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Could Vikings have navigated under foggy and cloudy conditions by skylight polarization? On the atmospheric optical prerequisites of polarimetric Viking navigation under foggy and cloudy skies

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In sunshine, the Vikings navigated on the open sea using sundials. According to a widespread hypothesis, when the Sun was occluded by fog or clouds the Vikings might have navigated by skylight polarization detected with an enigmatic birefringent crystal (sunstone). There are two atmospheric optical prerequisites for this alleged polarimetric Viking navigation under foggy/cloudy skies: (1) the degree of linear polarization p of skylight should be high enough and (2) at a given Sun position, the pattern of the angle of polarization α of the foggy/cloudy sky should be similar to that of the clear sky. Until now, these prerequisites have not been investigated. Using full-sky imaging polarimetry, we measured the p- and α-patterns of Arctic foggy and cloudy skies when the Sun was invisible. These patterns were compared with the polarization patterns of clear Arctic skies. We show here that although prerequisite (2) is always fulfilled under both foggy and cloudy conditions, if the fog layer is illuminated by direct sunlight, prerequisite (1) is usually satisfied only for cloudy skies. In sunlit fog, the Vikings could have navigated by polarization only, if p of light from the foggy sky was sufficiently high.

Keywords: Vikings; polarimetric Viking navigation; Arctic skies; sky polarization; fog; cloud

1. Introduction

There is archaeological evidence that the Vikings did not possess magnetic compass, and they navigated on the open sea with the help of a sundial composed of a wooden disc with a perpendicular gnomon in its centre (Thirslund 2001). In the disc, some hyperbolas were engraved, the shape of which corresponded with

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the curves described by the tip of the gnomon's shadow cast on the disc from sunrise through culmination to sunset during the sailing season from April to August on the 61° N latitude. Along this latitude was one of the Vikings' most frequently used ship routes between Hernam (north of Bergen on the western coast of Norway) and Hvarf (north of the southern tip of Greenland at Cape Farewell; Thirslund 2001). On this route, the Vikings always had to travel west or east. These directions were determined by means of the sundial in such a way that the disc was rotated around the vertical gnomon until the tip of the gnomon's shadow touched the appropriate hyperbola, when the direction of geographical north could be read from the disc.

Obviously, the Viking sundial could function only when the Sun was shining. Ramskou (1967, 1969) hypothesized that when the Sun was occluded by clouds, or fog, the Vikings might have also been able to navigate by means of skylight polarization. They were thought to have detected the direction of skylight polarization with the help of an enigmatic birefringent crystal, called 'sunstone'. Although there is no archaeological evidence supporting this hypothesis of polarimetric Viking navigation, it is frequently cited (Barfod 1967; Ramskou 1967, 1969; LaFay 1970; Binns 1971; Britton 1972; Kreithen & Keeton 1974; Schnall 1975; Wehner 1976; Walker 1978; Nussbaum & Phillips 1982; Können 1985; McGrath 1991; Schaefer 1997; Shashar *et al.* 1998; Thirslund 2001). The widespread belief is that the Vikings were able to navigate by skylight polarization under any weather conditions: under clear; foggy; partly cloudy or totally overcast skies.

Roslund & Beckman (1994) emphasized the lack of evidence for the hypothesis that Viking navigators used skylight polarization. They treated the usefulness of sky polarization for orientation with extreme scepticism. One of their qualitative counter-arguments was the assumption that solar positions or solar azimuth directions could be estimated quite accurately by the naked eye, even if the Sun is behind clouds or below the sea horizon. Barta *et al.* (2005) tested quantitatively the validity of this qualitative counter-argument, and their data, obtained in psychophysical laboratory experiments, did not support the common belief that the invisible Sun can be located quite accurately from the celestial brightness and/or colour patterns under partly cloudy or twilight conditions. Thus, the mentioned counter-argument of Roslund & Beckman (1994) cannot be taken seriously as a valid criticism of the hypothesis of polarimetric Viking navigation.

At northern latitudes frequently sailed by the Vikings, the Sun is often occluded by fog or clouds, especially at low solar elevations. The question is whether the Vikings could have also navigated under foggy or cloudy skies by celestial polarization, as Können (1985) has hypothesized. There are two atmospheric optical prerequisites for polarimetric Viking navigation under foggy/cloudy skies:

(1) the degree of linear polarization p of skylight should be high enough and
(2) at a given Sun position, the pattern of the angle of polarization α of the foggy/cloudy sky should be similar to that of the clear sky.

Until now, these prerequisites have not been investigated. To study this problem, we measured the patterns of the degree p and the angle α of linear polarization of Arctic foggy and cloudy skies when the Sun was invisible to a human observer.

