A note on a problem of Mahler

It is clear that the star bodies $S$ and $S_0$, defined in the proof of theorem 3, satisfy the conditions of theorem 4. This proves theorem 4, and shows that even this one-sided result is false for unbounded star bodies.

The results of this section should be compared with the results obtained by Mahler (1946) using a different definition of limit in the space $P$.

References


Extensive penetrating cosmic ray showers

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Extensive penetrating cosmic-ray showers were recorded with an arrangement consisting of a penetrating shower set $P$ and an extension $E$ containing shielded and unshielded counters. The following results were obtained: (1) The effective density integral spectrum of the showers observed is given by $D(x) = Ax^{-6+1}$, where $x$ is the number of particles per m.$^3$. (2) The rate of shower coincidences decreases only slightly when the extension is moved from a distance of 0-5 m. to a distance of 9 m. from the main set $P$. (3) The ratio of the coincidences obtained with shielded and unshielded extension counters does not change noticeably with increasing separation between $E$ and $P$. The results are based on approximately 10,000 hr. of recording.

1. Introduction

In a former paper (Broadbent & Jánossy 1947, referred to as I) it has been shown that penetrating showers are of two distinct types: (a) local penetrating showers, which probably consist of groups of mesons produced by fast nucleons, and (b) extensive penetrating showers, which are large air showers containing some penetrating particles.

It has been inferred by Cocconi, Loverdo & Tongiorgi (1942, 1943, 1946) that all extensive showers contain penetrating particles; the penetrating extensive showers under investigation will thus be assumed to be ordinary extensive showers, that is, large cascade showers containing some penetrating particles (see also Daudin 1942; and Clay 1943).

The present investigation deals with the production of penetrating particles in extensive air showers.
2. THE EXPERIMENTAL ARRANGEMENT

The arrangement consisted of the penetrating shower set $P$, described in I, together with an extension shown in detail in figure 1. The extension consisted of two counter trays, an unshielded tray $E$ of twelve counters in parallel of area 2500 cm.$^2$, and the tray $S$ of area 1000 cm.$^2$ covered on the top and sides by 15 cm. of lead and with 10 cm. of lead below. The tray $S$ consisted of eight counters in two groups $S_1$ and $S_2$ of four counters each, alternate counters being in the same group as shown in figure 1 (see also figure 2).

![Figure 1. The extension.](image)

![Figure 2. Scheme of the experimental lay-out.](image)

Coincidences of the following types were recorded:

(1) Coincidence between $P$ and at least one of the counters of the tray $E$; designated $(P, E)$. 
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(2) Coincidences between $P$ and at least one of the counters $S_i$; designated $(P, S)$. The coincidences $(P, S)$ were caused by extensive showers containing at least one penetrating particle at $S$.

(3) Coincidences between $P$ and at least one counter in each of the trays $S_1$ and $S_2$, designated $(P, S_1, S_2)$. The coincidences $(P, S_1, S_2)$ were due to extensive showers containing at least two penetrating particles at $P$ and at least two particles penetrating the extension.

During part of the experiment, an additional tray $E'$ of eight counters in parallel covering an area of 1000 cm$^2$ was used and the coincidences $(P, E')$ were recorded simultaneously with the coincidences $(P, E)$.

Various absorbers $T$ of lead and paraffin were placed close above the top tray of counters of the set $P$ during some of the observations. The values of $T$ are attached to the tables of results given below.

3. The experimental results

(a) The density distribution of the showers

The density distribution of penetrating showers was derived from counter observations, using a method somewhat similar to that used by Cocconi and also by Daudin (1942). In the present experiment the size $F$ of a counter tray was varied and a record of the coincidences $(P, F)$ as a function of the area $F$ was taken.

In Cocconi’s experiments the areas of several trays are changed simultaneously and the rate of coincidences is measured as a function of the simultaneous change of area of all trays.

In our experiments the area of the extension $F$ is changed but $P$ remains unchanged. Thus the interpretation of our results is different from the interpretation of the Cocconi type of experiment, and is as follows.

The set $P$ introduces a certain bias towards the showers of higher densities, and therefore those showers capable of setting off $P$ have a certain density spectrum $D(x)$ different from the unbiased density spectrum $C(x)$ of all showers. By varying only the area $F$ of the extension the biased spectrum $D(x)$ is measured, while changing the areas of all trays of the set simultaneously would give the unbiased spectrum $C(x)$.

The actual bias introduced by the set $P$ was found to differ according to the thickness and material of the absorber $T$.

In the actual experiment, instead of changing the area of one extension, the simultaneous coincidences with two extensions of areas $E$ and $E'$ were observed.

Thus we observed simultaneously: (1) coincidences $(P, E)$; (2) coincidences $(P, E')$; (3) coincidences $(P, E + E')$. The last type of coincidences are effectively the coincidences between $P$ and a tray $E + E'$, containing in parallel all the counters of the trays $E$ and of $E'$. The three coincidence rates thus represented the variation of the coincidence rate as a function of the size of the extension.
The coincidence rates obtained with $T = 5\text{ cm.} \ Pb$ and $T = 10\text{ cm.} \ Pb$ and the average of the two rates are collected in table 1.

