Influence of space charge on thermionic emission velocities

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A measuring method and experimental results are given on the distribution of tangential velocity components of the electron emission from oxide cathodes under space-charge conditions. This distribution is found either to be of the Maxwellian type with a temperature below that of the emitting cathode, or to consist of discrete velocity groups. Some of these groups include relatively large numbers of electrons with velocities far in excess of what could be expected from the temperature of the cathode. A possible explanation for this novel phenomenon is proposed; the splitting up is derived quantitatively as an oscillation phenomenon in the space charge. The frequency of the oscillation, as calculated from the properties of the space charge, is compared with the wave-length obtained from measurements of the distance of the velocity groups in the above angular distributions.

INTRODUCTION

Little was known about line-focus cathode systems for producing strip-shaped electron beams, when in 1941 some interest developed for their technical application. In consequence, a study of the electron optics of various types of these systems was started. Ray-tracing methods which have been extensively applied in the past for the investigation of circular systems were now used for the strip systems. Pepperpot-like diaphragms were used to cut out of the large beams some fine pencils which could be traced with a sliding fluorescent target. There, in some particular cases, a novel phenomenon was observed. Individual electron pencils appeared to be split up into a certain number of small elementary pencils. This phenomenon was interesting enough to be investigated in detail. If the electron pencil was cut out of the beam by a single narrow slot, extending in perpendicular direction to the cathode strip from which the emission started, the pencil could sometimes be observed to be split up into a multiple of up to 30 elementaries. The phenomenon reminded one of a diffraction pattern. Soon, however, it was found that the production of the elementaries depended greatly upon the space-charge conditions near the cathode surface.

Discontinuities in the velocity distribution are possibly caused by fluctuations in the space charge. Such fluctuations have already been suspected in earlier investigations on shot noise in valves. No indication of discontinuities, however, has been reported in the great number of investigations on velocity distribution of emission from thermionic cathodes which have been published in the literature. But all these investigations have only been made with relatively small currents and did not include the influence of the space charge. It thus seemed worth while to investigate the distribution of tangential velocities emitted from cathode systems under space-charge conditions. Line-focus systems appear to be particularly suitable
for such an investigation because focusing effects which would largely upset the
tangential velocities have there a negligibly small influence upon the velocity com-
ponent parallel to the surface in the direction of the longitudinal extension of the
cathode strip.

1. Description of Emission Systems with Line Focus

Line-focus emission systems are electron sources to produce flat, strip-shaped
beams. The simplest systems of this kind consist of a cathode, a slotted ‘grid
electrode’ and an anode. Two examples of these emission systems are shown in
figures 1 and 2. In these figures co-ordinate systems are drawn with the z-axis
pointing in the direction of the electron beam and with the x-axis parallel to the line
focus which could be produced with the help of such a system. The three drawings
in figure 1 represent cross-sections through the emission system in three different
planes; they show the flat oxide cathode (Ca), the slotted grid electrode (Gr) and the
open box-shaped anode (An). The emission system in figure 2 is identical with the
one of figure 1 except for the fact that the anode is no longer an open rectangular
box but is closed by a diaphragm facing the grid electrode, and having an elongated
slot aperture like the grid electrode. The rectangular shape of grid electrodes and
anodes of the system of figures 1 and 2 is not essential or characteristic for the
working of the system. These electrodes could just as well be of tubular shape with
a circular cross-section, as shown by figure 3. The two classes represented by the
open anode system of figure 1, and the two-diaphragm system of figure 2, however,
have very different characteristics as far as the electrode voltage to emission current
relation and the angular distribution of the emitted current are concerned. Moreover,
the characteristics in each class can be changed largely by changing the
spacings between the electrodes.

![Figure 1](image1)

![Figure 2](image2)

![Figure 3](image3)

**Figures 1 to 3. Emission system with line focus.**
2. Pencil tracing

In ray-tracing experiments such as described for instance by Klemperer & Wright (1939), electron rays are traced as the axes of narrow pencils. The apertures of these narrow pencils depend of course upon the width of the holes of the pepperpot diaphragms which are used for producing the pencils. The pencil aperture, however, also depends upon phenomena which introduce velocity components directed tangentially to the emitting surface. The tangential components are superimposed upon normal velocity components which are produced by the accelerating electric field.