2. Materials and methods

The polarization patterns of foggy, clear and partly cloudy skies were measured between 21 August and 21 September 2005 at different places during the third part (Leg 3) of the international Arctic polar research expedition 'Beringia 2005' organized by the Swedish Polar Research Secretariat. The expedition crossed the Arctic Ocean with the Swedish icebreaker Oden departing from the Alaskan Barrow (71°17′ N, 156°47′ W) and arriving in Longyearbyen (78°12′ N, 15°49′ W) on the island of Spitsbergen (Svalbard, Norway). The geographical coordinates, date and time (local summer time = UTC − 8 and UTC + 2 west and east of North Pole, respectively) of the measurements are summarized in table 1. The Sun was invisible to the human observer on both the foggy and the partly cloudy skies.

The skylight polarization was measured by full-sky imaging polarimetry, the technique and evaluation procedure of which have been described in detail by Gál *et al.* (2001). A 180° field of view (full sky) was ensured by a fisheye lens (Nikon–Nikkor, $F=2.8$, focal length 8 mm) with a built-in rotating disc mounted with three broadband (275–750 nm) neutral density linearly polarizing filters (Polaroid HNP'B) with three different polarization axes (0, 45 and 90° from the radius of the disc).

The detector was a photo emulsion (Kodak Elite Chrome ED 200 ASA colour reversal film; the maxima and half bandwidths of its spectral sensitivity curves were $\lambda_{\text{red}}=650\pm40$ nm, $\lambda_{\text{green}}=550\pm40$ nm and $\lambda_{\text{blue}}=450\pm40$ nm) in a roll-film photographic camera (Nikon F801). For a given sky, three photographs were taken for the three different directions of the transmission axis of the polarizers. The camera was set on a tripod such that the optical axis of the fisheye lens was vertical, i.e. pointed to the zenith. After 24-bit (3×8 for red, green and blue) digitization (by a Canon Arcus 1200 scanner) of the three chemically developed colour pictures for a given sky and their computer evaluation, the patterns of the radiance, I, the degree of linear polarization, p, and the angle of polarization, α (or E-vector alignment), of skylight were determined as colour-coded, two-dimensional, circular maps, in which the centre is the zenith, the perimeter is the horizon, and the zenith angle θ is proportional to the radius from the zenith (zenith $\theta=0°$, horizon $\theta=90°$). These patterns were obtained in the red, green and blue spectral ranges, in which the three colour-sensitive layers of the photo emulsion used have maximal sensitivity.

The theoretical α-patterns of the clear sky were calculated on the basis of the model of Berry *et al.* (2004) based on the neutral points, later derived using multiple scattering by Hannay (2004). This model provides a very good quantitative approximation of experimental clear sky α-patterns, particularly with respect to the existence of neutral points. The model has three parameters to be freely set: two for the Sun position (solar zenith and azimuth angles) and one for the angular distance (digression) between the Arago and the Babinet neutral points. In our case, the theoretical α-patterns (figure 1*j*–*l*) were computed according to the position of the Sun as it appears in the photographs (figure 1*b*) or estimated on the basis of the exact time and geographical coordinates of the site of measurements, when the Sun was not visible to the human observer (figure 1*a*,*c*). The digression between the Arago and the Babinet points in the theoretical model of Berry *et al.* (2004) was scanned throughout between 100 and 160° during the comparison with a measured α-pattern and the

Table 1. Sky condition (F, sunlit fog; C, clear sky; PC, partly cloudy sky), sky number, geographical coordinates (latitude and longitude), date, time (local summer time=UTC−8 or UTC+2) and solar elevation angle during the sky polarization measurements, the polarization data of which are given in tables 2–4.

