e. the mass of the cubic decimetre of water. The material representation of this standard was the Kilogramme des Archives, a simple cylindrical piece of platinum which was constructed in the latter part of the eighteenth century. The practical realization of the definition of the kilogram proved to be insufficiently precise, and it was accordingly superseded by the material standard itself, viz. the Kilogramme des Archives. Though there may be natural standards of mass which might be employed, there is no prospect as yet that any of these could be realized with a precision approaching that with which material standards of mass can be compared.

The Kilogramme des Archives remained the standard until 1889, when, under the auspices of the Metric Convention, it was replaced by the International Prototype Kilogram, the present fundamental standard in the metric system. The latter takes the form of a simple cylinder, of height equal to its diameter, and is made of the same 10 % platinum-iridium alloy as the international metre. At the time of this replacement the new kilogram was declared* to be equal to the earlier standard, to within the limits imposed by the uncertainty of the volume of the older standard, which had not been established so closely as that of the later one.

Some forty or more copies of the international kilogram were constructed. The principal copies, or ‘témoins’, and other working copies are preserved at the International Bureau of Weights and Measures, Sèvres, Paris. Other copies are distributed, as national reference copies, to the various countries signatory to the Metric Convention. One such copy, No. 18, designated the British Copy of the Kilogram, is kept in this country.

Balances for comparing standards

Consideration will next be given to the balances used for the comparison of precision standards. The accuracy attained with an ordinary type of knife-edge balance of good quality is surprisingly high, and a precision of the order 1 part in $10^7$ has for many years been attained in the comparisons of pounds or kilograms. The principle of double weighing is normally employed, each standard being weighed from each arm of the balance in turn. Due allowance is made for the atmospheric buoyancies of the standards, which are computed from a knowledge of the volumes of the standards, as experimentally determined, and the air density determined at the time of each weighing. The densities of the standards are determined by weighing them in air and in water. When a new standard is constructed, its density may with advantage be determined shortly before the final polishing of the standard.

Balances of special design for this class of weighing were in use on the Continent 50 years ago. These were provided with arrangements for interchanging the standards undergoing comparison without opening the shutters of the balance, thus enabling a double weighing to be made with the minimum of disturbance to the temperature of the balance. Balances of this type, constructed by the well-known firm of Rüprecht, Vienna, gained a high reputation and have long been in use at the International Bureau of Weights and Measures, Paris.* An accuracy of the order 1 part in $10^8$, or somewhat finer, was attained in the comparison of kilograms as the result of repeated weighings. In this country, however, no substantial progress was made until recent years, when a new balance, designed and constructed at the National Physical Laboratory, was installed. Some of the principal features of this balance will be described briefly here, including also a special technique of weighing which has enabled a striking improvement in accuracy to be made. Figure 1 (plate 6) gives a general view of the instrument with its cover removed. Provision is made for weighing masses up to 1 kg. in air or in vacuo, though there is no intention of carrying out in vacuo any standardizations which should properly be made under ordinary conditions of atmospheric pressure. The balance is installed in a closed vault and is so designed that the whole operation of weighing, including the interchange of the standards during a double weighing, can be controlled from outside the vault. To minimize wear on the standards two loose pans are provided, and once the standards are placed on these, they are not dis-

turbed during the whole of a sequence of weighings. Beneath these pans each pan suspension has an aperture cut in it in the form of a cross, open towards the centre of the balance, and to interchange the pans on the suspensions a ring surrounding the centre of the base, and bearing two arms with suitably shaped ends which can pass through these apertures, is first raised, lifting the pans off their suspensions, then rotated through 180°, and again lowered. Allowance, of course, has to be made for the difference between the unloaded pans, which can be determined by direct weighing, as often as is found necessary. Alternatively, the standards can be weighed on each pan in turn, and the mean result taken.

Another feature of the balance is the design of the two series of stirrups, supported on crossed knife-edges, through which the load is transmitted to the beam. These are arranged so that in combination they provide precise location of the loads transmitted by them on each terminal knife of the beam.

![Diagram of arrangement for steadying the beam.]

But even with these special features the improved accuracy which is available with the balance would not have been attained without the adoption of a special technique for carrying out the weighings. In ordinary circumstances a knife-edge balance would be fully arrested between successive determinations of its equilibrium point and also whenever the loads on the pans are being changed. During arrest the knives would be separated from the bearing planes. Many years ago Poynting (1878) showed that the residual errors associated with the use of the balance for fine weighings can be considerably diminished if the balance can be brought to rest, and the interchange of standards on the pans safely effected, while the knives are still in contact with their bearing planes. In the N.P.L. balance this principle is applied by directly steadying the beam after determining its equilibrium position. To enable this to be done, a slightly flexible metal strip is attached horizontally to the lowest part of the beam (figure 2). When the balance is swinging, a pair of agate points suitably mounted on an independent bridge below the beam can be raised so as just to meet the underface of the strip as the beam passes through the horizontal position, and so bring the balance temporarily to rest without separating the knives from their bearing planes. A separate control is provided for this operation, which can readily be carried out. In this condition of the
balance, the pan-hooks can be held in arrest, and the pan plates interchanged without disturbing the beam and the stirrups. The balance is then set free to swing, and the double weighing completed without separating the knives and bearings. An accuracy closely approaching 1 part in 10⁸ has been attained in the comparison of kilograms on this balance.

**Stability of the pound and the kilogram**

With regard to the stability of the primary standards of mass there is no ‘natural’ control to which reference can be made as in the use of the wave-length of light to check the standards of length. Deductions as to the stability of the pound and the kilogram have necessarily to be based on such evidence as may be found in the results of repeated comparisons of different standards over the period of time considered. So far as the pound is concerned, the results of intercomparison of the Imperial Standard Pound and its Parliamentary copies extend back to 1846, though only two intercomparisons were made prior to 1892. For various reasons the evidence is somewhat limited, but it clearly suggests that in relation to its copies the Imperial Standard Pound diminished by 1 part in 3½ million between 1846 and 1883 but remained constant to within about 1 part in 10 million between 1883 and 1933, when comparisons were last made.* The decrease in mass was attributed to wear resulting from too frequent usage. It should, however, be borne in mind that the Imperial Standard Pound and its original copies are somewhat inferior to the relatively modern kilogram standards in respect to the quality of the platinum and its surface finish. The Imperial Standard is the sole ultimate standard, whatever its mass may be; and only in the event of its replacement owing to loss or damage would the copies be used for reproduction of the standard.

Evidence as to the stability of the metric standard of mass will first be considered as from 1889, when the International Prototype Kilogram was installed. To avoid wear, this standard has been preserved unused for at least 50 years, and though arrangements have been made more recently for its recomparison with certain copies and also with the Kilogramme des Archives, the results are not yet available. Many of the copies have occasionally been re-verified by mutual intercomparisons, and the new values obtained for them, based on reference to certain standards at the Bureau (including the témoins), have been found to be in very close accord with those originally assigned to them (Guillaume 1927). Of sixteen copies so compared in 1889 and again in 1899, only four showed any change exceeding 0.02 mg. (viz. 2 parts in 10⁶). Most of the copies which had been kept out of service between the two sets of intercomparisons showed only minute changes, comparable with the order of accuracy of weighing. Similar results were obtained for the interval 1899 to 1911, and so on. It is of interest to note the high stability of the British Copy

* 1938, *Report by the Board of Trade on the comparisons of the Parliamentary Copies of the Imperial Standards.*
A discussion on units and standards

(no. 18), which has been standardized at the Bureau on three occasions. The resulting values, which are given in table 3, do not differ among themselves by more than 2 parts in $10^8$.

**Table 3. British copy of the kilogram**

<table>
<thead>
<tr>
<th>Year</th>
<th>Mass (g.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1889</td>
<td>1000-000 070</td>
</tr>
<tr>
<td>1924</td>
<td>1000-000 051</td>
</tr>
<tr>
<td>1933</td>
<td>1000-000 058</td>
</tr>
</tbody>
</table>

In addition to the evidence obtained from comparisons within the pound and kilogram systems respectively, there is also available,* as a check, a series of four well-established determinations of the relationship between the two systems, the results of which are given in table 4.

**Table 4**

<table>
<thead>
<tr>
<th>Date</th>
<th>Value of the Imperial Standard Pound in grams</th>
<th>Referred to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1846</td>
<td>453-592 652</td>
<td>Kilogramme des Archives</td>
</tr>
<tr>
<td>1883</td>
<td>453-592 428</td>
<td>International Kilogram</td>
</tr>
<tr>
<td>1922</td>
<td>453-592 343</td>
<td></td>
</tr>
<tr>
<td>1933</td>
<td>453-592 338</td>
<td></td>
</tr>
</tbody>
</table>

These results, when plotted, lie on a smooth curve showing a continuous diminution of the pound, as compared with the kilogram. From 1846 to 1883, the diminution amounted to 1 part in 2 million, of which rather more than one-half was suggested by the evidence of the comparisons of the pound and its copies. From 1883 to 1933, the diminution amounted to 1 part in 5 million, which again was more than that suggested by the other source of comparison. Even disregarding the value obtained in 1846, which depends on the now discarded Kilogramme des Archives, the diminution of the pound between 1883 and 1933 is more than can be attributed to experimental error. For this period there is good evidence of the relative stability of the kilogram copies amongst themselves. Moreover, the quality of the material of which these were constructed is superior to that of the platinum pounds, which are of much earlier date. Furthermore, experiments made at the International Bureau, in which one of the kilogram standards of platinum-iridium was kept for a year in a stream of air at $100^\circ$ C, demonstrated that the change in mass was almost negligible (Guillaume 1927). These considerations in combination give reasonable assurance that the kilogram can be regarded as a

* 1936, Report by the Board of Trade on the comparisons of the Parliamentary Copies of the Imperial Standards.
stable standard, and it must be concluded that the pound has changed. It is not proposed to speculate on possible causes of its changes.

In connexion with the determinations of the relationship between the pound and the kilogram, it is of interest to remark that in the United States, where both the pound and the kilogram systems are in use, the pound is defined as bearing a fixed ratio to the international kilogram. The value accepted for this purpose was that determined in 1883.* The British pound, having diminished by 1 part in 5 million since 1883 in relation to the kilogram, is therefore smaller than the U.S. pound by that amount.

Secondary and other standards

The fundamental standards serve as the ultimate basis of weighings for purposes of trade, industry and scientific work, for which secondary and other standards are needed. Before considering how these are derived from the fundamental standards, a brief reference will be made to the materials available for their construction. For most purposes these should be non-magnetic, highly resistant to corrosion, and not too difficult to machine and polish.

On account of its high stability, platinum, preferably alloyed with a suitable proportion of iridium or similar constituent, is pre-eminently the most suitable material for the construction of a precision standard of mass. Owing to its high cost, however, it is not often used for this purpose, except for standards of specially high status, and also for small denominations for which the cost of the metal is not such a serious consideration. Rock crystal has sometimes been used for the construction of precision standards but it is readily electrified and is by no means easily discharged completely. A nickel-copper alloy containing these metals in approximately equal proportions has also been used, but not extensively. Stellite, an alloy containing cobalt, chromium, and a small proportion of tungsten or molybdenum, has also been used, but owing to its hardness the cost of making standards of this material is relatively high. Brass, without any protective coating, cannot, of course, be regarded as a suitable material for a precision standard; its use is still far more extensive than is justified by its properties, and should be restricted to weighings of an appropriately lower class. Plated brass is sometimes satisfactory, but the ordinary process of plating does not always ensure protection from chemical action. Of the metals used for plating, nickel and gold are not found very satisfactory, while platinum has been less extensively employed since the introduction of chromium and rhodium. During the last 25 years, some experience has been obtained in the use of nickel-chromium alloy and stainless steel. The commercial variety of the former, containing 80 % nickel and 20 % chromium, is normally satisfactorily non-magnetic and is being used with some success for secondary standards, and also for analytical weights. Its stability, though reasonably satisfactory, is not, however, comparable with that of platinum.

* The value actually accepted was that quoted in full in the original report, viz. 453\text{-}592 4277.
A discussion on units and standards

The stainless steel which has been in use in recent years is one which purports to be reasonably non-magnetic, being of the austenitic variety containing 18 % chromium and 8 % nickel. This type, however, has not been found sufficiently non-magnetic to give complete confidence in its use and should be superseded by a more reliable variety, e.g. that containing 25 % chromium and 20 % nickel, which is far superior to the 18/8 type in regard to non-magnetic properties. In spite of metallurgical progress there are not many suitable materials for precision work generally, and the choice for the time being is largely restricted to nickel-chromium, stainless steel and plated brass.