Figure 4. Tracing gear.

Figure 5. Cross-section through tracing gear.

The production of an electron pencil by a line-focus cathode system is shown in the perspective drawing (figure 4) and in the cross-section (figure 5). In both drawings Ca represents the cathode, Gr the grid electrode with the slot aperture Sl, and An the open anode. Pp is the pepperpot diaphragm which selects (through a series of small apertures of about \(\frac{1}{10}\) mm. diam.) a series of narrow electron pencils. One of these pencils (El) is drawn in the figures. The shape of the pencil is traced by the
Influence of space charge on thermionic emission velocities

sliding fluorescent target \((Ta)\), which is kept at a common potential with \(An\) and \(Pp\). Co-ordinate systems drawn in both figures show the grid slot to extend in \(x\)-direction.

All pencils passing the pepperpot diaphragm appear to be spreading mainly in \(x\)-direction. In absence of any electron optical focusing effects, the angle of spreading \((= 2\gamma_x)\) should be given by the Maxwellian distribution of the thermal agitation and by the applied anode voltage. It is surprising, however, that the experimental angles of spreading \((2\gamma_x)\) are often found to indicate temperatures which are substantially different from the cathode temperature. Moreover, under certain conditions, the pencil appears to split up in a multiplet of ‘elementaries’. This is explained by figures 6a to c and 7a and b. Figure 6a shows a circular aperture in the pepperpot diaphragm of \(\frac{1}{10}\) mm. diam. Magnified about 50 times, figure 6b shows the picture on the fluorescent target. The intensity has a maximum in the middle and gradually fades out in \(\pm x\)-direction. Figure 6c illustrates the phenomenon of a pencil through a round hole, splitting up into the 7 elementaries which are marked \((A), (B), \ldots, (F), (G)\).

A still greater number of elementaries can be observed if the round pepperpot hole is replaced by a very fine slot \((A)\ \frac{1}{100}\) mm. wide which is arranged, as shown in figure 7a, in ‘crossed’ position to the cathode. The cathode which is behind the plane of the drawing is indicated by the dotted lines \((Ca)\). The picture observed on the fluorescent target is shown in figure 7b; there can be seen a great number of more or less complete slot pictures, each representing an elementary pencil. Under favourable conditions up to about 30 elementaries have been observed.
3. THE ELECTRON PATH

The splitting up of the electron pencils described in § 2 appears to be a novel phenomenon which has so far not been mentioned in literature. I propose to give here now a more detailed analysis of the relevant facts. The electron paths may be first considered in the $x-y$-plane, sufficiently far away from the edge of the grid electrode so that marginal focusing effects are avoided. The electric field, which accelerates the electrons, is supposed to be directed strictly parallel to the $z$-axis. Moreover, if it is assumed for the beginning that transverse velocities are not affected by space charge, all electrons emitted from the cathode surface are accelerated perpendicularly to it, i.e. in $z$-direction only.

With these preliminary assumptions, the spreading of an electron pencil should simply be due to a superimposition of the tangential electron velocities of emission over the normal velocities acquired by the acceleration in the field of the anode.

![Figure 8. Electron paths and velocity distribution.](image)

In figure 8 there is shown a two-dimensional co-ordinate system $x-z$ with its origin in the centre of the cathode surface ($Ca$). The $x$-axis is in the cathode surface and extends longitudinally to the cathode strip. The $z$-axis passes through the middle of an aperture in the pepperpot diaphragm ($Pp$) at the co-ordinates $(x = 0, z = z_1)$. The diameter of the pepperpot aperture is assumed to be very small in comparison with $z_1$ and $(z_2 - z_1)$. An electron may be emitted with a tangential component velocity

$$ u = x_0/t_1, \quad (1) $$

$x_0$ being the co-ordinate of the point of its emission on the cathode surface, and $t_1$ the time the electron travels from the cathode to the aperture $(x_1, z_1)$. If the voltage
Influence of space charge on thermionic emission velocities

distribution $V = V(z)$ is known, this time of flight can be calculated from the energy equation to be

$$t_1 = \left(\frac{m}{2e}\right)^{\frac{1}{2}} \int_0^{z_1} \frac{dz}{[V(z)]^{\frac{3}{2}}}$$

$$= bt'_1,$$  \hspace{1cm} (2)

where $t'_1$ is the time of flight for a homogeneous field $V(z) = V_1 z/z_1$, $V_1$ being the potential at the pepperpot diaphragm, i.e. at $z_1$.