sky condition	sky number	latitude	longitude	date (2005)	time	solar elevation
sunlit fog	F1	71°36.9′ N	152°6.0′ W	21 Aug	09.00 (UTC−8)	15.2°
	F2	80°54.7′ N	145°40.0′ W	28 Aug	21.50 (UTC−8)	4.7°
	F3	80°56.5′ N	145°43.8′ W	28 Aug	22.50 (UTC−8)	2.9°
	F4	86°38.7′ N	174°38.6′ E	4 Sep	21.45 (UTC−8)	7.4°
	F5	87°39.1′ N	151°15.3′ E	6 Sep	21.33 (UTC−8)	7.5°
	F6	88°27.8′ N	148°6.8′ E	8 Sep	14.12 (UTC−8)	6.3°
	F7	87°59.2′ N	59°25.1′ E	15 Sep	16.10 (UTC+2)	3.0°
	F8	86°14.2′ N	49°40.6′ E	19 Sep	14.45 (UTC+2)	3.3°
	F9	86°14.2′ N	49°40.6′ E	19 Sep	15.40 (UTC+2)	2.5°
	F10	86°14.2′ N	49°40.6′ E	19 Sep	16.28 (UTC+2)	1.7°
	F11	85°5.9′ N	44°39.7′ E	21 Sep	16.25 (UTC+2)	1.5°
	F12	85°5.9′ N	44°39.7′ E	21 Sep	17.25 (UTC+2)	0.4°
clear sky	C1	72°24.8′ N	151°31.0′ W	21 Aug	21.55 (UTC−8)	3.8°
	C2	78°27.8′ N	149°9.1′ W	25 Aug	21.20 (UTC−8)	6.3°
	C3	83°54.2′ N	149°12.8′ W	31 Aug	23.02 (UTC−8)	4.0°
	C4	88°27.8′ N	148°6.8′ W	8 Sep	12.25 (UTC−8)	7.0°
	C5	88°27.8′ N	148°6.8′ W	8 Sep	12.45 (UTC−8)	7.0°
	C6	88°25.4′ N	145°57.3′ E	8 Sep	20.43 (UTC−8)	6.7°
	C7	88°24.9′ N	150°31.1′ E	9 Sep	01.35 (UTC−8)	4.7°
	C8	89°45.1′ N	79°42.3′ E	12 Sep	20.50 (UTC−8)	4.2°
	C9	89°45.1′ N	79°42.3′ E	12 Sep	21.30 (UTC−8)	4.2°
	C10	89°45.1′ N	79°42.3′ E	12 Sep	22.30 (UTC−8)	4.2°
partly cloudy sky	PC1	82°30.6′ N	147°36.3′ W	30 Aug	21.37 (UTC−8)	5.4°
	PC2	82°30.6′ N	147°36.3′ W	30 Aug	22.25 (UTC−8)	4.1°
	PC3	82°30.6′ N	147°36.3′ W	30 Aug	23.30 (UTC−8)	2.7°
	PC4	83°54.2′ N	149°12.8′ W	31 Aug	21.14 (UTC−8)	6.3°
	PC5	83°54.2′ N	149°12.8′ W	31 Aug	21.58 (UTC−8)	5.3°
	PC6	88°59.2′ N	77°47.6′ E	14 Sep	15.12 (UTC+2)	3.4°
	PC7	88°45.7′ N	75°9.7′ E	14 Sep	21.30 (UTC+2)	2.2°
	PC8	87°59.2′ N	59°25.1′ E	15 Sep	14.36 (UTC+2)	3.8°
	PC9	87°59.2′ N	59°25.1′ E	15 Sep	15.35 (UTC+2)	3.3°
	PC10	85°5.9′ N	44°39.7′ E	21 Sep	15.45 (UTC+2)	2.3°

digression for which similarity between the theoretical and the measured patterns yielded the highest value was accepted. The theoretical α-patterns in figure 1*j*–*l* show how the celestial E-vector distribution at the time of measurements was expected to appear under clear sky conditions.

The noisiness n of a given α-pattern (tables 2–4) was calculated as follows: the α-patterns were scanned throughout with a window of 5 pixel×5 pixel, in which the standard variance (σ^2) of the angle of polarization, α, was calculated, and then the average of the standard variances of all 5 pixel×5 pixel windows