In order to determine and maintain a secondary standard pound or kilogram, occasional comparisons have to be made between it and a copy of the fundamental standard; these follow the usual procedure of double weighing, with strict allowance for air buoyancy. The density of the secondary standard, which is usually in the vicinity of 8 g./ml. differs widely from that of the platinum standard, which is about 21 g./ml. The differential buoyancy correction therefore amounts to 1 part in 10,000 of the mass being determined and accordingly requires to be assessed with great care. Assuming that both density values have been ascertained with sufficient precision, the accuracy of assessment of the buoyancy correction depends on the precision with which the density of the air is determined at the time of weighing. This is computed on the basis of the ascertained current values of the pressure, temperature and humidity of the air within the balance. Precautions are taken to introduce fresh air into the balance case but the chemical composition of the air is not actually determined. This may preclude the attainment of an accuracy finer than 1 part in 20 million in deriving a secondary standard of mass from the primary. Of course the buoyancy difficulty might be circumvented by carrying out the weighings in vacuo, but other sources of doubt would presumably be introduced depending for example on the variability of the adsorbed gases on the surfaces of the standards. Whether or not some suitable technique can eventually be evolved for overcoming such difficulties, it appears that the accuracy at present attained is ample for all ordinary purposes.

Further considerations of buoyancy and conventions for adjustment of weights

From the secondary standards are derived the more ordinary everyday standards of mass which for convenience will be referred to by the less explicit term 'weights'. In practice most weights, apart from the smallest denominations, are made of brass or of a material of a similar density. In general the range of densities of the materials in use rarely exceeds 7 % of their mean value. The differences in buoyancy between like denominations will therefore normally lie within 1 part in 100,000 of the mass. These differences, of course, vary somewhat with the air density but as the latter rarely varies by more than 10 % under most ordinary conditions of weighing, the buoyancy difference remains constant to within 1 part in a million of the mass. This enables a large amount of routine weighing to be made in practice without elaborate attention to buoyancy corrections. Thus it is the general practice
to adjust weights on the basis of balancing in air a standard of mass of some conventional density representing brass, the material most commonly used for the construction of weights. This system of adjustment is used for purposes of trade, industry and science, where ordinary and not exceptional precision is required. For trade, the conventional density of brass associated with the standard in this country* is 8.143 g./ml.† Inspectors’ standard weights and trade weights are adjusted to balance the standard in air of normal density; but as has been said before small changes in air density do not seriously affect the result.

Though buoyancy allowances are not normally made in trade transactions based on weight, a fine point arises in monetary transactions in converting from sterling (which is based on the pound) to the currency of a metric country (which is based on the kilogram). The conventional density 8.143 associated with the standard used for trade weighings in this country differs somewhat from that adopted abroad, and on this account the effective conversion from pounds to kilograms, or vice versa, will differ from the pure ratio of the respective units of mass. This difference, which is by no means negligible, appears to be a matter which ought to be referred to the International Committee of Weights and Measures, with a view to arriving at some agreed procedure.

Weights used in industry and science (as distinct from trade) are normally adjusted to balance in air a standard of mass of density 8.4 g./ml. and are calibrated on this basis in air whose density is sufficiently near the normal value not to affect the result appreciably. A typical example of this is a set of analytical weights, containing multiples and submultiples of a gram. These are usually so constituted that, once the value of the head weight is ascertained, the values of the remaining weights can readily be determined by intercomparisons among the weights of a set. Analytical weights are standardized on this basis at the N.P.L. to an accuracy of 1 part in a million for weights of 10 g. and larger, and to 0.01 mg. for smaller weights; this is quite adequate for the great majority of industrial and scientific purposes. As a result of this procedure the weights can, in effect, be treated as standards of mass of density 8.4 g./ml., even though their actual densities may differ slightly from this value. In any chemical weighings where allowances for buoyancy are of importance, these can be assessed so far as the weights are concerned by means of an inclusive correction for their buoyancy based on a density of 8.4 g./ml. This procedure is much simpler than that of calibrating weights according to their true masses, which would involve a knowledge of the actual density of each individual weight, and individual application of its buoyancy correction.

Receptacles for standards

In order to maintain the high stability of weights it is important that the receptacles in which they are stored should be appropriately designed and constructed,

† There is no need here to discuss the reason for the adoption of this particular density value, which was introduced 100 years ago.
A discussion on units and standards

and should not contain any material which is likely to affect adversely the general surface condition of the standards. Manley (1933, 1935, 1945) has investigated the conditions of housing of weights in their cases and has studied the growth of films on the surfaces of weights of different materials under a variety of conditions of storage. His work, which is a substantial contribution to the knowledge of this subject, should be consulted in connexion with the designing of receptacles for weights.

The temperature scale

By J. A. Hall

The direct realization of the thermodynamic temperature scale involves the use of either a perfect heat engine or of a perfect gas, neither of which can be attained experimentally. The scale, therefore, must be reproduced by the use of one of the ‘permanent’ gases, the appropriate corrections to give the thermodynamic scale being deduced from the measured departures from the $PV = RT$ relationship under the particular experimental conditions to be used. The most generally employed gases are hydrogen, helium and nitrogen, and measurements are usually made in terms of pressure at constant volume. A gas thermometer, however, is an inconvenient instrument, and the experimental difficulties are considerable, increasing greatly at high temperatures.

The first step towards making the gas scale readily available was taken by Chappuis in 1888, when he compared four standard mercury-in-glass thermometers with his constant-volume hydrogen thermometer. The corrections needed to reduce the readings of these thermometers (which were used as absolute instruments to reproduce the ‘mercury in verre dur’ scale) to those of the constant-volume hydrogen scale were determined over the range 0–100°C to an accuracy of about ±0.005°C, and similar thermometers made from the same glass were distributed among various national standardizing laboratories and similar institutions. So was the ‘échelle thermométrique normal’ made generally available.

The next stage was the development of other methods of measuring temperature (e.g. the platinum resistance thermometer) which are capable of being read, in certain parts of the scale, to much higher accuracy than the gas thermometer itself. Such instruments, however, have not, in general, any theoretical connexion with the thermodynamic scale, and they can only be used to define it in so far as they have been accurately compared with a gas thermometer. They do not, therefore, improve the position as regards the accuracy with which we can define the thermodynamic scale. If, however, secondary instruments of this type are compared directly (or through some suitable intermediary) with a gas thermometer, they enable us to define the thermodynamic scale as accurately as the gas thermometer observations permit, while at the same time allowing us to reproduce a practical scale to a higher order of accuracy.

Such a practical scale is the International Temperature Scale, which was brought into being at the 7th Conference of Weights and Measures in 1927. As an
example of the distinction between accuracy and reproducibility—a distinction which is always found to take a prominent place in temperature measurement—we may take the case of the boiling-point of sulphur. A study of all gas-thermometer data available at that time showed that the boiling-point lay between 444·50 and 444·60° C, and was almost certainly nearer the latter figure. By the use of a resistance thermometer, however, it is possible to reproduce the point to an accuracy of \( \pm 0.005^\circ \) C. On the International Temperature Scale, therefore, the sulphur point was fixed at 444·60° C by definition, though it was admitted that the second decimal figure was not accurately known. By this means we have a scale which can be reproduced to within \( \pm 0.005^\circ \) at different times and in different laboratories, and which is certainly in agreement with the thermodynamic scale to within \( \pm 0.1^\circ \). Since that date, Blaisdell & Kaye (1941) have analysed all the published work on the subject, including recent experiments at the Massachusetts Institute of Technology, and recommend a revision of the value to 444·7° C.

Some idea of the way in which the accuracy of realization and the reproducibility of the thermodynamic centigrade scale vary over a wide range of temperatures is given in the following table.

**Absolute accuracy of realization and reproducibility of the thermodynamic centigrade scale**

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Accuracy of Realization</th>
<th>Reproducibility</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>-182·97</td>
<td>( \pm 0.02 )</td>
<td>( \pm 0.02 )</td>
<td>resistance thermometer</td>
</tr>
<tr>
<td>0</td>
<td>---</td>
<td>( \pm 0.0003 )</td>
<td>&quot;</td>
</tr>
<tr>
<td>100</td>
<td>---</td>
<td>( \pm 0.003 )</td>
<td>&quot;</td>
</tr>
<tr>
<td>444·6</td>
<td>( \pm 0.1 )</td>
<td>( \pm 0.005 )</td>
<td>&quot;</td>
</tr>
<tr>
<td>1063</td>
<td>( \pm 1 )</td>
<td>( \pm 0.05 )</td>
<td>thermocouple</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \pm 0.1 )</td>
<td>optical pyrometer</td>
</tr>
<tr>
<td>2000</td>
<td>( \pm 6 )</td>
<td>( \pm 2 )</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

The method by which the International Temperature Scale seeks to reproduce the thermodynamic centigrade scale is as follows. A range of fixed points which had been the subject of gas-thermometer measurement was selected. They were chosen so as to cover the temperature range from -182·97 (boiling-point of liquid oxygen) to 1063° C (melting-point of gold), and thence up to the highest temperatures by the use of an agreed value of the radiation constant \( C_2 \). Between the fixed points the platinum-resistance thermometer and the platinum-10% rhodium-platinum thermocouple were chosen as interpolation instruments. The resistance thermometer was selected to cover the range -182·97 to 660° C. It is calibrated by the usual quadratic law based on the ice, steam and sulphur points for temperatures above 0° C, while an extra calibration point at -182·97° C provides the additional term required to define the gas scale accurately below zero.
The platinum thermocouple is used over the range 660 to 1063°C (melting-point of gold) and is calibrated by a quadratic equation through the gold, silver (960·5°C) and antimony (630·5°C) points. The antimony point is not a primary fixed point, and the value of the freezing-point of the particular specimen in use is determined by the resistance thermometer, thus ensuring continuity of the scale at this point. Unfortunately, however, the dividing point of the scale was fixed (for some reason which is not now clear) at 660°C. This has led to the curious result, pointed out by the Bureau of Standards at discussions on the revision of the scale which took place in 1939, that the freezing-point of pure aluminium cannot be measured on the International Temperature Scale. If a determination of the freezing-point is made with a resistance thermometer, the value found is just in excess of 660°C, and thus falls within the domain of the thermocouple. If, then, a thermocouple determination is made, the value is found to be just below 660°C and should therefore be measured by the resistance thermometer. The Bureau of Standards estimate that the discrepancy between the two parts of the scale is about 0·15°C, and in order to eliminate this difference, the 1939 meeting of the Comité Consultatif de Thermométrie* recommended that the scale should, in future, be divided at 630·5°C, the approximate value of the antimony point. The outbreak of war, however, made it impossible for the Conference on Weights and Measures to meet, and this recommendation has not been ratified.

Above the gold point extrapolation is by means of a radiation scale using monochromatic light, and taking 1063°C as the melting-point of gold and 1·432 as the value of $C_2$ in Wien's equation. These two constants are sufficient to define the scale.

There are one or two points which may be mentioned in amplification of the figures given in the table, which refer to the results obtained by using the apparatus and following the technique laid down in the specification of the International Temperature Scale.

It has been suggested (notably by the Physikalisch-Technische Reichsanstalt) that the triple point of water would provide a more reproducible fixed point than the normal freezing-point, but this proposal has not so far gained acceptance, as the view is held by some other workers that little is gained in accuracy to compensate for the loss in convenience. The value of the triple point is about 0·0098°C, of which about 0·007°C is accounted for by the change in pressure and the remainder by the absence of dissolved air (the international specification calls for an air-saturated mixture of ice and water).

It will be seen from the table that it is the steam-point which, almost alone, limits the precision obtainable in the determination of the fundamental interval of a thermometer, though the extremely careful work carried out by Beattie and his collaborators (1935–6) at the Massachusetts Institute of Technology has somewhat altered this aspect of the problem. By using a closed system with an artificial atmosphere of helium in place of the usual open type of hypsometer they were able to obtain a fourfold improvement in reproducibility—somewhat better than

* 1939, P.V. Com. int. Poids Mes.
± 0·001° C. By the use of the same technique they obtained a reproducibility of about ± 0·002° C in the sulphur boiling-point.

Variations in the amount of deuterium oxide present in the water will have an effect on both the freezing- and boiling-points. As far as I have been able to ascertain, the maximum range of concentration which has been measured in water from natural sources is from 1 in 5600 to 1 in 8900. The melting- and boiling-points of D₂O are +3·8 and 101·42° C, so one arrives at an elevation of the freezing-point above that of pure H₂O varying from 0·00043 to 0·00068° C, with the corresponding figures of 0·00016 and 0·00025° C for the boiling-point. Evidently, then, heavy water need not worry us at the present stage of thermometric precision.

When the International Temperature Scale was first adopted, there was no very reliable evidence as to how closely the resistance thermometer scale agreed with the thermodynamic scale in the region between 0 and 100° C where extremely high precision (of the order of 0·001° C) is frequently required. A thorough intercomparison of resistance thermometers with a large number of mercury thermometers of the same construction as those used by Chappuis in his original intercomparison with the gas thermometer was therefore undertaken at the National Physical Laboratory (Hall 1930). This work established identity of the scales to within two or three thousandths of a degree over the range 0–50° C, while the discrepancies between 50 and 100° C certainly did not exceed 0·01° C and were probably a good deal less. It is not possible to be more precise, as Chappuis’s observations above 50° C were confined to two temperatures only. Two pairs of observations were taken in chloroform vapour at 60° C and two pairs in alcohol vapour at 78° C. At 60° C the overall spread was 0·007° C, but there was a difference of 0·012° between the means of the two pairs of observations at 78° C. Giving weight to all these observations leads to a difference between the international scale and the thermodynamic scale of 0·007° at 80° C. One pair of the observations at 78° C is quite consistent with the resistance thermometer scale, and if Chappuis’s equation is modified by ignoring the other pair of observations, then the difference is reduced to 0·003° C.