From (1) and (2) the displacement of the electron at the cathode surface is calculated to be

$$x_0 = u \left(\frac{m}{2e}\right)^{\frac{1}{2}} \int_0^{z_1} \frac{dz}{\sqrt[3]{V(z)}}$$

$$\frac{x_0}{z_1} = b \frac{u}{\sqrt[3]{(\epsilon/e) V_1}}.$$  \hspace{1cm} (3)

Between plane, parallel electrodes, the accelerating electric field will be strictly homogeneous and $b = 1$. In the presence of space charge, however, this field will distinctly deviate from homogeneity. The time of flight of the electron through a space charge between plane electrodes has been calculated by W. R. Ferris (1936) to be

$$t = \left(\frac{m}{2e}\right)^{\frac{1}{2}} \frac{3z_1}{V_1} = 5 \times 10^{-8} \frac{z}{V_1^{\frac{2}{3}} \text{ cm. volts}^{\frac{1}{3}}}.$$  \hspace{1cm} (4)

As compared with equation (2), the time of flight $t$ is increased by the factor $\frac{3}{2}$ if the system is under space charge, and the factor $b$ is increased by the same amount.

For the box-shaped anode ($2x = 32$ mm., $2y = 16$ mm., $z = 50$ mm.) shown in figures 4 and 5, the evaluation of the potential distribution without space charge and its integration (for which I am indebted to Mr L. S. Goddard) leads to

$$b = 0.6.$$  

In practical arrangements using box-shaped or tubular anodes, the potential drop is likely to be concentrated near the cathode surface, and the value $b = 1$ thus represents an upper limit or near enough the correct figure for practical $b$-values.

After leaving the pepperpot diaphragm, the electron passes through a field-free space until it strikes the target at the co-ordinates $x = x_2$, $z = z_2$. The energy equation gives

$$z_2 - z_1 = t_2 \sqrt{\frac{2e}{m} V_1}.$$  \hspace{1cm} (5)

As in equation (1) the tangential velocity component is

$$u = x_2/t_2,$$

and elimination of $t_2$ from the last two equations leads to

$$\frac{x_2}{z_2 - z_1} = \frac{u}{\sqrt[3]{2e/m} V_1} = \frac{u}{w},$$  \hspace{1cm} (5)

$w$ being the constant-velocity component in $z$-direction after leaving the pepperpot diaphragm. According to equations (4) and (5) the arrangement of figure 8 is
comparable with a particular type of light-optical pinhole camera, the ‘image’ at the
target \((x_2)\) being the projection of an object at the cathode \((x_0)\). Different image points,
however, are produced by different velocities, \(u\). Moreover, any line element \(\Delta x_2\)
of the image appears to be scaled down in comparison to the line element \(\Delta x_0\) at
the cathode by a factor
\[
\frac{\Delta x_2}{\Delta x_0} = \frac{(z_2 - z_1)}{2bz_1}.
\] (6)

In order to obtain a full-scale projection of the picture at the cathode surface on to
the fluorescent target, i.e. to make \(\Delta x_0 = \Delta x_2\), the distance of the target from the
pepperpot has to be made \(2b\) times larger than the distance between cathode and
pepperpot. This distance \((z_2 - z_1) = 2bz\) may be called the ‘full-scale target distance’.

4. The velocity distribution

According to equation (5), the current distribution \(I(x_2)\) at the target is a complete
representation of the tangential velocity distribution \(I(u)\) of the electron emission.
This experimental distribution \(I(x_2)\) will now be compared with the well-known
Maxwellian distribution of the temperature agitation in the cathode.