**Table 1. Coincidence rate as function of the area $F$ of the extension tray. $T = 5\text{ cm.} \ Pb$ and $T = 10\text{ cm.} \ Pb$**

<table>
<thead>
<tr>
<th>$T$/cm. Pb</th>
<th>time (hr.)</th>
<th>$F = 1000\text{ cm.}^2$</th>
<th>$F = 2500\text{ cm.}^2$</th>
<th>$F = 3500\text{ cm.}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>280</td>
<td>26</td>
<td>46</td>
<td>51</td>
</tr>
<tr>
<td>rate</td>
<td>0.093 ± 0.018</td>
<td>0.164 ± 0.024</td>
<td>0.182 ± 0.026</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>308</td>
<td>17</td>
<td>27</td>
<td>33</td>
</tr>
<tr>
<td>rate</td>
<td>0.055 ± 0.013</td>
<td>0.088 ± 0.017</td>
<td>0.107 ± 0.019</td>
<td></td>
</tr>
<tr>
<td>average rate</td>
<td></td>
<td>0.073 ± 0.011</td>
<td>0.124 ± 0.015</td>
<td>0.143 ± 0.016</td>
</tr>
</tbody>
</table>

Note that the rate of coincidences increases noticeably with increasing area of the extension tray. It must be emphasized that the three types of coincidences are recorded simultaneously, and that therefore the comparatively small difference between the rates obtained for different values of $F$ are still statistically significant. The relative difference rates between the various extensions have the following numerical values.

**Table 2**

\[
\begin{align*}
(P, E) &- (P, E') = (P, E) + (P, E') - (P, E) \\
0.259 ± 0.053 &- 0.070 ± 0.021
\end{align*}
\]

Assuming that the coincidence rates vary as the $\gamma$th power of the area $F$, the comparison of $E$ and $E'$ gives $\gamma = 0.62 ± 0.12$. The comparison of $E$ and $E + E'$ gives $\gamma = 0.42 ± 0.13$. These two values for $\gamma$ do not differ significantly (their difference is equal to the standard error of their difference). The average of the two $\gamma$-values is therefore significant, and the best value obtained comparing all three areas is thus

\[
\gamma = 0.56 ± 0.10, \tag{1}
\]

assuming

\[
(P, F) \propto F^\gamma. \tag{2}
\]

The above equations refer to observations with thick lead absorbers (5 to 10 cm.) of lead above $P$.

Independent evidence for the density spectrum of the showers was obtained by Miss B. Choudhuri. An unshielded cloud chamber was controlled by coincidences $(P, E')$, and it was found that the numbers of photographs containing $n$ or more tracks was proportional to $n^{-0.95}$, confirming equation (2).

(b) Soft and penetrating particles

(1) Observations have also been made with various separations $r$ between the extension $E$ and the main set $P$, and with no absorber $T$ placed above the set $P$. The results are collected in table 3.
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Table 3. Rates of coincidences as function of the separation $r$ between $P$ and the extension. $T = 0$

<table>
<thead>
<tr>
<th>$r = 0.5$ m.</th>
<th>2.5 m.</th>
<th>9 m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P, E$</td>
<td>2934.7</td>
<td>1518.3</td>
</tr>
<tr>
<td>$P, S$</td>
<td>0.100 ± 0.006</td>
<td>0.100 ± 0.008</td>
</tr>
<tr>
<td>$P, S_1, S_2$</td>
<td>3877.2</td>
<td>3178.7</td>
</tr>
</tbody>
</table>

Note from Table 3 that the rates of coincidences $(P, S)$ are much smaller than the rates $(P, E)$. The difference between the two rates is partly due to the fact that the tray $S$ is smaller than the tray $E$, but this cannot account completely for the difference and the comparatively small $(P, S)$ rate must therefore be due to the lead absorber shielding the tray $S$.

Assuming that most of the shower particles are capable of penetrating the lead above the counters $S$, then one would expect the rates of coincidences to increase with the size of the extension tray not more than is given by equation (2). But

$$(E/S)\gamma = 1.6 \quad (\gamma = 0.5),$$

when $E$ and $S$ are the areas of the respective trays, while

$$(P, E)/(P, S) = 4 \text{ (observed)}.$$ 

Thus the coincidence rate is reduced more strongly than one would expect in terms of a mere change of area of the extension. It must be concluded therefore that the showers contain many soft particles which are incapable of discharging the shielded counters. The actual fraction of penetrating particles will be estimated in a subsequent paper where a detailed analysis of the present data will be given. It will be seen that this fraction is of the order $\frac{1}{50}$.