The question of the eventual elimination of the thermocouple as a standard interpolation instrument has been considered, and a certain amount of experimental work with the resistance thermometer has been done at the gold point. Moser (1930), at the Physikalisch-Technische Reichsanstalt, has obtained values ranging from 1061·92 to 1062·45° C, while some unpublished work carried out at the N.P.L. at about the same time led to provisional values in good agreement with the latter figure. The values found by Callendar (1899) and by Heycock & Neville (1895) when corrected to a sulphur boiling-point of 444·60° C (instead of 444·53° C), were 1061·2 and 1062·2° C. Bearing in mind the fact that the platinum available at that time was far from pure by modern standards, the agreement with the recent work is remarkable.

The internationally accepted value of 1063° C for the gold point is based on the gas-thermometer observations of Holborn & Day (1900) and of Day & Sosman (1911), which led to values of 1064 and 1062·4° C respectively. There is thus no justification for regarding the gold point as being known to a greater absolute
A discussion on units and standards

accuracy than $\pm 1^\circ$ C, and therefore no evidence that the quadratic extrapolation of the platinum-thermometer equation from the sulphur boiling-point does not hold up to the gold point. If 444·70$^\circ$ C is adopted as the value of the sulphur boiling-point, as suggested by Blaisdell & Kaye, the platinum-thermometer values for the gold point are increased by 0·7$^\circ$ C, and the agreement with the gas thermometer is even better. In view of the importance of the gold point as the start of the radiation scale, however, it is necessary to assign to it a definite numerical value, and a small correction term would be necessary to bring the readings of all platinum thermometers to the same value at the gold point, should the platinum thermometer become the recognized means of interpolation up to that temperature.

In using the radiation scale, one of the chief difficulties is to establish and verify the adequacy of suitable black-body conditions. Two methods are available for the optical pyrometer determination of the gold point (and also of the palladium point (1555$^\circ$ C)—a valuable secondary fixed point on the scale). One is the wire method, in which a short length of gold wire is used to complete the junction of a thermocouple inside the black-body enclosure. A second couple is used to control the rate of heating of the furnace and to maintain the furnace steady at the melting-point after the melting-point of the wire has been determined by observing the arrest in the thermo-e.m.f. and ultimate rupture of the circuit. This method, however, is not capable of as high precision as the use of a small ingot in which a black-body enclosure is immersed. The chief problem is to secure uniform temperature conditions in the furnace, and this has been solved in a variety of ways, such as by the use of multiple windings with or without the additional precaution of enclosing the crucible or black-body enclosure in a metal block. A reproducibility of $\pm 0·2^\circ$ C can be obtained at this point. This is equivalent to $\pm 0·6$ at 2000$^\circ$ C, but in practice the reproducibility at the higher temperature is only about $\pm 2^\circ$ C. The additional uncertainty arises from possible errors in the value of the mean effective wavelength to be used in comparing the brightnesses of 2000 and 1063$^\circ$ C and in the angles of the two sectors which have to be used in turn in order to effect the big reduction in brightness (about 1200 to 1), which is necessary to compare the two temperatures.

A measurement of the absolute value of a temperature above 1063$^\circ$ C on the thermodynamic scale by optical means, however, depends on an absolute knowledge of the gold point and of the constant $C_2$ in Wien's (or Planck's) equation. An uncertainty of 1$^\circ$ at 1063$^\circ$ C is equivalent to 3$^\circ$ at 2000$^\circ$ C, and there is still the uncertainty in $C_2$ to be considered. In 1927 the value of 1·432 cm.deg. was internationally adopted. In 1939 the Comité Consultatif agreed that this figure was undoubtedly too low, but decided not to change to the value of 1·436, which was then considered the most probable, on the grounds that the change was hardly significant and that it was better to defer the matter until it seemed improbable that the value would soon need to be changed again. Since then Birge (1941) has derived from atomic constants a value of 1·43845, to which he assigns an accuracy of $\pm 0·0003$. If, then, the uncertainty in $C_2$ be taken as $\pm 0·003$, one arrives at an additional uncertainty of $\pm 3^\circ$ C in temperature at 2000$^\circ$ C, making a total of
± 6° C. Here, again, the reproducibility attainable with the practical instrument is superior to the accuracy with which the scale is known.

The 1939 meeting of the Comité Consultatif de Thermométrie proposed that Planck's law should be substituted for Wien's law as the means of defining the radiation section of the International Temperature Scale, thus enabling it to reproduce the thermodynamic scale to higher temperatures. This proposal, which has not yet been ratified, would only affect a temperature of 2000° C by 0·3° or 3500° C by 0·7°. At 5000° C, however, the difference is 17° and at 6000° C, 50°.

Coming lastly to the most fundamental point, the absolute zero, the 1927 Conference did not attempt to relate the International Temperature Scale to the absolute scale, but only to the Centigrade thermodynamic scale. The position was reviewed by the Comité Consultatif in 1939 when the numerically lowest value considered was −273·144° C, obtained at Leiden, while the highest was a provisional value of −273·16 to −273·17° C relating to the work of Beattie, of which the final result had not then been published.

Considerable discussion took place as to the choice between −273·15 and −273·16° C, and finally the Committee passed a resolution that, in their opinion, 'according to the experiments completed at the present time', the most probable value was −273·15 ± 0·02° C. Since that date, Beattie (1941) has given a figure of −273·165 ± 0·015° C.

An interesting suggestion was placed before the 1939 meeting on behalf of the National Research Council of the United States, to the effect that the scale should be based on the absolute zero and the ice point instead of on the ice point and the steam point, the position of the absolute zero being fixed so as to give the steam point a value of 100·000° C at the present time. In this way a change in the value of the absolute zero would alter the numerical value of any point on the scale except the ice point; that is, the steam point would, in future, be just as liable to alteration in value as any other fixed point on the scale. While it was admitted that this plan would offer advantages to the worker at very low temperatures, it was agreed that, while the position of the absolute zero was still uncertain in the second decimal place, the inconvenience to the metrologist (who would be faced with the prospect of appreciable changes in the temperature known as 20° C) would of itself make it inadvisable to adopt the suggestion at the present time.

Finally, some mention may be made of the real need of industry for a temperature scale reproducible to the order of accuracy which has just been outlined. Mr Barrell has mentioned above the need for an accuracy of one or two thousandths of a degree at 20° C in order to secure adequate precision in the length of the yard and metre, and two other examples will show how the Laboratory is being pressed in other parts of the scale. At temperatures up to about 100° C, the oil technologists are regularly demanding an accuracy of 0·005° C for the determination of kinematic viscosity, while at high temperatures the steel industry is beginning to talk in terms of an accuracy of about 5° at 1600° C. The International Temperature Scale, therefore, has little enough in hand to satisfy modern industrial needs.
A discussion on units and standards

Electrical standards

By L. Hartshorn

Ideal units

To the theoretical physicist electrical standards are entirely unnecessary. A standard of some kind is of course essential if numerical values of the various concepts are to have any meaning in terms of practical experience, but if standards of length, mass and time have been established, the theoretical physicist can define the unit for any electrical property by simply writing down an equation. The theory of electromagnetism plays so important a part in electrical practice that the theoretical units derived in this way, by writing down the simplest possible sequence of equations that will cover the field under investigation, have long been recognized as an ideal system of units by both theoretical and practical men. Like many other ideals it was utterly impracticable when it was first conceived (by Weber in 1851 following Gauss in 1833) and for long afterwards, and it says much for the idealism of the electrical world that it followed the recommendations of the B.A. Committee of 1862, and adopted the system for all classes of work, theoretical and practical. It is curious when we look back to note that the unit first realized, the B.A. ohm of 1865, was in error by 1.3%. The accuracy with which these theoretical units have been realized has, however, steadily improved down the years, and, although it has never kept pace with that of the most precise purely electrical work, the margin between the two is now so small that it is of no significance to anybody outside standardizing laboratories. An accuracy of 1 part per 100,000 has now been realized for all the more important electrical quantities. The present note outlines some of the more interesting metrological problems that have arisen in the course of this work.

Practical standards

The most precise purely electrical work mentioned above consists largely of measurements made by means of the Wheatstone bridge and its various modifications, and the potentiometer. These devices have now been brought to such a state of perfection that it is a fairly simple matter to compare resistances and standard cells with an accuracy of 1 part per million. It follows that with equipment of this kind, and various combinations of resistors and standard cells, measurements of resistance, voltage, current, power, quantity can all be made without much difficulty with an accuracy about ten times as great as that with which the theoretical units are known. Moreover, modern resistors and standard cells appear to be stable over a period of at least a few years with something like the same accuracy, although this is a point not easily substantiated. It follows that if the highest possible precision at any one time were the chief requirement, the best we could do would be to adopt a selected group of resistors and standard cells as primary standards, i.e. on the same footing as the metre kilogram, and secondly, to relate all other electrical measurements to them. This is, indeed, the
course followed in the standardizing laboratories. Each national laboratory sends representative coils and cells, with their estimated values in theoretical units (or whatever other unit is called for in the current agreed international specification), to the International Bureau of Weights and Measures, where they are compared, and as a result of the comparison ‘mean international units’ are adopted with a precision of 1 part per million. In this way arbitrary units are accepted by the national laboratories for the purpose of securing the greatest possible degree of international uniformity, but the procedure ensures that the differences between these arbitrary units and the theoretical (or specified) unit are as small as it is possible to make them; for when assigning their values the national laboratories take into account all the available evidence concerning the theoretical or other specified units.

At the present time the specified units are the ‘international units’ defined by reference to a column of mercury and the silver voltameter, these units having been adopted by the International Conference held in London in 1908, contrary to the opinions of the President, Lord Rayleigh, because the Conference as a whole could not be persuaded that the theoretical units were really practicable.

Recent work has, however, shown that the theoretical units are now more practicable than the specified units, and there is little doubt that the international ohm, ampere, etc., will soon be dropped in favour of the theoretical ohm, ampere, etc. The object of the pioneers, viz. strict consistency between theory and practice, will then have been achieved as far as is possible, though the arbitrary element in the units employed at the standardizing laboratories is always likely to remain.

Relations between electrical and mechanical quantities

Returning now to the work that has proved beyond reasonable doubt that the theoretical units are practicable, note that the problem to be solved is to establish links between the mechanical quantities that are measurable by reference to the standards of mass, length and time, and the various electrical quantities. In theory, the link is energy which is common to the two systems, but unfortunately this quantity, in spite of its dominating importance in physical science, can only be measured by indirect and correspondingly inaccurate methods; and this link, introduced by Thomson (Kelvin) in 1862, is of no direct value in modern metrological practice.

The links that would be of most value in practice are those directly connecting the three fundamental quantities length, mass and time with some electrical quantity. The simplest of such relations is that between time and frequency, a property of alternating currents, but this barely takes us into the electrical world. Length is directly linked to capacitance and inductance. Of these only the inductance link has been established with the accuracy now required, and the unit of inductance is therefore the starting-point of the theoretical system. Several electrical quantities, including resistance, can be measured in terms of inductance and frequency by the technique of the a.c. bridge, but they are all what the electrical
A discussion on units and standards

engineer calls ‘dead’ quantities, mere ratios of the ‘live’ quantities, and to complete the system we must have a link between a ‘live’ quantity and mass. There is no direct link with mass alone, but in the current balance there is a system in which current is related to force and the geometry of the circuits. It therefore gives a somewhat indirect link between current and mass, length and time, expressed by the equation

\[ I^2 = mg/D \quad \text{or} \quad I^2 \frac{dM}{dz} = mg. \]

where \( I \) denotes current, \( m \) mass, \( g \) gravitational acceleration, and \( D \) the dynamometer constant of the fixed and moving coils, which is a derivative of mutual inductance and directly linked with length.

These few links are sufficient to determine the whole system of electrical and magnetic quantities: standard resistors are calibrated in terms of mutual inductance and frequency, standard cells in terms of current and resistance, and everything else is measured by reference to those standards and the standard of inductance.

Mechanical to electrical transfer standards

The problem of establishing these links between the electrical and mechanical units consists essentially in constructing apparatus on which it is possible to make measurements of the highest precision of some electrical quantity and also of the mechanical quantities to which it is related by theory. It is necessary to satisfy at the same time the conditions for precise mechanical measurements, for precise electrical measurements, and those postulated in the theoretical equations. These are very severe conditions, and there is some excuse for those who hold that a system of units which was dependent on the realization of at least two such links was impracticable. For mechanical precision attention must be confined to materials that are sensibly rigid and of low temperature coefficient of expansion. For electrical precision only the best conductors and the best insulators must be used, and for theoretical precision there must be the closest possible approach to linear circuits of zero resistance in a medium that is perfectly non-conducting and non-magnetic.