The Maxwellian distribution for a tangential velocity component of emission
can be written as (see, for instance, Richardson 1921)
\[
N(u)\, du = \frac{S}{\alpha \sqrt{\pi}} \exp \left[-\left(\frac{u}{\alpha}\right)^2\right] du,
\] (7)

where \(N(u)\, du\) is the number of electrons emitted per unit time with tangential
velocity components between \(u\) and \(u + du\), \(S\) being the total number of electrons
per unit time and
\[
\alpha = \sqrt{\frac{2kT}{m}},
\] (8)
k being the Boltzmann constant and \(T\) the absolute temperature.

The maximum of the distribution (7) occurs at \(u = 0\), where the exponential
function reaches unity. Dividing (7) by its maximum value and writing currents
\(I(u)\, du\) instead of numbers of electrons per sec., the following relative distribution
is obtained:
\[
\frac{I(u)\, du}{I(0)} = \exp \left[-\left(\frac{u}{\alpha}\right)^2\right] du.
\] (9)

It now remains to express this distribution equation in terms of the directly
observed variables \(x_2\) and \(dx_2\). By changing the variable, equation (9) is transformed into
\[
I(x_2) = \frac{I(0) \exp \left[-\left(\frac{u}{\alpha}\right)^2\right]}{(z_2 - z_1)/w},
\]
using equations (5) and (8), replacing \(w\) by \(\sqrt{\left(\frac{2e}{m} V_1\right)}\) and dividing again by \(I(0)\)
in order to get rid of the constant factor in front of the exponential function, then
\[
\frac{I(x_2)\, dx_2}{I(0)} = \exp \left[-\left(\frac{eV_1}{k \gamma^2}\right)\right] dx_2,
\] (10)
Influence of space charge on thermionic emission velocities

where \( \gamma_x = \frac{x_2}{(z_2 - z_1)} \). According to equation (10) it should be possible to represent the angular current distribution as a straight line, if the log of the current density is plotted against the square of the angle \( \gamma_x \). At a given temperature (e.g. 1000° K for oxide cathodes) the slope of this line should depend upon the anode voltage only. Neither the field distribution between cathode and pepperpot nor the distance between these electrodes has any influence upon the distribution equation (10).

5. Measurements and results

For the experimental test of equation (10) the pepperpot diaphragm (\( PP \)) of figure 4 may be replaced by a diaphragm with one very fine slot, extending in \( y \)-direction as indicated in figure 7a. In a particular example, for which measuring results are given below, the slot is worked by a cold chisel into 0·3 mm. thick Eureka sheet, its dimensions \( 2x \times 2y = 0·05 \pm 0·01 \times 5·0 \) mm., its distance from the grid slot is \( z_1 = 74 \) mm. In the same distance from the ‘pepperpot slot’ is placed the target (\( z_2 - z_1 = 74 \) mm.). Behind the target there is arranged a Faraday cage.

Figure 9. Cathode system and equipotential lines.

Figure 10 shows cross-sections of the general arrangements in the \( y - z \)-plane and in the \( x - y \)-plane respectively. In the middle of the target (\( Ta \)), there is arranged a very fine slot (\( St \)); its dimensions are \( 2x \times 2y = 0·1 \pm 0·01 \times 2·7 \) mm. The Faraday cage (\( Fa \)) is fixed on the back of the target and spaced from it a fraction of 1 mm. by means of an insulating ring (\( Is \)) made from porcelain. In the front face of the Faraday cage (\( Fa \)) there is arranged a slot (\( Sf \)) which is aligned with the target slot (\( St \)), but its dimensions, \( 2x \times 2y = 0·5 \times 3 \) mm., are slightly larger than those of the latter one. The Faraday cage is mounted on a mica disk (\( Mc \)) which is supported by a three-rod assembly, one rod of which (\( Rd \)) is shown in the figure. The whole assembly is surrounded and shielded electrostatically by a closed cylinder, the target shield (\( Ts \)). The front of the target (\( Ta \)) is covered with fluorescent substance which is protected by the tubular ring (\( Tr \)) so that it may be handled safely when the gear is assembled. Any pattern produced by the impinging electrons on the fluorescent target is reflected by the stainless steel mirror (\( Mr \)) through a glass window in the base-plate (\( Bp \)) and can be observed from outside by means of a microscope.