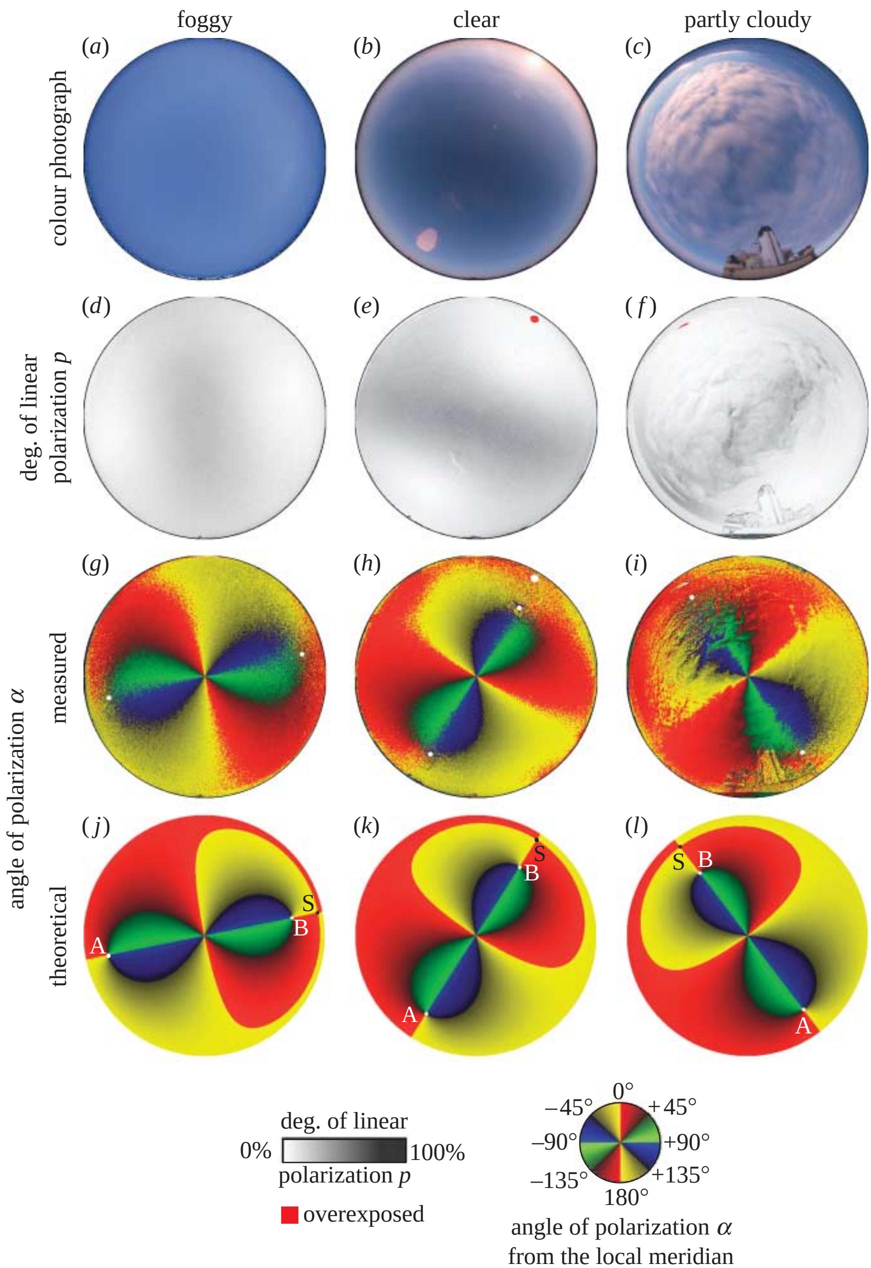

Figure 1. Colour photographs (*a*–*c*) and patterns of the degree of linear polarization p (*d*–*f*) and angle of polarization α (*g*–*i*) of the sunlit foggy Arctic sky F7, clear Arctic sky C9 and partly cloudy Arctic sky PC1 measured by full-sky imaging polarimetry in the blue (450 nm) part of the spectrum. The polarization data of these skies are given in tables 2–4. The optical axis of the fisheye lens was vertical, thus the horizon is the perimeter and the centre of the circular patterns is the zenith. (*j*–*l*) Theoretical α-patterns of the clear sky calculated on the basis of the model of Berry *et al.* (2004) for the same Sun position as in the skies F7, C9 and PC1. The positions of the Sun as well as the Arago and Babinet neutral points are marked by dots in the α-patterns. The abbreviations in patterns *j*, *k* and *l* are as follows: S, Sun; A, Arago neutral point; B, Babinet neutral point.

Table 2. Polarization characteristics of Arctic sunlit fog measured by full-sky imaging polarimetry in the red (R, 650 nm), green (G, 550 nm) and blue (B, 450 nm) parts of the spectrum. The degree of linear polarization p (average ± s.d.) is averaged over the entire sky. The noisiness n of the angle of polarization α, the similarity s, dissimilarity d and under-/overexposure u of the α-pattern in comparison with the theory are computed for the whole sky, and their definitions are given in §2. The longitude, latitude, date, time and solar elevation angle during the measurements are given in table 1.

							similarity to theory								
	deg. of polarization, p (%)			noisiness n (%) of α			R			G			B		
sky no.	R	G	B	R	G	B	s (%)	d (%)	u (%)	s (%)	d (%)	u (%)	s (%)	d (%)	u (%)
F1	7±4	7±5	7±3	27	38	19	20	72	8	23	69	8	40	52	8
F2	5±3	6±3	6±3	27	23	14	19	75	6	46	48	6	47	47	6
F3	5±3	6±3	6±2	20	21	15	35	59	6	45	49	6	54	40	6
F4	6±4	5±3	4±3	30	40	30	27	73	0	23	77	0	27	73	0
F5	8±5	7±4	5±3	31	45	26	27	73	0	24	76	0	27	73	0
F6	12±6	12±7	15±8	11	10	8	46	43	11	61	32	7	69	23	8
F7	15±7	14±6	13±6	13	8	5	62	38	0	52	48	0	47	53	0
F8	9±4	8±3	8±3	16	10	8	60	40	0	71	29	0	68	32	0
F9	10±5	8±4	8±3	21	16	8	60	40	0	68	32	0	64	36	0
F10	14±7	11±5	12±5	17	14	6	45	54	1	54	45	1	62	37	1
F11	6±4	6±3	5±2	20	22	14	46	54	0	39	61	0	48	52	0
F12	6±4	6±3	4±2	29	28	24	50	50	0	42	58	0	47	53	0
average±s.d.	8.6±4.7	8±4.1	7.8±3.6	21.8±6.8	22.9±12.5	14.8±8.4	41.4±14.5	55.9±14.3	2.7±4.0	45.7±16.6	52.0±16.6	2.3±3.3	50.0±14.2	47.6±15.2	2.4±3.4

Table 3. As table 2 for clear Arctic skies.