As is well known three such links have been established at the N.P.L., the Campbell standard of mutual inductance, the Ayrton-Jones current balance, and the Lorenz apparatus. A detailed consideration of the way in which the difficulties have been met in each of these is beyond the scope of a short paper, but it may be noted that mutual inductance is a common factor in the three links, and it follows that the accuracy of the whole system of measurements turns very largely on the accuracy with which inductors of suitable size and form can be constructed and measured. The main features of this side of the work will therefore be noted by way of illustrating its general character.

The type of construction employed in all this work at the N.P.L. is that of the single-layer solenoid, wound with thin bare copper wire under tension on a marble
cylinder. The tension in the wire ensures that the geometry of the coil is almost entirely determined by that of the marble, this material having been chosen for its rigidity, low temperature coefficient of expansion \((4-6 \times 10^{-6} \text{ per } ^1\text{C})\), and tolerably good insulating and machining properties. Lord Rayleigh recently showed that the dimensional stability of marble is far from perfect, and its insulating properties may become poor owing to its hygroscopic character, but nevertheless under suitable conditions it has given satisfactory results, surpassed only by the most recent American work with cylinders of pyrex glass, the working of which is, of course, far more laborious. Accurately defined radial dimensions are automatically obtained by winding on to a cylindrical surface, but the axial dimensions of the coils are equally important, and the precise axial location of the wire is far more difficult. It becomes essential to fix the position of every turn of the coil by winding the wire in helical grooves cut into the surface of the marble cylinder. The machining of these grooves with the necessary precision is one of the most important steps in realizing the units.

It is of the greatest importance that the finished coil shall have the highest possible degree of geometrical uniformity, not only because the theoretical formulae can only be applied to circuits of regular form, but also because only in such a case can the geometry of the circuit be completely specified by a finite number of measurements of length. In order to secure this uniformity, not only must the helical groove be uniform but the wire must be of uniform circular cross-section. One coil was found to show irregularities of diameter because the wire was of elliptical cross-section and the ellipse had rotated during the winding. In the most recent American work, the final drawing of the wire and winding of the coil were carried out in one operation, the wire passing through specially selected dies straight on to the cylinder.

**Corrections for departure from theoretical conditions**

Appreciable departures from uniformity are inevitable when dealing with a material like marble, and it is therefore necessary to make measurements of diameter and axial length for a very great number of pairs of points throughout the length of the helix, and to make allowance for the observed irregularities by making approximate calculations of the effect of each and applying them as ‘corrections’ to the theoretical formula for a uniform helix. The whole process of both measurement and calculation is very laborious, but quite straightforward: the various turns of the helix are regarded as equivalent to circles of a diameter and axial position equal to the mean measured value for the corresponding portion of the helix.

This non-uniformity of the helix is not the only feature in which some departure from the conditions postulated in the theory is inevitable, and in every instance the corresponding ‘correction’ must be estimated and applied. Thus the wire although thin (0.6 mm. diameter) is not a linear conductor. As a first approximation one naturally regards the current as concentrated in the axis of the wire, but in some cases
this approximation is not close enough. In order to find a better it is necessary to know the distribution of current throughout the cross-section, and this is not known. If the wire is regarded as equivalent to a bundle of parallel filaments, allowance must be made for the fact that the outer ones will be longer than the inner ones, and therefore of higher resistance if the resistivity of the material is uniform over the cross-section of the wire. Thus the outer filaments must be credited with less current than the inner ones so that the equivalent single filament is not in the axis of the wire. But here again an unjustifiable assumption has been made: the wire is under tension and the inner filaments are therefore compressed by pressure on to the marble, while the outer ones are stretched; the assumption of uniform resistivity is not justified. The best one can do here is to calculate the correction on the most plausible assumptions. Fortunately, the uncertainty is not large enough to be of any real consequence, though this effect of variable resistivity is believed to affect the temperature coefficient of the best American inductors. Such corrections are less important in mutual inductors than in self-inductors, because in the mutual inductance the uncertainty is confined to large quantities, the distances between the primary and secondary, while in the self-inductor it appears also in much smaller quantities, e.g. distance between adjacent turns.

A correction must also be applied to allow for the fact that materials like marble and pyrex glass are not strictly non-magnetic. Marble is diamagnetic, its permeability being about 10 parts per million less than unity, but since the cylinder is always hollow the ‘medium’ is by no means all marble, and the ‘correction’ for permeability only amounts to less than —3 parts per million in practice.

The effect of the finite resistance of the coils must also be considered. A definite voltage is required to drive the current against this resistance, and the coil possesses an electric field, as well as the magnetic field, in terms of which the theoretical units are defined. This field may give rise to attractive forces in the current balance or to induced voltage in the Lorenz apparatus, or to ‘capacitance effects’ when an inductor is used with a.c. These effects can usually be made negligible by a suitable choice of the electrical conditions, e.g. the potential distribution with respect to earth, or the frequency of the a.c., which must be kept sufficiently low; 10 cyc./sec. is usual.

Conclusion

This very brief survey is sufficient to show the complexity of the problems that have arisen in attempts to approximate as closely as possible to an ideal system of electrical units. So many disturbing factors are discovered in any one piece of apparatus that one is always asking how many have been missed. It is a melancholy historical fact that the experimenter’s estimate of his ‘probable error’ is almost invariably far less than his actual error proves to be. There is, however, one good criterion of accuracy. If observers in different countries get consistent results for the same quantities by different methods of measurement, then they must have realized the same system of units, and it is but reasonable to suppose it is the one they were both looking for.
All the determinations of the ohm and the ampere that have been published from 1870 to the present time are summarized in figures 3 and 4. They have, as far as possible, been reduced to common bases by reference to the surveys given by Smith (1922) in Glazebrook’s *Dictionary of Applied Physics*, vol. 2 and by Curtis (1944). These charts illustrate well for both the ohm and the ampere the steady approach to definite values that are consistent with all the observations made by all the methods within what may reasonably be regarded as their limits of error.
There is good reason to believe that the points in double rings obtained in the most recent measurements at the N.P.L. and N.B.S. represent the most accurate determinations, and it is seen that these deviate from the final value marked on the charts by little more than 1 part in 100,000. The maximum deviation of the three values for the ampere is only 4 parts per million, and of the four values for the ohm only 12 parts per million. It is exceedingly improbable that any of these results are significant to 1 part per million, and there is no reason to suppose that the ampere is known with greater certainty than the ohm. It may, however, be concluded that the uncertainty in both values is approximately 1 part per 100,000, or about one-tenth of the smallest amount that is significant in industrial measurements.

**Figure 4.** Determinations of the ampere. The symbol ' & ' following a name has the same meaning as in figure 3. For further details see Smith (1922) and Curtis (1944).
THE ACCELERATION DUE TO GRAVITY

BY J. S. CLARK

Most determinations of the acceleration due to gravity are relative measurements, the resulting values of \( g \) being made to depend on some one reliable determination of the absolute value. The relative values of \( g \) in such a system are in general more accurate than the computed absolute values.

The values of \( g \) at two stations are usually compared by swinging a pair of so-called 'invariable' pendulums in each of the stations in turn. At each station, the two pendulums, which usually have semi-periods of about half a second, are set swinging in opposite phases so as to eliminate the effect of the motion of the support on which the pendulums are swung. Other methods of comparing the values of \( g \) at two stations have been used, but the above method is the one most generally employed, and is capable of giving an accuracy of about one part in a million.

Only a very few absolute determinations of \( g \) have been made in recent times, and the method used in each case was essentially that of the Kater 'reversible' pendulum, wherein the pendulum was adjusted until its period of oscillation about one of two alternative points of support was made very closely equal to its period about the other point of support when the pendulum was reversed end for end. The points of support may be knife-edges attached to the pendulum (as in the original Kater pendulum) or planes attached to the pendulum, the knife-edge being on the support. In either case, the length of the equivalent simple pendulum depends on the distance between the two points of support.

The accuracy of determination of the absolute value of \( g \) by the reversible pendulum method is inferior to that of relative determinations because, in addition to the corrections which also have to be applied to the observed periods of the invariable pendulums, there are a number of corrections peculiar to the reversible pendulum which are difficult to estimate with precision. These arise from the flexure of the pendulum itself, and from the fact that the pendulum does not oscillate about a geometrical point, but swings about a material knife-edge in contact with a plane surface. The knife-edge usually has a small, but finite, radius of curvature, and it undoubtedly suffers deformation under load. It is likely that the principal cause of uncertainty in the results obtained by means of reversible pendulums is due to the use of knife-edges.

Since 1909, the accepted international base for gravity measurements has been Potsdam. The accepted value, 981.274 cm./sec./sec., is the result of an absolute determination made by Kühnen & Furtwängler (1906) in the ‘Pendelsaal’ at the Königliche Preussische Geodätische Institut in Potsdam, and described by them in 1906. It corresponds with the value 981.275 cm./sec./sec. for the 'Ostkeller' of the Geodetic Institute.

Recent absolute determinations of \( g \) indicate that the Potsdam value is too large.
A discussion on units and standards

In 1936, Heyl & Cook published the result of an absolute determination made at the National Bureau of Standards, Washington, viz. 980-080 ± 0-003 cm./sec./sec. This result is 0-020 cm./sec./sec. lower than the value derived from the Potsdam value by means of a direct connexion between the two stations made in 1933 by Brown (1936) of the United States Coast and Geodetic Survey.

In 1939, the author published the result of an absolute determination made at the N.P.L., viz. 981-181.5 ± 0-001.5 cm./sec./sec. This result is 0-0138 cm./sec./sec. lower than the value derived from the Potsdam value by means of number of transfer measurements adjusted by Bullard & Jolly in 1937.

In 1940, Browne & Bullard published the results of direct intercomparisons made between the N.P.L., Teddington, and the N.B.S., Washington. These relative measurements gave the difference

\[ g(\text{Teddington}) - g(\text{Washington}) = 1-0969 \text{ cm./sec./sec.} \]

the difference between the two absolute determinations being 1-1015 cm./sec./sec.

In 1942, Dryden published the results of his analysis of the corrections which Kühnen & Furtwängler applied in the course of their original Potsdam determination. He suggested that the systematic errors present in the individual Potsdam determinations arose from other causes than those discussed by them, and that in the absence of further knowledge these errors should be considered as of a random nature, in which case a straight mean value, about 12 parts in a million lower than the value adopted by Kühnen & Furtwängler, would better represent the true value of \( g \) at Potsdam.

The following figures summarize the above absolute and relative determinations:

<table>
<thead>
<tr>
<th>(A) Potsdam (amended by Dryden) = 981-262</th>
<th>Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B) N.B.S. (Hely &amp; Cook) = 980-080</td>
<td>± 0-002.5</td>
</tr>
<tr>
<td>(C) N.P.L. (Clark) = 981-181</td>
<td>± 0-003</td>
</tr>
<tr>
<td>A-B (E. J. Brown) = + 1-174</td>
<td>± 0-000 25</td>
</tr>
<tr>
<td>B-C (Browne &amp; Bullard) = - 1-096.9</td>
<td>± 0-001 1</td>
</tr>
<tr>
<td>C-A (Bullard &amp; Jolly 1936) = - 0-078.7</td>
<td>± 0-000 2</td>
</tr>
</tbody>
</table>

The least-square solution of these six results gives

\[ A \ (\text{Potsdam}) = 981-259.5 \ \text{cm./sec./sec.} \]
\[ B \ (\text{N.B.S.}) = 980-083 \]
\[ C \ (\text{N.P.L.}) = 981-181 \]

from which the residuals quoted above have been calculated. Using the original (Kühnen & Furtwängler) Potsdam value 981-274, the least-square solution gives

<table>
<thead>
<tr>
<th>Potsdam 981-265.5</th>
<th>Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>N.B.S. 980-086</td>
<td>± 0-008 5</td>
</tr>
<tr>
<td>N.P.L. 981-184</td>
<td>± 0-006</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>± 0-002.5</td>
</tr>
</tbody>
</table>
No attempt has been made to weight the individual results contributing to these calculations. There would be obvious difficulty in doing so, and the smallness of the residuals indicates that no important change in the calculated values would be likely to result from any reasonable degree of weighting that might be suggested.

According to Dryden (1942), the Sub-Committee on Gravity of the National Research Council Committee on Fundamental Physical Constants has recommended a reduction of values in the Potsdam system by 17 parts in a million in those cases where the most precise absolute values are desired. The above analysis, however, supports the view that values in the Potsdam system should all be reduced by 0·014₃ cm./sec./sec. to give the closest approximation to the absolute values.

It seems unlikely that any future absolute measurements made with reversible pendulums will lead to appreciable improvement in accuracy, despite the very high precision with which measurements of length and time can now be made. There is, however, at least one other possible method of determining $g$ absolutely, which does not involve the corrections associated with the use of a pendulum and knife-edges, and that is by observing the time of free fall of a body in vacuo.