The whole target assembly can be shifted in \( x \)-direction by means of a sliding arrangement, the principles of which are shown in figure 11. The three-rod assembly
O. Klemperer

holding the Faraday cage is fixed in the holding stem (Hs), one rod (Rd) of the assembly being shown. This holding stem is connected with a slide (Sl) which can be shifted in x-direction by means of the calibrated micrometer screw (Mm). A spring (Sp) pressed the slide against the micrometer screw, thus avoiding backlash. While

![Figure 10. Target and Faraday cage.](image)

cathode system, Faraday cage, etc., are in the highly evacuated tube (Gt), the slide moves in air, the evacuated space being separated by the flexible copper bellows (Be).

Cathode and grid electrode are at earth potential, all other electrodes, including the Faraday cage, are connected with the anode voltage \( V_A = V_1 \).
Influence of space charge on thermionic emission velocities

For the measurement of the velocity distribution curve, the target is moved step by step across the pencil, and for each position the current received by the Faraday cage is measured with a sensitive mirror galvanometer. Various results shown in figures 12 to 14 are all obtained with an open anode system as shown in figure 1 having a grid slot $20 \times 1$ mm. In these figures, abscissae are the micrometer slide positions which give the relative positions of the target slot in mm., ordinates are the currents received in the Faraday cage, measured in $10^{-10}$ amp.

![Figure 12](image)

Figure 12 shows three curves, all measured under full space charge. The two solid curves are both taken with the same cathode to top-grid spacing ($t_c$) equal to two grid semi-apertures ($y_{gr}$) but with two different anode voltages: $V_A = 725$ and $V_A = 1200$ V. Under these conditions, the total emission from the cathode is 1 and
2 mA respectively. The broken curve is taken with \(x = y_{gr}\), at 275 V and 1 mA. The area under each curve is very nearly 15 cm.\(^2\)/mA total emission.

On the other hand, the measurements plotted in figures 13 and 14 are not taken under full space-charge conditions. Anode voltages and total emission currents are noted at each curve. The areas under these curves range from 4 to 11 cm.\(^2\)/mA total emission; they are appreciably reduced as compared with the curves taken under full space charge.

Figure 14 is remarkable in so far as it shows five distinct maxima which correspond to the elementary pencils shown in figures 6 and 7.

In order to check whether the curves of figures 12 and 13 correspond with Maxwellian velocity distributions, the ordinates \(I\) have to be divided by the maximum ordinate \((I(0))\) of each curve. The abscissae are measured here as \(x\)-distances from the maximum \(I(0)\) of the curves and divided by the distances of 74 mm. between pepperpot diaphragm and target (see figure 8(z - z\(_1\))). In this way the semi-apertures \(\gamma_x\) of the pencils are obtained. These \(\gamma_x\) values are then still reduced to an anode voltage of 1000 V by multiplying with \(\sqrt{V_A/1000}\) in accordance with equation (5):

\[
\gamma_0 = \frac{x_2}{z_2 - z_1} \sqrt{\frac{V_A}{1000}}.
\]  

(11)

In figure 15 the values of \(\gamma_0^2\) are plotted as abscissae against the ordinates \(I/I(0)\) in logarithmic scale. The various symbols in this graph represent the values calculated from four different curves with the same symbols in figures 12 and 13. Voltages \(V_A\), currents of the total emission \(I_{em}\), and the characteristic constants \(D = V_{gb}/V_A\) (= penetration factor) and \(G = I_{em}/V_{gb}^4\), are noted in the figure for each symbol, \(V_{gb}\) being the negative 'black out' voltage which has to be applied to the grid electrode in order to cut off the total emission.
Influence of space charge on thermionic emission velocities

The values of these four examples, and as a matter of fact, all values calculated from any of the curves of figures 12 and 13 closely fit to a straight line which reaches the ordinate \( I/I(0) = 0.1 \) at the abscissa \( \gamma_0^2 = 1.4 \times 10^{-4} \). Thus, the experimental values represent a purely Maxwellian distribution. The temperature \( T \) of the distribution is obtained from the slope of the straight line which according to equation (10) is

\[
T = \frac{(eK_i/k)\gamma_0^2}{\log \text{nati} I/I(0)}. \tag{12}
\]

