

halved. As, however, less accurate predictions would be required to place the fan beam so as to intercept the path of the satellite, such an equipment could be used to make observations at shorter range. On the other hand, the angular measurement derived in the plane of the fan will be less accurate than with the pencil beam. It is clear that with available techniques a balance of many factors is involved, and not the least are the practical considerations of effort and expense in constructing the equipment.

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## Method for computations of satellite orbits

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The orbital program for satellites at the Astrophysical Observatory of the Smithsonian Institution was originally built around three main routines, programmed for the IBM type 704 calculator.

(1) Routine for the determination of an approximate orbit using three visual observations.

(2) Improvement of the first approximate orbit by a method of differential corrections to the osculating elements, using any number of observations.

(3) Ephemerides of various sorts, to suit different observing groups.

As it turned out, a very important routine had to be added—a routine for the analysis of the incoming observations. Given a set of elements, this program derives subsatellite points from observations and computes the corresponding times and positions of the ascending-node crossings, which then can be tabulated and used for predictions.

The method for the approximate orbit determination was developed by R. E. Briggs and J. W. Slowey. It is based on the variations of topocentric distances and includes the first-order secular perturbations.

The differential-correction method was developed by D. A. Lautman. Starting from a set of osculating elements, residuals are computed for each observation by making use either of first-order perturbation theory (i.e. with inclusion of linear terms in  $J$ ; equations worked out by L. Cunningham) or of numerical integration

of the differential equations of motions. Each residual is expanded in a Taylor series in the unknowns and corrections to the original elements are computed by imposing the condition that the sum of the squares of the final residuals be a minimum. The inclusion of the numerical-integration program in the differential-correction method makes it possible to allow for drag when the available observations are scattered over a longer period of time.

The differential-correction program is planned to be general enough so that any quantities may be taken as unknowns, including the coefficients of the expansion of the earth's gravitational potential in spherical harmonics or the actual station positions.

The various ephemerides include subsatellite points at equal time intervals, time and longitude of pre-established parallel crossings and azimuths and elevation of the satellite at meridian and parallel crossings of individual stations. They are all computed using empirical equations to represent the precessions of node and perigee and the time of the ascending node crossing; allowance is made for a continuous variation of the major axis of the orbit consistent with the orbital accelerations.

As an example of empirical equations, the following represent the nodal crossings of satellites 1957  $\alpha 1$  and 1957  $\alpha 2$  as derived from Moonwatch observations.

1957  $\alpha 1$  (valid to 26 November 1957):

$$t_{\Omega} = \text{October 1957 } 9.82058 \text{ (U.T.)} + 0^{\text{d}}0686822n - 1.47721 e^{0.00134n} - 1.769 \times 10^{-5} e^{0.0102n};$$

$$\alpha_{\Omega} = 334.3 - 3.058(t - \text{October } 7.0) - 0.0041(t - \text{October } 7.0)^2.$$

1957  $\alpha 2$  (valid to 29 October 1957):

$$t_{\Omega} = \text{October 1957 } 4.87138 \text{ (U.T.)} + 0^{\text{d}}0668120n - 8^{\text{d}}84 \times 10^{-7}n^2;$$

$$\alpha_{\Omega} = 306.7 - 3.18(t - \text{October } 16.0).$$

In the equation for the time of the ascending-node crossing  $n$  represents the number of revolutions ( $n = 0, 1, 2, \dots$ ).

Six visual observations of a faint object in the interval 1957, 15 to 24 October give values of  $\alpha$  in agreement with the equation for  $\alpha 2$ , but values of  $t_{\Omega}$  systematically off by a few minutes. Under the assumption that this is a distinct object (1957  $\alpha 3$  (the nose cone?)) we obtain for it

$$t_{\Omega} = \text{October 1957 } 4.8722 \text{ (U.T.)} + 0^{\text{d}}0667072n - 6^{\text{d}}08 \times 10^{-7}n^2.$$