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Kolmogorov's ideas 50 years on**

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J. C. R. Hunt, O. M. Phillips and D. Williams

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PREFACE

This special issue of the *Proceedings* is to celebrate some aspects of the work of A. N. Kolmogorov (1903–87), who was elected a Foreign Member of the Royal Society in 1964 ‘for his researches in analysing the foundations of the theory of probability and of stochastic processes’. Of course this brief sentence gives no hint of the enormously wide range and depth of his research in many aspects of mathematics and its applications, such as the theory of the turbulent motions of fluids, diffusion theory, theory of automata, communication theory, and stochastic processes.

An excellent collaborative paper summarizing his work and listing all his publications has recently been published by Kendall (1990). The first volume of his selected works, with new translations into English and commentaries by Kolmogorov himself and his colleagues, edited by Nikol’skii & Tikhomirov (1991), is to be published by Kluwer. Two further volumes will be edited by Yu. V. Prokhorov and A. N. Shiryaev.

This issue is being published in 1991 to coincide with 50 years since the publication of probably the most famous paper ever written on turbulence, by Kolmogorov (1941*a*). (Two of his 1941 papers and his 1942 paper are published in this issue because they are referred to widely in this issue and because they cannot be found in most libraries.) 1991 is also the 30th year since his second most well-known paper on turbulence (presented in Marseille in 1961, but published in English in 1962) and the 60th year since his important paper on probability theory in 1931. Most of the papers in this issue are concerned with the subject of the small-scale structure of turbulence; they show how a deeper understanding of turbulence has resulted from the recent developments in theory and in research techniques, involving the increased speed and capacity of computers and new measurements (over scales ranging from that of the laboratory to even that of the galaxy). But also these developments stem from a better understanding of the many implications of Kolmogorov’s ideas in his 1941 and 1961 papers.

The paper by Spalding focuses on Kolmogorov’s 1942 paper, which proposed a heuristic model for the large-scale motion of inhomogeneous turbulence that enables the stresses in the turbulence to be calculated in complex flow. Spalding explains how this paper and a similar one written by Prandtl in 1945 in Germany have become the basis for most modern engineering calculations of turbulent research. (Spalding’s translation is included as an appendix.) The paper by Bray & Cant is concerned with Kolmogorov’s 1941 theory to modelling turbulent combustion, but he also refers to the modelling of branching processes in diffusion processes by Kolmogorov *et al.* (1937). In the papers on small-scale turbulence, Hunt & Vassilicos, Mandelbrot and Sreenivasan refer to the more recent approach, using the ‘capacity’, or ‘box counting’, measures for the analysis of multiscale, non-differentiable, self-similar, or ‘fractal’, variables which Kolmogorov & Tikhomirov introduced in 1959 and that, following Mandelbrot’s (1978) exposition, has led to techniques for direct investigation of the self-similar structure in turbulent flows. Mandelbrot’s and Frisch’s papers analyse turbulence by using the more recent ideas of multifractals. The first paper is a brief paper by V. I. Arnol’d, who is a Foreign Member of the Royal Society and a former student and colleague of Kolmogorov. He points out that most of the fundamental problems in turbulence we are thinking about, even now, Kolmogorov

was well aware of 30 years ago. Arnol'd indicates how recent developments in nonlinear dynamical systems theory may provide important new insights into turbulence. Perhaps before too long will these lead to insight or even models that can be appreciated by those working at the experimental and modelling 'end' of the subject?

Finally, Williams reviews progress in some of the important areas of probability theory to which Kolmogorov contributed. We note that there is one paper of Kolmogorov referred to both here and by two fluid mechanics authors (Bray & Cant), namely the paper by Kolmogorov *et al.* (1937).

Finally, as editors, we thank the authors for producing their papers on time, to a tight schedule. We regret that there are many learned experts on these and other aspects of Kolmogorov's work whom we would have liked to invite to contribute to this volume. We and our fellow authors certainly cannot claim to be the most expert. We just happen to be included!

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April 1991

J. C. R. Hunt
O. M. Phillips
D. Williams

CONTENTS

	PAGE
PREFACE	5
A. N. KOLMOGOROV	
The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers	9
Dissipation of energy in the locally isotropic turbulence	15
V. I. ARNOL'D	
Kolmogorov's hydrodynamic attractors	19
R. TEMAM	
Approximation of attractors, large eddy simulations and multiscale methods	23
D. WILLIAMS	
Transition functions, paths and path integrals	41
R. H. KRAICHNAN	
Turbulent cascade and intermittency growth	65
B. B. MANDELBROT	
Random multifractals: negative dimensions and the resulting limitations of the thermodynamic formalism	79
U. FRISCH	
From global scaling, <i>à la</i> Kolmogorov, to local, multifractal scaling in fully developed turbulence	89
Z.-S. SHE, E. JACKSON AND S. A. ORSZAG	
Structure and dynamics of homogeneous turbulence: models and simulations	101
O. M. PHILLIPS	
The Kolmogorov spectrum and its oceanic cousins: a review	125
C. VAN ATTA	
Local isotropy of the smallest scales of turbulent scalar and velocity fields	139
C. H. GIBSON	
Kolmogorov similarity hypotheses for scalar fields: sampling intermittent turbulent mixing in the ocean and galaxy	149
K. R. SREENIVASAN	
On local isotropy of passive scalars in turbulent shear flows	165
J. C. R. HUNT AND J. C. VASSILICOS	
Kolmogorov's contributions to the physical and geometrical understanding of small-scale turbulence and recent developments	183
D. B. SPALDING	
Kolmogorov's two-equation model of turbulence	211
K. N. C. BRAY AND R. S. CANT	
Some applications of Kolmogorov's turbulence research in the field of combustion	217
A. N. KOLMOGOROV	
Equations of turbulent motion of an incompressible fluid (Appendix to preceding paper)	214