With \( e/k = 11,600{\text{°}}/\text{volts} \), and with \( \log \text{nati} I/I(0) = 2.30 \) for \( I/I(0) = 0.1 \), equation (12) yields

\[ T = 5.05 \times 10^6\gamma_0^2. \]

For example, with \( T = 1000{\text{°}} \text{K} \), the straight line representing equation (10) in the semi-log plots of figures 15 and 16 would intersect the \( I/I(0) = 0.1 \) abscissa at \( \gamma_0^2 = 2 \times 10^{-4} \); with \( T = 700{\text{°}} \text{K} \) it would intersect this abscissa at \( \gamma_0^2 = 1.4 \times 10^{-4} \). Both these examples are plotted in figure 15 as well as in figure 16. \( T = 1000{\text{°}} \text{K} \) corresponds to the temperature of the emitting cathode while \( T = 700{\text{°}} \text{K} \) represents well the experimental results plotted in figure 15. On the other hand, the experimental values plotted in figure 16 do not correspond to an electron gas of homogeneous temperature but rather to discrete groups of electrons of different temperatures ranging from below 700° K to far above 1000° K. These somewhat surprising results have been confirmed here by various experiments under different conditions, and the possibility of substantial influences of spurious effects seems to be ruled out. For instance, the small diverging influences upon the pencils caused by electron optical effects in the box anode and near the ‘pepperpot slot’ are negligible. Ray-tracing results suggest that pencils which are sufficiently near to the axis (z) of the box are not focused by the field in the box. Moreover, the accelerating field is sufficiently concentrated near the cathode, that relatively small fields can be expected near the pepperpot slot, and that the fully accelerated beam is hardly influenced by these small fields.

Suspicion of surface charges on the jaws of the pepperpot slot should be discarded, since the same reduced velocity distribution is measured for anode voltages as different as 200 and 4000 V.

6. Discussion and Theory

The velocity distribution of thermionic emission has been measured by a great number of observers (see, for instance, Germer (1925) or Nottingham (1932)). All these investigations have been made in the retardation region; results as shown in the figures given here have not been reported. Richardson (1908), in particular, investigated the tangential velocity components of emission; his experimental arrangement (movable cathode strip, Faraday cage with one slot) is sufficiently similar to that given here to enable a close comparison of results to be made.
Influence of space charge on thermionic emission velocities

Richardson's investigations, however, refer to relatively small electron currents in the region of temperature limited emission, and there is obtained a Maxwellian distribution of the proper temperature of the emitting cathode.

The present results—i.e. either a reduction of the temperature of emission with respect to the cathode temperature, or under other circumstances a splitting up of the continuous distribution in discrete velocity groups—thus appear to be caused by space-charge effects in front of the cathode. The effect of temperature reduction might be related to the effect of reduction of shot noise by space charge, which has been discussed repeatedly in the literature (see, for instance, Thompson & North 1940).

The only explanation for the splitting-up effect which is free of contradictions and can be given at present is based upon the possibility of space-charge oscillations. Fluctuations in the space charge have already been suspected in earlier investigations on shot noise in valves (see, for instance, Thompson & North 1940). Bull (1940) was the first to my knowledge to have suggested that oscillations, set up in a space charge, might give rise to an uneven withdrawal of current over the cathode surface. Moreover, some useful notions can be derived from a theoretical treatment of the electronic space charge given by Laue (1918).

The pressure in a space charge is composed of a kinetic and of an electrostatic component. In a direction normal to the cathode surface, both components exactly balance each other. In tangential direction, however, they both add. For a plane cathode both components are equal. Thus the total pressure in tangential direction is

\[ p = 2NkT, \]

where \( N \) is the number of electrons per c.c. The propagation of a wave in tangential direction should be possible, and the order of magnitude of its velocity \( U \) may be estimated from Newton's formula

\[ U^2 = \frac{\text{pressure}}{\text{density}} = \frac{2NkT}{Nm} = \alpha^2, \]

where \( m \) is the mass of an electron. It is remarkable that \( N \), the number of electrons per unit volume, cancels out, and that the velocity \( U \) of the wave equals, according to equation (8), \( \alpha \), the most probable velocity of the temperature agitation of the electron gas.