	deg. of polarization p (%)			noisiness n (%) of α			similarity to theory								
							R			G			B		
sky no.	R	G	B	R	G	B	s (%)	d (%)	u (%)	s (%)	d (%)	u (%)	s (%)	d (%)	u (%)
C1	29 ± 16	27 ± 16	26 ± 14	5	5	6	62	24	14	65	26	9	65	23	12
C2	34 ± 25	25 ± 16	21 ± 15	5	3	6	50	41	9	62	30	8	45	40	15
C3	22 ± 15	18 ± 11	16 ± 9	5	4	3	78	22	0	78	22	0	78	22	0
C4	33 ± 20	27 ± 17	24 ± 15	5	3	5	62	25	13	69	21	10	63	24	13
C5	34 ± 22	27 ± 18	24 ± 16	4	3	5	51	38	11	56	35	9	55	32	13
C6	30 ± 18	25 ± 15	21 ± 12	6	5	6	68	17	15	74	16	10	66	18	16
C7	30 ± 19	23 ± 14	19 ± 11	4	3	6	63	27	10	67	26	7	62	23	15
C8	26 ± 14	21 ± 11	19 ± 9	4	3	3	75	20	5	79	17	4	79	17	4
C9	25 ± 15	22 ± 13	19 ± 10	4	3	3	77	18	5	81	15	4	79	17	4
C10	19 ± 11	20 ± 12	17 ± 10	4	3	3	72	24	4	76	21	3	78	19	3
average ± s.d.	28.2 ± 17.5	23.5 ± 14.3	20.6 ± 12.1	4.6 ± 0.7	3.5 ± 0.8	4.6 ± 1.4	65.8 ± 10.0	25.6 ± 8.0	8.6 ± 4.9	70.7 ± 8.2	22.9 ± 6.4	6.4 ± 3.4	67.0 ± 11.6	23.5 ± 7.3	9.5 ± 6.0

Table 4. As table 2 for partly cloudy Arctic skies.

	deg. of linear polarization p (%)			noisiness n (%) of α			similarity to theory								
							R			G			B		
sky no.	R	G	B	R	G	B	s (%)	d (%)	u (%)	s (%)	d (%)	u (%)	s (%)	d (%)	u (%)
PC1	12±9	13±8	13±8	15	11	7	40	54	6	51	43	6	66	28	6
PC2	23±14	19±11	18±11	8	7	6	57	37	6	62	32	6	61	33	6
PC3	18±13	17±12	17±11	11	11	7	52	42	6	49	45	6	49	45	6
PC4	10±5	11±6	12±6	9	8	6	47	50	3	61	39	0	71	28	1
PC5	10±6	11±6	12±6	10	10	5	55	45	0	61	39	0	70	30	0
PC6	15±9	16±9	15±8	6	6	6	59	29	12	69	23	8	72	20	8
PC7	11±6	11±5	10±5	11	7	9	44	47	9	57	35	8	59	29	12
PC8	15±9	13±7	11±6	14	9	7	36	64	0	45	55	0	54	46	0
PC9	25±17	20±12	20±13	9	4	5	53	46	1	63	37	0	58	38	4
PC10	19±12	17±11	16±10	11	8	5	47	53	0	54	46	0	58	42	0
average±s.d.	15.8±10	14.8±8.7	14.4±8.4	10.4±2.7	8.1±2.2	6.3±1.3	49.0±7.5	46.7±9.6	4.3±4.2	57.2±7.4	39.4±8.7	3.4±3.7	61.8±7.7	33.9±8.5	4.3±4.1

was obtained. Finally, this value was normalized to that of white noise calculated with the same method. Thus, noisiness n of an α-pattern denotes how noisy it is compared with the white noise ($n=0\%$, no noise; $n=100\%$, white noise).

The measured α-pattern of a given sky was compared from pixel to pixel with the corresponding celestial α-pattern calculated on the basis of the model of Berry *et al.* (2004) for the same Sun position. The solar azimuth angle was assumed to coincide with the symmetry axis of the α-pattern, and the solar elevation angle (from the horizon) was calculated on the basis of the known geographical coordinates, date and time of the measurement (table 1) by the online solar position calculator of US Naval Observatory, Astronomical Applications Department (http://aa.usno.navy.mil). At a given celestial point, the measured α_m and the theoretical α_{th} were considered to be similar and dissimilar if $|\alpha_m - \alpha_{th}| \leq 5°$ and $|\alpha_m - \alpha_{th}| > 5°$, respectively. The numbers N_s and N_d of 'similar' and 'dissimilar' points were counted and divided by the total number $N \approx 290\,000$ of celestial points considered in order to obtain the following quantities: similarity $s = N_s/N$ and dissimilarity $d = N_d/N$. If the number, N_u, of the unevaluable celestial points (under- or overexposed points, or points of the image of the icebreaker Oden visible on the periphery of some circular pictures of the sky) is divided by N, we obtain the quantity, $u = N_u/N$. Note that $s + d + u = 1$, because $N_s + N_d + N_u = N$.