(2) On the suggestion of Dr Ferretti, we have carried out observations with lead and iron shields above the counters $S$. These experiments are analogous to experiments carried out by Cocconi & Festa.* Coincidences $(P, S)$ with $T = 0$, $r = 9$ m. were observed, the counters $S$ being shielded either with 16 cm. Pb or with 15 cm. Fe placed above 11 cm. Pb. The results of the observations are given in Table 4.

Table 4

<table>
<thead>
<tr>
<th>Material over the counters $S$</th>
<th>Rate of coincidences $(P, S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 cm. Pb</td>
<td>0.013 ± 0.004</td>
</tr>
<tr>
<td>15 cm. Fe + 11 cm. Pb</td>
<td>0.016 ± 0.003</td>
</tr>
<tr>
<td>Difference</td>
<td>0.003 ± 0.005</td>
</tr>
</tbody>
</table>

* We are indebted to Professor Cocconi for communicating unpublished results.
The ratio $Q$ of the two coincidence rates is thus

$$Q = \frac{(P, S)_{Fe}}{(P, S)_{Pb}} = 1.23 \pm 0.38.$$  

(3)

The ratio is not significantly different from 1, so there is no indication of a variation of the coincidence rate with shielding material.

A similar result was obtained by Mura, Salvini & Tagliaferri (1947). Comparing their data referring to 20 cm. Pb and to 36 cm. Fe + 5 cm. Pb, the corresponding $Q$ value is found to be

$$Q = 1.37 \pm 0.27.$$  

The statistical significance of the results is greatly enhanced when they are taken together.

We discuss briefly the interpretation of the ratio $Q$.

(c) **Significance of the iron-lead ratio $Q$**

The physical significance of the ratio $Q$ depends on the mechanism by which the penetrating particles are produced in air showers. There are two main possibilities which can be considered.

(i) It may be assumed that the penetrating particles are mainly produced in the air. In this case one is led to expect $Q = 1$; indeed, the number of penetrating particles with ranges exceeding 16 cm. of lead must be almost equal to the number of penetrating particles with ranges exceeding 15 cm. Fe + 11 cm. Pb.

Thus the observed value of $Q$ is compatible with the assumption that the penetrating particles are produced in the air.

(ii) Alternatively, it may be assumed that the penetrating particles are produced in the absorber covering the counters $S$. In this case the material of the absorber might be of importance. Denote by $\Phi_p(Z)$ the cross-section for production of the penetrating particles and by $\Phi_A(Z)$ the cross-section for the absorption of the primaries of the penetrating particles by other processes. The probability of a penetrating particle being produced by a suitable primary is thus proportional to $\alpha(Z) = \frac{\Phi_p(Z)}{\Phi_A(Z)}$. Assuming that the primaries producing the penetrating particles are electrons, we may put

$$\Phi_A \propto (Z^2/A) \log(183Z^{-4}) \text{ per nucleon}$$

and thus

$$\frac{\Phi_A(Pb)}{\Phi_A(Fe)} = 2.5.$$  

Assuming further $\alpha(Fe)/\alpha(Pb) = Q$, we find with help of the observed value of $Q$

$$\frac{\Phi_p(Fe)}{\Phi_p(Pb)} = 0.49 \pm 0.15.$$  

Thus, if we reject fluctuations exceeding three times the standard error as improbable, we conclude

$$\Phi_p(Pb) > \Phi_p(Fe).$$

This is very strongly supported by the results of the Italian school, as their results are statistically independent and have about the same weight as our own.
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Thus, assuming that the penetrating particles are produced in the local absorbers by electrons or photons, the cross-section $\Phi_p(Z)$ for production must increase with $Z$. The observations would be in agreement with the assumption of a cross-section $\Phi_p(Z)$ proportional to $Z^2/A$.

The possibility of the penetrating particles being produced, not by electrons, but by nucleons present in the shower, will be discussed in detail in a subsequent paper.

(d) The lateral spread of shower particles

It is seen from table 3 that the rate of coincidences $(P, E)$ decreases only slightly as the distance between $P$ and $E$ is increased from 0·5 to 9 m. The rate of coincidences $(P, S)$, which is due to showers containing penetrating particles at the extension, was only observed from 2·5 to 9 m., but the ratio between soft and penetrating particles does not change greatly between these separations as can be inferred from the values of the ratio $(P, S)/(P, E)$ for the two separations obtained from table 3. Table 5 shows the values of this ratio.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$(P, S)/(P, E)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2·5 m.</td>
<td>$0·27 \pm 0·03$</td>
</tr>
<tr>
<td>9 m.</td>
<td>$0·20 \pm 0·03$</td>
</tr>
</tbody>
</table>

These results are compatible with the assumption that the density of the penetrating particles is proportional to the density of the soft particles throughout the shower, although a slow change of this ratio throughout the shower is not excluded.

A detailed discussion of the physical significance of the above results will be given in a subsequent publication.

We are greatly indebted to Miss B. Choudhuri for communicating unpublished results. We are also indebted to Dr B. Ferretti for valuable discussion.

References

Clay, J. 1942 Physica, 9, 897.