**Standard gravity**

For certain purposes, e.g. the measurement of barometric height, which varies with the acceleration due to gravity, the need for relating the results to some gravity standard has long been recognized. Originally this standard was taken to be gravity in latitude 45°, at mean sea-level. No particular value of gravity was at first associated with this standard, and the reduction of results on this basis was made by means of simple theoretical formulae providing for changes of gravity with latitude and altitude. Later, when the need arose for associating a particular value with the gravity standard, the value 980·665 cm./sec./sec. was accepted by the Third International Conference for Weights and Measures in 1901. This value was subsequently found to be somewhat too high, but it was deliberately retained by the Fifth International Conference in 1913 as the value of standard gravity for reference purposes, and was thereby given a conventional status distinct from the original conception of standard gravity. The same value was used in the definition of the standard atmospheric pressure to which boiling-points on the International Temperature Scale are referred.†

A different view of standard gravity, however, was taken in regard to meteorological work under the auspices of the International Meteorological Committee, and the geophysical basis, viz. gravity in latitude 45° at mean sea-level, was retained. Associated with this is the rounded value 980·62 cm./sec./sec.‡

A discussion on units and standards

The concurrent use of two different conceptions of standard gravity, each supported by the authority of an international organization, is unsatisfactory and is a cause of confusion which should be eliminated. The whole subject of gravity in its bearing on geodesy, physics, meteorology, etc., needs to be reviewed with due regard to changes which now appear necessary in the absolute basis of gravity.

THE BAROMETRIC STANDARD

BY F. A. GOULD

Fundamental considerations

In connexion with the establishment of some of the standards discussed in other papers of this series, an accurate knowledge of atmospheric pressure is necessary. Examples of this may be found in the determination of the steam point and other basic points on the international temperature scale; also in the determination of air density in connexion with certain precision weighings. The mercury barometer provides the most accurate means of determining atmospheric pressure. Barometer scales were originally graduated in units of length, and the process of determining pressure consisted, in principle, of two stages, viz.

(i) the determination of the true height of the mercury column from the scale reading, and

(ii) the translation of that height into a conventional pressure unit.

The first stage is largely a matter of instrumental convention and technique, which will be considered in a later section of this paper. In the second stage it is evident that if the density of mercury and the value of gravity in the vicinity of the barometer are known, the pressure can be expressed in absolute units, e.g. dynes/sq.cm., in accordance with the fundamental hydrostatic equation: \[ p = \rho g h \]. Further, the scale can be graduated in units of length so chosen that the reading taken under specified conditions of temperature and gravity is identical with the pressure expressed in absolute units. In effect, such a scale would represent the result of multiplication of the three quantities, \( h \), \( \rho \) and \( g \), in the hydrostatic equation. This principle is adopted in the graduation of the ‘millibar’ scale (1 millibar = 1000 dynes/sq.cm.).

Apart from the millibar, which is of comparatively recent introduction, the conventional pressure unit adopted is a column of mercury of unit length (inch or millimetre) measured at 32° F (0° C) and subject to standard gravity.

The use of this reference temperature follows a long-established practice, but there are, unfortunately, two conceptions of standard gravity, each supported by the authority of an international organization.* The International Meteorological Committee retain the original conception of standard gravity, i.e. gravity at mean sea-level in latitude 45°, and now associate with it the value 980-62 cm./sec./sec.

* See ‘The acceleration due to gravity’ in this series of papers.
On the other hand, the International Conference for Weights and Measures has defined standard gravity as being equal to 980.665 cm./sec./sec., regarding it as a conventional reference standard distinct from the original geophysical conception. This value is used by physicists, chemists and engineers generally in work of high precision carried out in this country. In addition, the International Conference for Weights and Measures* has defined standard atmospheric pressure as the pressure exerted by a column of mercury, 760 mm. high, and of density 13.5951 g./c.c., subject to an acceleration of 980.665 cm./sec./sec. due to gravity. This definition is followed in precision pressure measurements made in the determination of physical standards and constants.

* Fundamenta standard barometers

(a) Early history. For many years the standardization of barometers was carried out at Kew Observatory where there were two mercury barometers, both having tubes of large diameter and designed to be read by cathetometer. These were installed between 1855 and 1860, and served as the fundamental standards of the Observatory. In 1912, when barometer testing was transferred to the N.P.L., the two standards remained at Kew, and a Fortin type barometer, having a tube of internal diameter 19 mm., was used as the principal reference standard at the Laboratory pending the design and construction of a new primary standard barometer, to be installed at Teddington. The Fortin barometer was compared with the Kew standards before being put into service and was subsequently standardized occasionally from first principles at Teddington. It was later compared with the new primary standard barometer, which was installed at the N.P.L. in 1931.

During the interval, 1912–31, the Fortin barometer remained in exceptionally good condition without need of overhaul, and no appreciable change in its error was found. A high degree of accuracy was attained in the maintenance of a barometric standard in rather difficult circumstances, and no appreciable discontinuity was introduced by the use of different standard barometers in 1912 and 1931.

(b) Primary Standard Barometer at the N.P.L. In this barometer a departure has been made from the usual conception of an instrument based on the use of a glass barometer tube containing the mercury column (see figure 5, plate 6). Instead of glass, a block of stainless steel is used, suitably bored to take the barometric column of mercury, which is of large diameter at the top and bottom, where the mercury surfaces are viewed through optically flat and parallel glass windows sealed to the body of the barometer. Provision is made for the vacuum space above the mercury column to be renewed, as required, by a suitable pumping system. The measurement of the length of the column is referred to a line-standard of length mounted vertically near the column, and is made by means of a vertical comparator which carries a pair of micrometer microscopes disposed one above the other. The comparator can be traversed so that the microscopes are brought opposite either to the mercury surfaces or to the line-standard as required.

The steel body of the barometer is also bored vertically to take a mercury thermometer having a bulb which is adjacent to and extends over the whole length of the barometric column. This thermometer, being immersed in mercury and surrounded by the same mass of steel as the barometric column, records very closely the mean temperature of the latter. The reader is referred to the original account (Sears & Clark 1933) of this instrument for further details, including the optical system for reading the position of each mercury surface.

(c) Reduction of measurements. The first stage in the determination of pressure with this barometer is the measurement of the true height of the mercury column at the prevailing temperature, which can be controlled thermostatically. The height is then corrected to give the equivalent height of the column at 0°C. The coefficient of thermal expansion of mercury which is used for this purpose is sufficiently well established to provide for this correction without any appreciable loss in accuracy.* Further, to express the pressure in terms of a column of mercury at 0°C and under standard gravity 980·665 cm./sec./sec., the height of the column at 0°C is multiplied by the ratio of gravity at the N.P.L. to standard gravity, i.e. by \( \frac{981·181_5}{980·665} \).

Alternatively, if the pressure is desired in dynes/sq.cm., the height of the column, reduced to 0°C but subject to gravity at the N.P.L., is multiplied by the density of mercury at 0°C, which is taken to be 13·5951 g./c.c., and also by the value of gravity at the N.P.L., viz. 981·181_5 cm./sec./sec.

(d) Accuracy. It is estimated that under reasonably steady conditions the barometric height can be determined to an order of accuracy of \( \pm 0·002 \) mm., viz. to 1 part in 400,000 approximately. The accuracy associated with the absolute determination of gravity made at the N.P.L. in 1937 is \( \pm 0·0015 \) cm./sec./sec., (Clark 1939), corresponding to 1 part in 700,000. So far, however, the density of the mercury used in this barometer has not been ascertained, and this is the chief cause of uncertainty in the determination of pressure. The ordinary method of purifying mercury by distillation under reduced pressure results in a partial separation of its isotopes. For this reason the densities of different specimens of mercury vary slightly according to their history. The evidence available from relative and absolute density determinations suggests that the density of mercury should not be regarded as constant to closer than \( \pm 1 \) part in about 150,000 of the mean value 13·5951 g./c.c. assumed. When due allowance for all sources of error has been made, it is considered that an accuracy of \( \pm 1 \) part in 100,000 is attained in the determination of the atmospheric pressure with the Primary Standard Barometer.

Secondary standard and other mercury barometers

(a) Usual types. There are numerous varieties of design of mercury barometers, and these vary considerably in different countries. It is proposed here to refer

* See, for example, *Proc. Amer. Acad. Arts Sci.* 74, 371, 1941.
briefly to two general types of barometers which have long been in use in this
country, viz. the Fortin and Kew types. In the Fortin barometer, settings are
made at the lower and upper mercury surfaces. For the former, a fiducial point
fixed in the ceiling of the cistern serves as the zero to which the mercury surface is
brought through the agency of Fortin’s leather bag and an adjusting screw. The
level of the top of the barometric column is determined by a setting with a cursor
carrying a vernier which is read against the scale of the instrument. In the Kew
barometer only a single setting is required, viz. at the top of the column, this being
effectuated as in the Fortin barometer. To compensate for changes in level of the mer-
ccury in the cistern, the scale is contracted uniformly. The amount of contraction
depends on the diameters of the tube and cistern.

Of the two types, the Fortin barometer is often preferred as an observatory or
laboratory standard, but the Kew barometer is much more widely used for general
purposes owing to its greater simplicity of design. For high accuracy, the tubes of
barometers should be of large diameter. Small tubes are used where portability is
of greater importance.

(b) Scale units and conventions employed. Thé scales of barometers of the Fortin
and Kew types are normally graduated on brass. The inch scale of a mercury
barometer is nominally correct at the temperature 62° F associated with the
definition of the Imperial Standard Yard. Under this convention the scale mea-
sures correctly in inches the current length of the mercury column when the
temperature of the barometer is 62° F.* The convention is applied to all inch
barometers, irrespective of their type. Thus the readings of all inch barometers
should agree at 62° F.

In conformity with the temperature 0° C at which the international metre is
defined, the millimetre scale of a mercury barometer measures correctly in milli-
metres the length of the mercury column when the temperature of the barometer
is 0° C. Accordingly, the readings of all millimetre barometers should agree at 0° C.

The millibar scale is employed almost exclusively for meteorological purposes,
and the current conditions underlying its use were formulated by the Meteor-
ological Office in 1914. The scale is so graduated that its reading gives the pressure
correctly in millibars when the barometer is at a temperature of 285° A (12° C),†
and is at mean sea-level in latitude 45°.

It is unfortunate that the conventions governing the graduation of the inch,
metric and millibar scales lack uniformity. This causes some confusion in the
correlation of corresponding readings on the scales and also in their reduction; and
renders it difficult to prescribe for a fourth scale, viz. a scale of altitudes, which is

* Effect of capillary action. In practice, barometers are adjusted to agree ultimately with
a fundamental standard instrument in which the effect of capillary action is negligible. This
adjustment involves a slight displacement of the ‘zero’ of the scale, so that the reading corre-
sponds, not to the length of the actual mercury column, but to the length which the column
would have in the absence of capillarity.

† For meteorological purposes, temperatures expressed in °A are derived from temperatures
measured in °C by adding 273°.
used on one type of long-range barometer. Consideration should therefore be
given to the possibility of simplifying the current system of scale conventions.

(c) **The testing of mercury barometers.** The testing of mercury barometers is
undertaken at the N.P.L., where they are compared directly with a secondary
standard of the Laboratory. The standard used for this purpose, a long-range
instrument of appropriate design, is compared as required with the Primary Stan-
dard Barometer at the prevailing atmospheric pressure. This comparison is supple-
mented by others made from first principles, using as basis a barometric column
in a large-bore tube in which the mercury levels are read with a cathetometer.
It is estimated that an accuracy of \( \pm 0.03 \) mm. is attained in the determination of
the errors of the secondary standard.

In the routine tests, mercury barometers are set up in a large airtight chamber
in which five instruments of ordinary design are compared with the secondary
standard at the same temperature and pressure. The pressure in the chamber is
set and controlled barostatically at any value in the range required, and the
settings on the barometers are effected by suitable controls from outside the
chamber, enabling the barometers to be read in the usual manner by means of their
cursors and verniers. An accuracy of the order \( \pm 0.1 \) mm. is attained in the stan-
dardization of barometers whose design admits of this precision. In many instru-
ments, particularly those having tubes of small diameter in which the amount of
capillary action is both considerable and variable, the accuracy attainable cannot
be guaranteed to closer than \( \pm 0.2 \) mm., or even \( \pm 0.3 \) mm.

(d) **Reduction of barometric readings.** The term 'reduction' relating to baro-
metric readings is normally understood to cover the corrections which have to be
applied to the readings in order to express the pressure in some conventional form,
either in absolute pressure units or in terms of a column of mercury measured
under specified conditions of temperature and gravity. If the latter form of expres-
sion is chosen, it is important that these conditions should be clearly stated. For
example, the definition of the gallon lacks precision on account of the omission to
state the conditions.*

In the more precise determinations the reduction is made from first principles
with due allowance for errors in measuring the height of the column, for its tem-
perature, and for the value of gravity at the station where the barometer is read.
So far as the temperature reduction is concerned, the Fortin and Kew types of
barometer have a somewhat different temperature coefficient. To avoid misunder-
standing, the appropriate reduction formulae for the respective types are given in
the certificates issued for barometers tested at the N.P.L. unless provision for
effecting the reduction is otherwise made. If the value of gravity at the station is
not known it can be assessed to a reasonably good approximation, in general, by
reference to *International Critical Tables.*† With this information the equivalent
height of the column under 'standard' gravity can readily be computed.