3. Results

Figure 1 shows examples for the polarization patterns of foggy, clear and partly cloudy Arctic skies measured by full-sky imaging polarimetry in the blue (450 nm) part of the spectrum. Comparing the p- and α-patterns of foggy (figure 1*d*,*g*; table 2), clear (figure 1*e*,*h*; table 3) and cloudy (figure 1*f*,*i*; table 4) skies, we can establish that the polarization pattern of foggy and cloudy skies is qualitatively the same as that of the corresponding clear sky:

— p of skylight increases with increasing angular distance from the Sun and anti-Sun and reaches its maximum at 90° from the Sun and anti-Sun. Skylight from the Arago and Babinet neutral points is unpolarized ($p=0\%$). These neutral points are placed along the solar and anti-solar meridians in the vicinity of the Sun and anti-Sun. p of skylight and the position of the neutral points depend on the wavelength.

— The sky region with $+45 \leq \alpha \leq +135°$ (i.e. nearly horizontal direction of polarization, called 'positive polarization', and shaded by green and blue colours in figure 1*g*–*l*) is an 8-shaped area within the celestial region with $-45 \leq \alpha \leq +45°$ (i.e. approximately vertical direction of polarization, called 'negative polarization', and shaded by yellow and red colours in figure 1*g*–*l*). The perimeter of this 8-shaped region is defined by the so-called 'neutral line' characterized by $|\alpha| = 45°$. The long axis of this 8-shaped area coincides with the solar and anti-solar meridians. The neutral points are positioned at the tips of this 8-shaped celestial region, where positive polarization switches to negative polarization crossing the neutral points along the solar and anti-solar meridians. The celestial α-pattern also depends on the wavelength.

Tables 2–4 contain the degree of linear polarization, p, the noisiness, n, of the angle of polarization, α, and the similarity, s, of α to the theory of foggy, clear and partly cloudy Arctic skies averaged over the entire sky. Depending on the cloudiness and the wavelength, the average degrees of linear polarization $p_{\text{cloudy}}=10$–25% and noisiness $n_{\text{cloudy}}=4$–15% of partly cloudy skies are between those of the clear ($p_{\text{clear}}=16$–34%, $n_{\text{clear}}=3$–6%) and foggy ($p_{\text{foggy}}=4$–15%, $n_{\text{foggy}}=5$–45%) skies.

The direct quantitative comparison between the measured α-patterns of foggy, clear and partly cloudy skies was not possible owing to the different solar positions. Thus, the α-pattern of a given (foggy, clear or partly cloudy) sky measured in the red, green and blue spectral ranges was compared with the corresponding theoretical α-pattern calculated on the basis of the model of Berry *et al.* (2004) for the same Sun position (figure 1*j*–*l*). This resulted in the similarity, s, and dissimilarity, d, of α to the theory. According to tables 2–4, the average similarities of the clear, partly cloudy and foggy skies are: $s_{\text{clear}}=65.8$–70.7%; $s_{\text{cloudy}}=49.0$–61.8%; and $s_{\text{foggy}}=41.4$–50.0%. The similarity is usually highest in the blue part of the spectrum for partly cloudy and foggy skies. The minima and maxima of s for clear, partly cloudy and foggy skies are: $45 \le s_{\text{clear}} \le 81\%$; $36 \le s_{\text{cloudy}} \le 72\%$; and $19 \le s_{\text{foggy}} \le 71\%$. This shows that if the fog is not too thick, then the celestial α-pattern can be as similar or even more similar to the theoretical α-pattern than those of certain clear skies. However, according to the above, the following relations are true for the averages: $p_{\text{foggy}} < p_{\text{cloudy}} < p_{\text{clear}}$; $n_{\text{clear}} < n_{\text{cloudy}} < n_{\text{foggy}}$; and $s_{\text{foggy}} < s_{\text{cloudy}} < s_{\text{clear}}$. Figure 2 shows the maps of similarity of α to the theory for the foggy, clear and cloudy skies of figure 1 computed in the red (650 nm), green (550 nm) and blue (450 nm) parts of the spectrum. The percentages s and d of the similar and dissimilar sky regions are given in tables 2–4.

4. Discussion

According to our polarimetric measurements, the degree of linear polarization p of foglight is more or less reduced relative to the p of light from the clear sky, but the E-vector pattern of sunlit fog remains qualitatively the same as that of the clear sky. Sunlit fog means that the fog layer is illuminated by direct sunlight, because the Sun is not occluded by clouds. In the meteorological situations investigated by us, the fog was sunlit, but it was so thick and/or dense that the Sun's disc was invisible. Our results can be explained as follows: in the single-scattering Rayleigh model, the E-vector of scattered skylight is always perpendicular to the main plane of scattering determined by the observer, the Sun and the celestial point observed. This type of polarization is called 'positive polarization' (Coulson 1988). Multiple scattering results in that the E-vector of scattered light has a component parallel to the main plane of scattering. This type is called 'negative polarization' (Coulson 1988). Hence, multiple scattering introduces negative polarization into the atmosphere. This depolarizes the skylight, i.e. decreases its p. The stronger the multiple scattering, the larger the amount of negatively polarized light added to the positively polarized single-scattered light, and thus the lower the net p of skylight. Neutral (unpolarized) points occur where the amounts of positively and negatively polarized skylight are equal. Apart