With suitable boundary conditions, a standing wave may be formed in the space charge, producing a stable pattern of nodes and antinodes of constant pitch. I believe that the periodic pattern formed by the splitting up of the electron pencils, which is shown in figures 6, 7, 14 and 16, is an image of the nodes and antinodes at the cathode. If \( \gamma_e \) be the experimental average angular distance of the elementaries, then the distance \( \frac{1}{2} \lambda \) of two nodes and the wave-length \( \lambda \) of the space charge oscillation is given according to equation (8).

\[ \lambda = 2\gamma_e z_1 2b. \]
Moreover, the frequency of the oscillations is

$$f = \frac{U}{\lambda},$$

with

$$U = 1.7 \times 10^7 \text{ cm./sec. at } 1000^\circ \text{ K and}$$

$$0.6 < b < 1.2,$$

$$0.01 < \gamma_\varepsilon z_1 < 0.04 \text{ cm.},$$

$$0.02 < \lambda < 0.2 \text{ cm.},$$

the frequency is found to be $10^8 < f < 10^9 \text{ cyc./sec.}$

Standing oscillations can be detected with the great majority of all cathode systems, including systems with complete axial symmetry, provided that the resolving power of the used 'pinhole camera' is chosen high enough and that the right conditions of $J_{em.}/V_A^2$ ($J_{em.} =$ emission current, $V_A =$ anode voltage) are properly adjusted. The wave-length of the standing wave is substantially the same for different geometrical arrangements, but the amplitude of the oscillations which is indicated by the number of elementaries into which the pencil appears to be split up varies over a wide range.

The amplitude of the oscillations is high: (1) if the current density in the system as operated under full space-charge conditions is high; (2) if the potential distribution near the cathode contains approximately straight parts of equipotentials which are long in comparison with the wave-length of the oscillations. A long, straight path, apparently, allows the oscillation amplitude to increase.

An illustration of a straight part of an equipotential line is given in figure 9. There $Ca$ represents the cathode and $Gr$ the grid electrode of a cathode system. Equipotentials are drawn as broken lines ($Eq$), and at one of these equipotentials, the straight part is marked by the points $A$ and $B$. Figure 9 represents a section in the $y-z$-plane. There the straight parts of the equipotentials in $y$-direction are relatively short. Much longer are the straight parts which extend in the $x$-direction. This is in agreement with the observation that elementaries are much more easily formed with their axes in the $x-z$ than in the $y-z$-plane.

The experimental limits for the wave-length $\lambda$, as given above, differ by a factor 10. From the following argument, however, it appears that the true value of $\lambda$ which, owing to equation (14) should be unique for a given cathode temperature, is probably nearer to 0.02 than 0.2 cm.

The experimental values of $\gamma_\varepsilon z_1$ vary by a factor 4, but when a relatively wide separation of the centres of apparent elementaries is observed, such apparent elementaries can often be resolved as consisting of clusters of true elementaries. Higher resolution of the image may be obtained by increasing the target distance or by a greater magnification of the observing microscope or quite often by adjustment of the filament temperature and/or anode voltage which may result in a greater amplitude of the oscillation.
Influence of space charge on thermionic emission velocities

The shortest wave-length, revealed by the resolution of a cluster, is, however, never found smaller than 0.02 cm. Clusters, on the other hand, can often be mistaken as elementaries because the intensities of the different elementaries belonging to one pencil vary by an order of magnitude. A pencil may be split up in a certain pattern of elementaries of periodically varying brightness, the brighter elementaries overlapping while the darker ones are mistaken as gaps. A given intensity pattern often appears to change spontaneously in the course of a few minutes' observation. Moreover, an apparently stable pattern can often be changed greatly by applying a very small bias to the grid electrode of the cathode system.