* See 'The measurement of volume' in this series of papers.
For ordinary purposes published tables of temperature and gravity reductions are available.* These lack satisfactory provision for the temperature reduction of Kew barometer readings in inches and millimetres, and are confusing in regard to the two different conceptions of standard gravity employed. More comprehensive reduction tables for inch and millimetre barometers are needed, but before these can be prepared the whole subject of gravity values needs reconsideration with due regard to changes which now appear necessary in the absolute basis of measurement of gravity.†

Note. The scope of this paper has been restricted to determinations of pressure covering broadly the whole natural range of atmospheric pressures and based on the mercury barometer as standard. For the measurement of pressures of a higher order of magnitude, pressure gauges are ordinarily employed. These are standardized by determining the pressure applied by a piston loaded with known weights. Pressure gauges of all ranges up to 12 tons/sq.in. are tested on this basis at the N.P.L. to an accuracy lying within ±1 part in 2000.

THE MEASUREMENT OF VOLUME

BY VERNEY STOTT

Units of volume

There are two kinds of units of volume, those directly derived from the corresponding unit of length, and those defined in terms of the volume occupied, under specified conditions, by a definite mass of water. In any system of units, once the fundamental unit of length has been defined, a hypothetical cube with edges of unit length clearly becomes a unit of volume. For practical purposes, however, hollow vessels are very convenient, and have long been used for measuring volumes of liquid, but to determine their capacity from linear measurements is not nearly so simple as to determine the weight of water required to fill them. Consequently units of volume came into existence defined in terms of the space occupied by a definite mass of water under specified conditions. The two types of unit are defined independently and the relation between them is a matter of experimental determination.

In the older systems of units there is no simple numerical relation between the two types. For example, the number of gallons in a cubic foot is not a whole number. In the metric system an attempt was made to link the two types of unit in a simple manner through the definition of the kilogram, and was achieved to a high degree of accuracy, but, as will be seen in the next section, even in the metric system the two types of unit are now defined separately and the relation between them is one based on experiment.

† See ‘The acceleration due to gravity’ in this series of papers.
**Metric units**

The founders of the metric system sought to ensure a simple numerical relationship between the two types of unit by defining the kilogram as the mass of a quantity of water occupying 1 dm. at its temperature of maximum density. Since, however, a cubic decimetre of water is manifestly unsuitable as a standard of mass, practical realization of the object in view necessitated making a standard of mass, of suitable material, strictly conforming to the above definition of the kilogram. The original Kilogramme des Archives was accepted as fulfilling this condition, and the litre, the volume of a kilogram of water at its temperature of maximum density, was regarded as equal to the cubic decimetre. Subsequently, however, the original definition of the kilogram was abandoned because doubt had arisen as to whether the Kilogramme des Archives did conform precisely to that definition, and the kilogram is now simply the mass of the International Prototype Kilogram.

The litre was separately defined* as being the volume occupied by a mass of 1 kg. of pure water at its temperature of maximum density and under a pressure of 1 atm. The reference to atmospheric pressure was introduced because water is slightly compressible. For pressures in the neighbourhood of 1 atm. and at ordinary room temperature a change in pressure of 1 mm. of mercury causes a change in volume of 0.000066 ml./l. Since the litre is defined on a mass basis, the buoyancy effect of the air will always result in the apparent weight in air of 1 l. of water being less than 1 kg. For example, against weights of density 8.4 g./ml. and in air of density 0.0012 g./ml., the weight of the quantity of water of temperature 4°C which occupies 1 l. at that temperature (and hence has a mass of 1 kg.) is 998.943 g. At higher temperatures the weight will be still less owing to the decrease in density of the water. For example, under the same conditions of weighing as above, the weight of a quantity of water of temperature 20°C and occupying 1 l. at that temperature is 997.176 g.

The present definitions of the kilogram and the litre contain no reference to the cubic decimetre, and when the litre was redefined arrangements were made for the accurate determination of the relation between the litre and the cubic decimetre. Three separate determinations were made.† The principle of each method was to determine by linear measurements the volumes of bodies of simple geometrical form and then to determine the mass of water displaced by the bodies by weighing them first in air and then in water. The methods differed only in the means employed to determine the linear dimensions. Guillaume used three hollow bronze cylinders, closed at the ends, of volumes approximately 780, 1300 and 2000 cm. and having diameters approximately 10, 12 and 14 cm. respectively and determined their dimensions by mechanical means. Chappuis used three glass cubes of 4, 5 and 6 cm. edge and employed a Michelson interferometer to measure the distances between the faces. The third determination was made by De Lepinay, Buisson and

Benoit using two quartz cubes of 4 and 5 cm. edge. They also utilized an interference method to determine the separation of the faces of the cubes but one differing from that employed by Chappuis. The final means of the individual determinations were:

- using bronze cylinders: $1 \text{l.} = 1000.029 \text{ cm}^3$,
- using glass cubes: $1 \text{l.} = 1000.026 \text{ cm}^3$,
- using quartz cubes: $1 \text{l.} = 1000.027 \text{ cm}^3$,

and the value finally accepted on the basis of all the results was

$1 \text{l.} = 1000.027 \text{ cm}^3$

It is a tribute to the skill with which the Kilogramme des Archives was constructed in the eighteenth century that the litre approximates so closely to the cubic decimetre.

The difference in magnitude between the litre and the cubic decimetre and consequently between the millilitre and the cubic centimetre may often be ignored in comparison with the accuracy desired in a determination, but the distinction between the units should be clearly borne in mind and the difference taken into account when necessary. For example, densities expressed in g./ml. will differ numerically by approximately 3 parts in 100,000 from the same densities expressed in g./cm.³, and so when densities of liquids are given to five decimal places the results expressed in g./ml. and those expressed in g./cm.³ will differ significantly in the fifth place.

To avoid much of the confusion which exists at present, it would be an advantage to express density—mass per unit volume—in g./ml. on all occasions, not only because of the difference just noted but also as affording a much better basis than specific gravity. This view is gaining acceptance; for example, density expressed in g./ml. is used throughout the *International Critical Tables*, and British Standard hydrometers are adjusted to indicate density in g./ml. at 20°C.

At one time confusion existed with volumetric glassware because some vessels were marked ‘c.c.’ although the unit used was the volume of a quantity of water having an apparent weight in air of 1 g. It has been stated previously that 1 l. of water of temperature 20°C has a weight in air of 997.176 g., so that a ‘1000 c.c.’ vessel containing water of temperature 20°C having a weight in air of 1 kg. would be nearly 3 ml. greater in capacity than one containing either 1000 cm.³ or 1000 ml. at 20°C. Volumetric glassware, however, is now calibrated in terms of the millilitre, marked ‘ml.’ and also with the standard temperature 20°C; with such glassware no confusion can arise.

*Imperial units*

The gallon is the same type of unit as the litre. It is the volume of ten Imperial standard pounds of distilled water weighed in air ‘against brass weights with the water and the air at the temperature of sixty-two degrees on Fahrenheit’s thermometer and with the barometer at 30 in.’ (§ 15, Weights and Measures Act, 1878).
A discussion on units and standards

The definition does not give precisely the conditions of weighing, and the Board of Trade assumes values of 0.00121698 g./ml. for the density of air and 8.143 g./ml. for the density of the brass weights to define the conditions under which 1 gal. of water of temperature 62° F has a weight of 10 lb. On this basis the gallon is equivalent to 4.54561. For conversion to units based on the Imperial units of length the relation 1 gal. = 277.42 cu.in. may be used. *

Heavy water

Prior to the discovery of isotopes, distilled water was regarded as a perfectly definite and homogeneous substance, and this assumption is implicit in the definitions of the litre and of the gallon. The average ratio of D₂ to H₂ in water is 1 to 6500, and if a sample of water contained D₂ in this ratio the complete removal of the D₂ would decrease the density by 17 parts in a million (Dorsey 1940). The variation in the D₂ content of ordinary distilled water is considerably less than that corresponding to the complete removal of D₂, and the lack of precision in the definition of the litre, in the light of the discovery of isotopes, is not likely to cause any practical difficulty in the measurement of volume.

Values of the density of water have, however, been published giving seven significant figures, e.g. to take only the values at one particular temperature from the tables, 0.9982336 g./ml. at 20° C (Chappuis as recalculated by the National Bureau of Standards (Tilton & Taylor 1937)) and 0.9982303 g./ml. at 20° C (Thiesen, Scheel & Dieselhorst 1900). The difference between these two values exceeds 3 parts in a million. The density of 'pure' water is therefore not known accurately to seven places of decimals, and the isotopic constitution would have to be taken into account in any attempt to establish the density to this degree of accuracy.

Volumetric measurements

Measurements of volume at the National Physical Laboratory have been mainly concerned with the verification of volumetric glassware. There is a great variety of such apparatus, some of which is limited in use to specific purposes, e.g. butyrometers, haemacytometer dilution pipettes, etc. The following details of the accuracy of certification offered for certain simple types of apparatus in general use serve to indicate the precision of measurement attainable in practice.

<table>
<thead>
<tr>
<th>Type of Apparatus</th>
<th>Total Capacity (ml.)</th>
<th>Accuracy of Certification (v) (ml.)</th>
<th>v as Fraction of Total Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>measuring flask</td>
<td>1000</td>
<td>0.1</td>
<td>1 in 10,000</td>
</tr>
<tr>
<td>measuring flask</td>
<td>100</td>
<td>0.02</td>
<td>1 in 5000</td>
</tr>
<tr>
<td>pipette</td>
<td>100</td>
<td>0.02</td>
<td>1 in 5000</td>
</tr>
<tr>
<td>pipette</td>
<td>10</td>
<td>0.01</td>
<td>1 in 1000</td>
</tr>
<tr>
<td>pipette</td>
<td>1</td>
<td>0.005</td>
<td>1 in 200</td>
</tr>
</tbody>
</table>

* The present U.S.A. gallon derives from an old 'wine gallon' and is defined as being 231 cu.in.—inches being defined in the U.S.A. by the relation 1 m. = 39.370000 in.
The accuracy is lower for the smaller capacities than for the larger ones. The development of micro-analysis has created a demand for more accurate measurement of small volumes of liquid. A syringe-pipette recently designed at the Laboratory, in which liquid is expelled from a glass barrel by means of a well-fitting piston working between stops, enables an accuracy of 1 part in 1000 to be obtained on volumes of 1 ml. An accuracy approaching 1 part in 5000 is obtainable with a micrometer operated burette of similar capacity recently made at the Laboratory.

**The photometric units and standards**

*Historical*

From the Light Division, National Physical Laboratory

The candle was legalized by the Metropolitan Gas Act of 1860 which referred to the testing of gas in the metropolitan area. Apparently no unit was ever legalized for any other part of the country or for any other purpose. It had many disadvantages, and the Gas Referees who, under the Board of Trade, were responsible for gas testing under the Act, in their published ‘Notification’ for 1898 discarded the sperm candle in favour of the pentane lamp. What legal power they had to do this is not clear, but it might be held to derive from their position under the Act of 1860. The value of the unit was unchanged, at any rate to the order of accuracy with which it had been maintained.

Now that there is no test for the illuminating power of gas in the metropolitan area, the last vestige of legal status for the standard sperm candle or, in fact, for the pentane lamp, seems to have disappeared.

The unsatisfactory nature of the flame standards used by the different countries, viz. the pentane lamp in Great Britain and the U.S.A., the Carcel lamp in France and the Hefner lamp in Germany, led to proposals to use electric lamps as custodians of the unit of candle-power (Fleming 1902). A large number of comparisons of carbon-filament lamps were carried out in the various national laboratories (Paterson 1909), and as a result it was found that the unit employed in the U.S.A. was 1·6% greater than the British unit, while the French ‘bougie décimale’ was 1% greater. Both these countries agreed to adjust their units to conform with the British unit, and in 1909 the three laboratories concerned issued an identical statement in their respective countries to the effect that from 1 April 1909 they would all use the same unit. The text of this statement is given in a paper by Paterson (1910).

The constancy of the unit was assured by repeated exchanges of specially constructed electric lamps between the national laboratories so that the position was fairly satisfactory, at any rate from the practical point of view, although there was no primary standard. In 1921 the name ‘International Candle’ was given to this agreed unit. Germany, which had not been invited to co-operate, did not come into the scheme and continued to use the Hefnerkerze, which was approximately ninetieths of a candle.
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The present position

The definition of the international candle is incomplete, in that it does not specify how the unit is to be realized in light of any colour other than that of the electric lamps referred to in the definition. These lamps run at a colour temperature slightly below 2000° K, whereas a commercial gas-filled tungsten lamp may run at a colour temperature of 2800° K or more. The completion of the definition had been accomplished implicitly by the Commission Internationale de l'Eclairage in 1924 when it adopted a table of values of the visibility function (the relative sensitivity of the eye to radiant energy of different wave-lengths in the visible spectrum). For some time, however, no attempt was made to use this as a basis for heterochromatic photometry, with the result that the practical units at higher colour temperatures, derived from the carbon-filament lamps at the various national laboratories, showed disagreements of various amounts, the most serious being a progressive change in the ratio of the international candle to the Hefnerkerze from 1·11 to 1·17 on stepping up to a colour temperature of 2600° K.