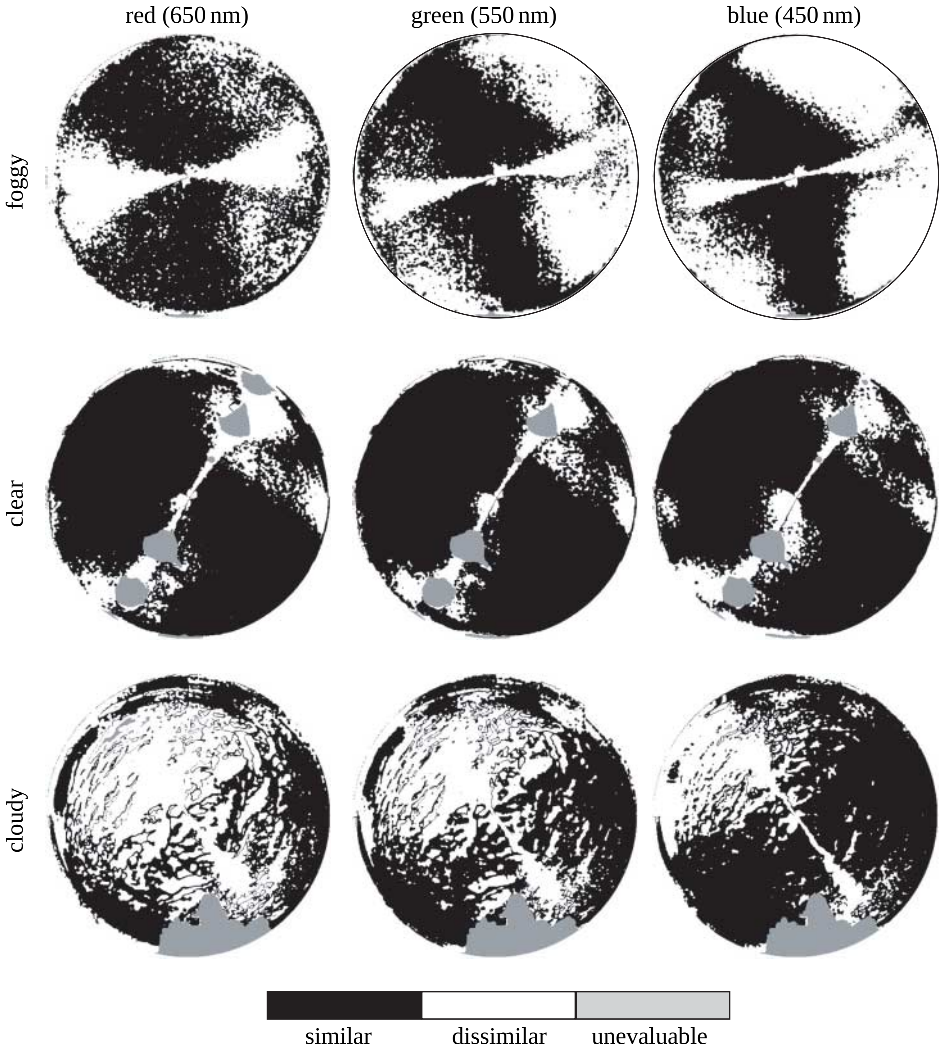

Figure 2. Maps of similarity (and dissimilarity) of the angle of polarization α to the theory for the sunlit foggy (F7), clear (C9) and partly cloudy (PC1) skies shown in figure 1 computed in the red (650 nm), green (550 nm) and blue (450 nm) parts of the spectrum. Celestial regions are shaded by black and white, the α-patterns of which are similar and dissimilar in comparison with the theoretical α-patterns calculated on the basis of the model of Berry *et al.* (2004) for the same Sun positions. The grey sky regions were unevaluable due to under- or overexposure. The percentages s, d and u of the similar, dissimilar and unevaluable sky regions (defined in §2) are given in tables 2–4.

from the neutral points, the E-vector pattern of multiple-scattered skylight remains similar to that characteristic of the single-scattering Rayleigh atmosphere, as long as $p > 0$.

In sunlit fog, scattering of sunlight happens on the tiny water droplets (of water fog) or ice crystals (of ice fog). On the one hand, the E-vector pattern of sunlit fog is not the result of light scattering in the air between the observer and the fog. Note that the observer is often within the fog layer. On the other hand, the reason for the E-vector pattern of sunlit fog is not that the E-vector pattern

of the clear sky above the fog is visible through the fog layer. The E-vector pattern of the sunlit fog is the result of the scattering of sunlight on the fog particles, rather than the transmission of polarized light from the clear sky, or scattering of light below the fog layer.