7. Observations with the fluorescent screen

For an investigation of the patterns of elementaries a visual observation on the fluorescent target is much superior to the current measurement with the Faraday cage. These often unstable and changing patterns require more rapid means of observation than could be provided by a point-by-point current measurement with a mirror galvanometer. Moreover, small, but steep, contrasts as occurring in the patterns of elementaries are much more easily detected visually than by current measurement through a slot of finite width. On the other hand, a continuous Maxwellian distribution is more easily investigated by means of the Faraday cage than by means of a fluorescent target. In some cases, however, it seems worth while to inspect the fluorescent spots which are due to pencils with a Maxwellian distribution of intensity such as shown in figure 12.

There the brightness distribution which actually fades away gradually towards the margin appears to be visible on the target as a spot of definite dimensions. Moreover, all measurements of spot width seem to be well reproducible by different observers on different days. The spot width is measured here on the scale of a reading microscope, this scale being illuminated by a dim yellow light. The brightness of the light is controlled by hand with a potentiometer to make the figures of the scale just readable.

I propose here to explain the experimental spot width by the assumption that the green light emitted by the fluorescent Willemite target ceases to be visible when its intensity drops below a certain level which substantially is given by the background illumination. The essential correctness of this assumption is borne out by the results plotted in figure 17. There $\gamma^2 = \text{the square of the visual semi-divergence of the pencils, as reduced for 1000 V with equation (11)}$ is plotted against $I_{\text{em.}} = \text{the total emission from the cathode in log scale.}$

The current densities in the pencil are proportional to the total emission shown by the ordinate.

All points plotted in figure 17 are taken at the constant anode voltage $V_A = 2 \text{kV}$. The total emission, however, varies between 30 and 7000 $\mu\text{A}$. Observations are made with an open anode system as shown in figure 1. The cathode to grid distance is varied, but always full space-charge conditions are maintained. It is known from
an earlier investigation that the angular current distribution of this open anode system does not much depend upon the grid to cathode spacing; thus the ratio of pencil current and total emission is practically unchanged over the whole range of measurements. Near the maximum of the distribution, $10^{-2}$ amp./m$^2$ are observed

$$\text{visible intensity ratio} = \frac{I_{\text{edge}}}{I_{\text{max}}}.$$  

**Figure 17.** Width of spots produced by pencils on fluorescent target. 
Values $\bigcirc$ measured at 2 kV target voltages.

on the target per mA total emission. All measurements are made at the same cathode temperature, and the two distances of cathode to pepperpot diaphragm and of pepperpot diaphragm to target respectively are kept constant and equal throughout. Thus exactly the same constant angular current distribution should be found in all observed spots: the great range of observed spot width, however, has to be explained solely in terms of the characteristics of the fluorescent target.
The brightness of fluorescence in the region of the measurement given here (2 kV, 10^{-2} amp./m.²) is proportional to the current density, and also it is practically about proportional to the voltage of the impinging electrons. Thus it follows that of a given distribution a different range becomes visible when the voltage or the current of the pencil are varied. The experimental values plotted in figure 17 are close enough on a straight line excepting those for the lowest currents. Proportionality between current and brightness thus is sufficient to explain the observed spot width within current variations of the order of 1:100. The excessive width of the low current spots is not quite understood. It may be connected with the relatively small contrast of the visible part of the spot if it is not due to a widening of the Maxwellian distribution. This, however, has not yet been checked up with the Faraday cage method, since the galvanometer used was not sensitive enough to measure these very small currents (10^{-12} amp.) with sufficient accuracy.

Some measurements on spot width made at 1, 3, and 4 kV are in sufficient agreement with an assumed proportionality between voltage and brightness. These results are not plotted in the figure but they would group sufficiently well about the broken lines shown there, just as well as the points taken at 2 kV are grouping about the solid line.

In agreement with the above statement, the divergence of the visible part of the pencil discloses the ‘visible intensity ratio’ (= intensity of the edge of the spot, where it merges with the background compared with the maximum intensity in the middle of the spot). Thus the abscissa of figure 17 can be recalibrated with the help of the straight line of figure 15 which represents a Maxwellian distribution of emission temperature. This new scale is shown on the top of figure 17 and, according to it, for the weakest and for the brightest observed spots, the visible intensity ratio in an individual spot may vary between \( \frac{1}{5} \) and \( \frac{1}{100} \) respectively.

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