After a preliminary discussion in 1927, the four national laboratories began co-operative work on the problem of devising a practical means by which the unit at higher colour temperatures could be derived in accordance with the agreed visibility function. As a result of this work it was found that the unit in use in Great Britain, U.S.A. and France for measuring tungsten filament vacuum lamps (i.e. at the colour temperature 2360° K) was too small by about 1 ½ % and that at higher colour temperatures it was smaller still.

While this work was going on, important developments were taking place, at first at the Bureau of Standards in America and later at the other national laboratories, towards the establishment of a primary standard of light, i.e. a standard quite independent of electric lamps and one reproducible from a written specification.*

This standard consists essentially of a small tube of thoria immersed in a crucible of pure platinum. The tube forms a ‘cavity’, or ‘black-body’ radiator, the brightness of which depends solely on its temperature. The temperature of the whole crucible is first raised until the platinum is melted and it is then slowly cooled. The brightness of the mouth of the tube is measured and the value obtained at the ‘arrest point’, which marks the stage at which the platinum is solidifying, is used to define the unit of luminous intensity. It is, perhaps, fortunate that this standard has a colour temperature, 2046° K, which is close to that of the carbon-filament lamps used to maintain the international candle. The average value found at the various laboratories for the brightness of this standard is 58·9 ± 0·1 international candles per sq.cm.

Until 1929 photometry was not among the subjects dealt with by the International Committee of Weights and Measures, but in that year it was added to the terms of reference of the Consultative Committee on Electricity, and in 1935 a Consultative

* P.V. Com. Int. Poids Mes. 1931, 14, 249; 1933, 16, 254, 256, 261; 1937, 18, 247.
Committee on Photometry was formed. This Committee recommended the adoption of a new unit of intensity, such that the above black-body standard should be exactly 60 units instead of 58.9. Though this sensibly alters the candle at this temperature the change is not serious for the temperature 2360° K, since the alteration almost exactly compensates the error at that temperature previously referred to.

The international adoption of the new unit was prevented by the war, though it came into force independently in Germany. It is expected that as soon as international co-operation is restored it will become universal.

The other photometric units, viz. the lumen (luminous flux), the foot-candle or metre-candle (illumination; flux received per unit area) and the foot-lambert or apostilb (brightness; flux emitted per unit area of perfect diffuser) are derived by formal definition from the unit of luminous intensity. The only one of these for which material working standards are required is the lumen. The most convenient method of preparing such standards is first to make point-to-point measurements of luminous intensity for a source which is, as nearly as possible, symmetrical about one axis and which can be rotated rapidly about that axis. A tungsten filament vacuum lamp of suitable design is used. The total luminous flux from the lamp can be found by integration from the mean polar curve of luminous intensity thus obtained. Other working standards of luminous flux can then be prepared by means of a spherical photometric integrator.

**Standards of radiation**

Radiation covering the ultra-violet, visible and infra-red regions of the spectrum can be standardized on a common basis, namely, that of energy. It is convenient to use the electrical units of energy for this purpose, so that the standards of radiation are ultimately based on the units of voltage and resistance.

The evaluation of radiant flux density in absolute units can be obtained by the use of a variety of radiometers. In all of the precision instruments the underlying principle involved is that the radiant energy is degraded into heat by absorption by lamp black or a black-body cavity, and the resulting heat measured either by comparison with heat produced electrically or by the absorption of the heat by Peltier cooling. The efficiency and accuracy of the instruments depend on the closeness with which the conditions of radiative and electrical heating approximate to each other as regards heat loss and distribution.

The accuracy to which the radiant flux density can be determined is very much lower than that attained in the establishment of electrical standards; consequently, the small errors associated with the latter are of no consequence. Scales of radiation maintained at the National Bureau of Standards, the Smithsonian Institute and Uppsala have been compared with the scale maintained at the N.P.L. (Guild 1937) and the general measure of disagreement among these scales probably does not exceed 0.5%. The internal consistency in the maintenance of the N.P.L. scale is of the order of 0.2%.
A discussion on units and standards

The absolute radiometer, like the majority of absolute instruments, is not convenient as a working instrument and is consequently replaced by thermopiles or calibrated lamps for substandards.

For radiation measurement generally the achievement of an accuracy of even 1% demands much careful work so that a better accuracy than this is rarely required except by standardizing laboratories.

Spectral energy distribution

In addition to measurements of total radiation it is important to be able to determine the way in which the energy is distributed in the spectrum. This is particularly important in the visible region. When this distribution is known the three response functions, which are internationally agreed as a definition of the normal observer, enable brightness and colour to be derived. Unfortunately, in the visible spectrum the accuracy of direct spectral energy determinations is very much lower than that attained in related measurements such as spectral transmission or reflexion, and is also lower than the discriminating power of the eye. To ensure internal consistency up to visual standards spectral distributions are obtained indirectly by means of tungsten lamps used as working standards of energy distribution. They are calibrated on a colour temperature scale based on visual comparisons with a black-body furnace.

Acoustical standards of measurement

By R. S. Dadson

Introduction

The last few decades have seen rapid advances in the field of applied acoustics which are of profound significance to the community, but which need no enumeration here. In these advances, the increasing perfection of acoustical measuring instruments, and developments in the absolute measurement of sound, have played an important part and are still finding new applications. The present paper attempts to summarize the principles of those absolute methods of measurement which form the basis of the modern science of acoustics, with some discussion of the practical techniques adopted to meet different requirements and the degree of precision now attainable. No attempt is made to treat of the details of individual methods, in regard to which full references are given. Except where otherwise stated, the discussion is restricted to the measurement of sounds of simple-harmonic type of audible frequency, in air under normal atmospheric conditions.

The measurable quantities which together determine the oscillatory conditions at a point in a sound field are:

(a) the sound pressure, i.e. the alternating component of the total pressure at the point;

(b) the three components of the displacement, or of the particle velocity, relative to the co-ordinate axes.
In practice, however, it is seldom necessary for all of the four quantities referred to in (a) and (b) to be specified. Largely owing to the fact that the response of the ear is most readily interpreted in terms of sound pressure, the latter is the quantity of greatest significance in practical problems. Measurements of sound pressure are commonly carried out by means of microphones, and the calibration of such microphones in absolute terms is one of the most important applications of the basic measurements. Reference will be made in the section on ‘Calibration of microphones’ to the methods used at the National Physical Laboratory for this purpose.

Determinations of the acoustical pressure in sound fields are seldom direct; generally they are derived indirectly by calculation from measurements of particle velocity or displacement. In order to obtain satisfactory results by this procedure it is necessary to work with sound fields in which the theoretical relations between pressure and particle velocity are of reasonably simple form and capable of accurate realization in the laboratory. The conditions commonly employed are plane stationary waves in tubes, and progressive spherical waves in an absorbent-lined cabinet. These two procedures are appropriate for the calibration of standard microphones in terms of their ‘pressure sensitivity’ and ‘field sensitivity’ respectively, as will be further discussed in the section dealing with this work. By the use of devices such as the pistonphone it is possible to generate known acoustical pressures in a small enclosure over the lower part of the audio-frequency range, and thence to obtain absolute pressure calibrations of suitable microphones. This method is independent of the above-mentioned indirect methods, and since the ranges of frequency over which the two procedures are applicable overlap, a useful check of accuracy is made possible.

It is not within the scope of this paper to give details of the different varieties of microphones in use for practical sound measurements. They may be of the condenser, moving-coil, piezo-electric, or pressure-gradient types, all of which have their special applications. Of these the condenser microphone, introduced by Wente (1917), has been found to be particularly suitable for use in the laboratory for standardization purposes. This it owes to its simple and comparatively rigid construction, the possession of a flat and extremely stiff diaphragm which, for many practical purposes, may be regarded as a rigid boundary, its reasonably high sensitivity, and above all its great stability of calibration over long periods of time. There has lately been an increasing tendency to use piezoelectric microphones also as standards. Thus in practice a condenser microphone carefully calibrated in absolute terms may be regarded virtually as having the status of an absolute standard for the measurement of sound pressures under the appropriate experimental conditions. The unit of sound pressure is the dyne/cm.².

**Absolute measurement of particle velocity: the Rayleigh disk**

For the purpose of the absolute measurement of particle velocity at a point in a sound field the Rayleigh disk is widely used. The development of accurate acoustical measurements has been very closely bound up with the study of this most useful
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and simple device. Its action is based on the well-known fact that a flat object suspended in a stream of fluid is acted upon by a couple which tends to turn it at right angles to the direction of flow. It was proposed by Lord Rayleigh (1880, 1882) that this effect should be turned to account for the absolute measurement of sound, and Koenig (1891) obtained the following expression for the torque \( T \) on an infinitely thin circular disk, which has formed the basis of subsequent work:

\[
T = \frac{1}{2} \rho d^3 v^2 \sin 2\theta.
\]  

In this expression \( d \), the diameter of the disk, is assumed to be small compared with the wave-length of the sound, and the density of the material of the disk is assumed to be very large compared with \( \rho \), the density of the medium. \( v \) is the root-mean-square value of the particle velocity in the sound wave, and \( \theta \) the angle between the normal to the disk and the direction of flow. The medium is assumed to be frictionless and the flow laminar.

The disk, carefully shielded from draughts, is suspended in the sound field on a fine fibre usually of glass or quartz, measurements being made either of the torque required to maintain the disk at a fixed inclination to the lines of flow, or of the actual deflexion of the disk in response to the sound. The value of \( \theta \) is normally chosen to be 45°, at which value the sensitivity of the device is a maximum. Practical details of the methods of calibrating and using Rayleigh disks will be found in a particularly useful paper by Barnes & West (1927).

In the application of the Rayleigh disk a number of small departures from Koenig’s formula are encountered which have been studied in detail by many workers. A very comprehensive investigation has been made by Scott (1945). Zernov (1908) and Scott have shown experimentally that on account of the finite thickness of the disk the torque tends to exceed the value predicted by Koenig’s formula to an extent which increases with the ratio of thickness to diameter. According to Scott the excess varies from about 1 to 10% in torque (equivalent to 0.5–5% in particle velocity) over a range of the ratio of thickness to diameter of from 0.006 to 0.12. To minimize this effect, therefore, the disks should be as thin as possible. A small tendency in the opposite direction is also found which originates from the fact that, owing to lack of infinite inertia, the disk is not completely immobile under the influence of the fluctuating torque. Appropriate corrections have been derived by Wood (1935) and King (1935), but, for most practicable types of disks used in air, would only amount to the order of 1% in torque (0.5% in particle velocity). King has also given a comprehensive theoretical treatment of the effect of the diffraction of the sound by the disk. The departure from Koenig’s formula so arising, which is only appreciable at the higher frequencies, reaches about 1% excess in torque (0.5% in particle velocity) when the wave-length falls to some 15 times the diameter of the disk, increasing approximately as the square of the frequency. In the case of disks used in a plane-wave system in a tube, there is a further small effect due to the proximity of the walls, which, according to Scott’s measurements, may amount to an excess of some 2% in torque (1% in particle
velocity) when the diameter of the disk reaches about one-third of that of the tube. Barnes & West have shown that appreciable errors may arise over narrow bands of frequency close to the internal resonances of the disk; these errors, however, are very localized in frequency and may be eliminated if care is taken to avoid using any particular disk at frequencies close to these resonances.

Barnes & West have concluded that if normal precautions are taken to ensure that the conditions presupposed in Koenig’s formula may reasonably be expected to be valid, the measurement of particle velocity may be effected with the Rayleigh disk to an accuracy within 2%.

Merrington & Oatley (1939) and Scott have suggested as a result of their investigations that there is a residual uncertainty in the constant in Koenig’s formula which may lead to over-estimation of particle velocity of up to about 3% over certain ranges of frequency. The origin of this effect is so far obscure and appears to deserve further study. Nevertheless, this conclusion is not greatly at variance with the figure of 2% estimated by Barnes & West. These estimates of the order of accuracy of Rayleigh disk measurements are in line with the experience over a period of years at the N.P.L. in the use of the instrument as a reference standard for the calibration of microphones. In the use of a disk for the comparison of sounds of identical frequency it is possible that the accuracy may be somewhat improved, in that uncertainties in the constant in Koenig’s formula will largely be eliminated.

During the history of its development and use the Rayleigh disk has proved itself within the limits mentioned to be a reliable instrument, the performance of which justifies its continued use as a reference standard.

Absolute measurement of displacement: the smoke particle method

Carrière (1929), Andrade (1931), and subsequently Andrade & Parker (1937), have investigated the use of illuminated smoke particles for the measurement, by direct observation, of the displacement amplitude of sound waves in a tube over a wide range of frequency. Theoretical considerations indicate that particles of sufficiently small cross-section and inertia introduced into the sound field may be regarded as vibrating with very nearly the full amplitude of the wave motion. In practice the observations are made in a plane-wave system in a tube. The particles are illuminated by a source such as an arc lamp and their excursions observed by microscope or by photographic means. Of various substances tried, magnesium oxide smoke has proved especially suitable.

Scott employed this method as a reference standard in his investigation of the residual errors of the Rayleigh disk, and it has also been used by Cook (1946) for assessing the accuracy of the thermophone and reciprocity methods for the absolute pressure calibrations of microphones, which will be discussed in a later section.

As regards precision, it appears that an accuracy of 1% or better, in the measurement of displacement amplitude, may be realized over the range of frequency for which the motion of the smoke particles may reasonably be regarded as indis-
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tinguishable from that of the medium. For particles of magnesium oxide smoke of radius about 0.3 \( \mu \), which is typical of the sizes found in practice, this condition would be expected to hold up to a frequency of about 4000 cye./sec., with an error of perhaps 1 or 2% at 5000 cye./sec. (Scott). Since, for given sound pressure or particle velocity, the displacement amplitude is inversely proportional to the frequency of the sound, it becomes necessary to use somewhat intense sounds at the upper frequencies in order to obtain sufficiently long traces for measurements of the highest accuracy. It is also worthy of note that the smoke method as at present used yields the value of the total excursion (peak to peak) of the particles. Purity of wave form is therefore of particular importance if it is desired to relate the results to quantities such as the root-mean-square pressure or particle velocity determined by other methods.

The technique of observation with smoke particles, although of comparatively recent introduction, has already proved of great value in sound measurement, and gives as close an approach to a truly absolute measurement as any method yet available.

Absolute determination of sound pressure and allied measurements

In so far as the techniques described in the sections on absolute measurement of particle velocity and displacement relate to measurement of pressure, they have relied on the absolute determination of the particle velocity or displacement in sound waves of simple form (such as stationary plane waves or progressive spherical waves) and the calculation therefrom of the sound pressure making use of the known relations between the various quantities. Consideration is now given to an independent class of absolute measurements based on the generation of known acoustic pressures in a small rigid-walled enclosure, by means of which it is possible to obtain directly the pressure calibrations of microphones of certain types. Two pressure generators of this nature, known respectively as the thermophone and pistonphone, have been widely used for the calibration of condenser microphones; it is convenient to consider these two instruments together.

The satisfactory operation of this method depends on obtaining a high degree of uniformity of sound pressure throughout the enclosure. The range of application is thus restricted to frequencies at which the wave-length is large compared with the dimensions of the enclosure.

The thermophone, introduced by Arnold & Crandall (1917) and further developed by Wente (1922), Ballantine (1932) and others, consists of a small metal-walled enclosure containing strips of gold or platinum foil through which is passed a.c. superposed on a much larger direct current. The resultant heating and cooling of the strip generates an alternating pressure in the enclosure which can be calculated provided the many constants of the instrument are accurately known and correct assumptions regarding the modus operandi of the system are made. In practice, however, these calculations are very complicated, and some of the constants and correction factors involved are not altogether free from uncertainty.
The pistonphone operates on much simpler principles. The alternating pressure in the enclosure is generated by the simple-harmonic vibration of a piston, the amplitude of which is either prescribed by mechanical means or directly measured. The calculation of the pressure is comparatively straightforward, and the physical constants involved are few in number and easy to determine.

Several authors (Wente, Ballantine, Cook, Ernsthausen and others) have assessed the performance of the thermophone and pistonphone by using them to calibrate condenser microphones which have also been calibrated by independent means. The pistonphone has generally proved a reliable instrument, and there appears to be general agreement that it may be relied upon to give an accuracy within 3 or 4%. On the other hand, published accounts of the use of the thermophone are inconsistent as to its precision. Whilst certain workers (e.g. Wente, Ballantine) have succeeded in using the thermophone as an absolute instrument with a precision comparable with that of the pistonphone, others (e.g. Cook) have reported considerable discrepancies which they attribute to uncertainties in the behaviour of the thermophones they employed.

Two other methods, which will only be mentioned briefly, may conveniently be grouped with the pistonphone and thermophone; both these methods allow of the calibration of microphones of suitable types by means of purely electrical measurements. The first, introduced by Gerlach (1923) and further developed by Ballantine, is often called the electrostatic actuator method, and was designed for the calibration of condenser microphones. An additional electrode in the shape of a grille is placed close to the diaphragm and an alternating e.m.f. superimposed on a large, steady polarizing e.m.f. is applied between the two. This results in an alternating force on the diaphragm which may be calculated from the form and dimensions of the electrode. This method is reputed to be comparable in accuracy with the pistonphone. The second method, which has received increasing attention during recent years, is based on the principle of reciprocity as applied to linear reversible electro-acoustic transducers.* Accounts of the method have been published by Maclean (1940), Foldy & Primakoff (1945), Cook and others; the latter author has made an extensive investigation of the application of reciprocity to the calibration of microphones of the condenser and crystal types, and has found the results to agree generally with those afforded by the pistonphone and electrostatic actuator to an accuracy of the order 4 or 5%. The principle of reciprocity appears to offer considerable promise as a supplement to the existing methods.

*In its simplest form the method is analogous to the well-known Hopkinson test of the performance of electrical machines.

Calibration of microphones

The absolute measurements discussed in the foregoing sections are suitable only for fundamental laboratory determinations, and most practical sound measurements are carried out by means of microphones, associated with electrical amplifying and indicating equipment. The calibration of such microphones is therefore a
Figure 1. Precision balance at The National Physical Laboratory. Cover removed.

Figure 5. National Physical Laboratory primary standard barometer.
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particularly important application of the basic measurements. Even for purposes of microphone calibration it is not always necessary to refer directly to absolute methods, and it has been the practice at the N.P.L. to maintain certain high-quality microphones, carefully calibrated in absolute terms, as reference standards against which other microphones may be compared. This procedure gives ample accuracy for ordinary purposes as well as resulting in simplification and saving of time. The microphones chosen as standards are of the condenser type (Wente 1917; Oliver 1930), which have been found by repeated tests to maintain stability of calibration over very long periods.

Owing to the disturbance of the sound field caused by the presence of a microphone, including the resonance of the cavity normally existing in front of the diaphragm, it is necessary to distinguish between two different ways of expressing the sensitivity (Aldridge 1928; West 1929). There is, first, its ‘field sensitivity’, which is the ratio of the response of the microphone when placed in a progressive sound wave to the sound pressure which existed at the same position before the introduction of the microphone. This quantity determines the performance of the microphone when employed to investigate free sound fields, and is the calibration appropriate to most ordinary sound measurements. Secondly, there is the ‘pressure sensitivity’ which is the ratio of the response of the microphone to the actual pressure actuating its diaphragm. This form of calibration determines the performance of the microphone when used to determine the pressure set up in a small enclosure, as, for instance, in the case of an artificial ear for the measurement of the acoustical output of telephone receivers.

The determination of the field sensitivity of the standard microphone is carried out, between the frequencies 300 and 10,000 cyc./sec., in a progressive wave in a cabinet heavily lagged with sound-absorbing material, in one side of which is situated a loud-speaker serving as the source of sound. Measurements of the sound particle velocity are made by means of the Rayleigh disk at a series of points on the axis of the loud-speaker and are converted into the corresponding sound pressures from the known relations in the progressive wave. The microphone is substituted for the disk and its response measured at the same positions. At frequencies below 300 cyc./sec. reflections from the walls of the cabinet are no longer negligible, with the result that the method ceases to be valid. However, at these low frequencies the dimensions of the standard microphone are small compared with the wavelength of the sound, and since the field and pressure sensitivities then become practically indistinguishable, the determination of the pressure sensitivity suffices to determine the field sensitivity also. The Rayleigh disks employed at the N.P.L. are normally of glass, 1 cm. in diameter and of the order of 0.015 cm. thick, and are suspended on quartz fibres about 5 μ in diameter. Smaller disks, 0.5 cm. in diameter, are also employed at the upper frequencies. For the accurate determination of frequency, reference is made to the N.P.L. Primary Frequency Standard (Essen 1938.)

The pressure sensitivity is determined from Rayleigh disk measurements in a stationary wave system set up in a cylindrical tube, the diameter of which is
usually chosen to be approximately equal to that of the microphone diaphragm, which closes one end of the tube. The stationary waves are set up by a pure-tone source at the other end of the tube, the Rayleigh disk being situated at an odd number of quarter wave-lengths from the microphone diaphragm, i.e. very closely at a velocity antinode. From the particle velocity determined by the deflexions of the disk the sound pressure actuating the diaphragm of the microphone may be deduced from the known relations in the stationary wave, and compared with the microphone response. Care must be taken in practice to avoid transverse resonances in the tube; in the case of the particular design of microphone concerned, the diaphragm of which is about 5 cm. in diameter, such resonances may be encountered in the frequency range above about 3500 cyc./sec. The use of stationary waves in hydrogen, in which the wave-length at a given frequency is nearly four times its value in air, may provide a means of extending the range of such measurements to higher frequencies.

The stationary wave method is not employed at the N.P.L. at frequencies below 62.5 cyc./sec. on account of the inconveniently long tube required, and at the lower end of the frequency range, from 10 to 400 cyc./sec., the pressure calibrations of the standard microphones have been determined by means of a moving-coil pistonphone.

![Diagram](http://rspa.royalsocietypublishing.org/)

**Figure 6.** Calibration curves of standard condenser microphone.

Calibration curves determined by Mr N. Fleming for one of the standard condenser microphones in use at the Laboratory are shown in figure 6. Over the range 62.5–400 cyc./sec. the pressure sensitivity has been determined both with the Rayleigh disk and pistonphone, the agreement between the results of the two methods being of the order of 3% which may be considered satisfactory. Good agreement is also shown between the pressure and field sensitivities, determined by the Rayleigh disk in stationary and progressive waves respectively, in the region 300–400 cyc./sec., where the two curves tend to become indistinguishable.

*The applications of absolute measurements: measurement of noise*

The application to industrial and other practical problems of the absolute measurements discussed in the preceding sections virtually reduces to the use of
microphones which have been calibrated in absolute terms, usually by comparison with microphones maintained as laboratory standards. These applications vary so widely in nature that the question of the precision of the measurements involved can only be treated in the broadest possible way. Frequently the sounds concerned are very complex, and the circumstances in which they arise can only be approximately specified. An accuracy closer than about 10 %, that is to say of the order 1 db.,* is rarely demanded in such measurements.

In recent years much attention has been paid to the problems of measurements associated with the suppression of noise, and in 1936 the British Standards Institution laid down a standard procedure for the measurement of noise by subjective means, and defined the terms to be adopted. In brief, the loudness of noise is judged by comparison, in free air, with a standard pure tone of frequency 1000 cyc/sec. under certain specified experimental conditions. If the 'normal' observer (i.e. an average of a group of persons of normal hearing) decides that a noise is equally loud to a 1000 cyc/sec. tone of intensity $n$ db. above a reference level corresponding to a root-mean-square sound pressure of 0.0002 dyne/sq.cm. (which is very close to the normal threshold of hearing for the standard frequency), the noise is said to have an 'equivalent loudness' of $n$ British Standard phons. An International Commission held in Paris in 1937 reached provisional agreement on the international standardization of the phon scale. It would be outside the scope of this paper to discuss the question of the standard procedure for such noise measurements in any detail, in which connexion the British Standard Glossary of Acoustical Terms and Definitions (1936) and a paper by Churcher & King (1937) should be consulted. The need for suitable objective instruments for the measurement of noise has also long been realized, and useful progress has been made in the design of objective meters employing a microphone, amplifier and suitable networks simulating the characteristics of the ear, and designed to give readings in phons for various types of noises of common occurrence (Davis 1938; King, Guelke, Maguire & Scott 1941). The precision with which such measurements may, in general, be made and interpreted is difficult to assess owing to the widely differing types of noise encountered, and the variety of circumstances, psychological and otherwise, in which they occur. In the more restricted case of reasonably steady noises, an accuracy of the order 2 to 3 phons should usually be possible if suitable precautions are taken.

The author is indebted to Mr N. Fleming for information concerning some of the past work on acoustical standards at the National Physical Laboratory.

* The decibel notation is commonly employed in acoustics on account of the very wide range of intensity which can be accommodated by the ear. Two powers $W_1$ and $W_2$ are said to be separated by an interval of $n$ db. where $n = 10 \log_{10} (W_1/W_2)$. If the conditions are such that the ratio of the pressures $P_1$ and $P_2$ (or of the corresponding particle velocities) is the square root of the power ratio, then $n = 20 \log_{10} P_1/P_2$ also. In most practical circumstances a variation of less than 1 db. is scarcely noticeable to the ear.
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Figure 1. Precision balance at The National Physical Laboratory. Cover removed.
Figure 5. National Physical Laboratory primary standard barometer.