The theory of polarimetric Viking navigation (Ramskou 1967, 1969; Thirslund 2001) is based on the following assumptions, which have not been tested experimentally until now.

- (i) The Viking navigators possessed a certain kind of birefringent crystal (sunstone, e.g. cordierite, tourmaline or calcite) functioning as a linear polarization analyser.
- (ii) With the use of this crystal, they were able to determine the direction of polarization of skylight at least at two celestial points A and B with high enough degree of linear polarization p ($>p^*=$threshold).
- (iii) They could set the two great circles of the sky dome, passing through points A and B, parallel to the direction of skylight polarization at A and B.
- (iv) They knew that the invisible Sun (occluded by fog or clouds) is positioned at the above-horizon cross-section of these two great circles.
- (v) At a given Sun position, the pattern of the angle of polarization α of the foggy or partly cloudy sky is similar to the α-pattern of the clear sky.

If all these conditions are fulfilled, then, according to the hypothesis, with certain accuracy the Viking navigators could determine the position of the Sun occluded by clouds or fog. Unfortunately, until now, no archaeological evidence has been found that could support assumption (i). Similarly, it is unknown whether the Viking navigators held possession of the knowledge necessary to fulfil the conditions (iii) and (iv). However, on the basis of our results presented here, the validity of assumptions (ii) and (v) can be quantitatively investigated.

(*a*) *Doubtful validity of assumption (ii) and atmospheric optical prerequisite (1)*

Let us consider assumption (ii). Looking at the sky through a cordierite crystal (sunstone), for example, that is rotated periodically in front of our eyes along an axis of rotation pointing to a given celestial location, the direction of polarization of skylight can be determined with an error ε on the basis of the periodical change of the intensity and/or colour of light transmitted through the crystal. It is the task of future psychophysical experiments to measure ε as a function of the degree of linear polarization p of light. As long as this error function $\varepsilon(p)$ is unknown, the most we can assume is that $\varepsilon(p)$ may be monotonical, that is the lower the p, the weaker the intensity/colour change of light transmitted through the sunstone, consequently, the larger the error ε. Obviously, if p is lower than (unknown) threshold p^*, the direction of polarization cannot be determined by this method. Hence, under a given weather condition, the lower the average p of skylight, the smaller the chance that the polarimetric Viking navigation can function.

We measured the polarization characteristics of Arctic foggy and cloudy skies, because fog and clouds commonly occur at the northern latitudes ruled and sailed by the Vikings for several hundreds of years. In table 2, we can see that depending on the wavelength, as well as on the density and thickness of the fog layer

occluding the Sun, the degree of linear polarization, p_{average}, averaged over the entire foggy sky ranges from 3 to 15% with $p_{\min}=1\%$ and $p_{\max}=23\%$. As long as the above-mentioned threshold p^* is unknown, we cannot decide whether these p_{average} and $p_{\max}$ values of foggy skies are high enough for polarimetric Viking navigation.

The characteristics of the instrument used to analyse skylight polarization determine the minimum levels of linear polarization that can be detected. On the basis of our own experience (G. Horváth, 2004, unpublished data), p^* is about 10%, if the sky is viewed through a common linear polarizer. Thus, assuming $p^*\approx 10\%$, on the basis of table 2 one can establish that the condition $p_{\text{average}} > p^* \approx 10\%$ is fulfilled for the foggy skies F6, F7, F9 and F10 investigated, while the condition $p_{\max} > p^* \approx 10\%$ is satisfied for the foggy skies F1, F5–F9 and F10. This would mean that in the majority of the studied foggy skies, the polarization of light from certain sky regions would have been strong enough to fulfil the atmospheric optical prerequisite (1) ($p > p^* \approx 10\%$) of polarimetric Viking navigation mentioned in the introduction. However, we derived this $p^* \approx 10\%$ limit by viewing the sky through modern polarization filters. But what could be expected by the technical means which were possessed by the Vikings? The quality of the polarization analysers that Vikings are likely to have used (sunstones) should have been rather poor. In fact, the average degree of polarization p on foggy days was only approximately 8% (table 2). If the limit p^* were increased to 15%, for example, Viking navigation would not be possible under the majority of foggy skies (table 2). Consequently, in our opinion, it remains doubtful whether the polarization of foggy skies is strong enough to be detected by some kind of birefringent crystal that was possibly available to the navigating Vikings.

According to tables 2–4 and figure 1, the partly cloudy sky is an intermediate between the clear sky and the foggy sky: depending on the cloudiness and the wavelength, in the partly cloudy sky there are regions with as high p as that in the clear sky, and where the α-pattern is the same as that of the clear sky, but in the clouded celestial regions, p of skylight is drastically reduced and the α-pattern can considerably deviate from that of the clear sky. This means that under partly cloudy conditions, both atmospheric optical prerequisites (1) and (2) of the polarimetric Viking navigation are fulfilled in certain (smaller or larger) celestial regions. From this, it follows that condition (v) is also satisfied for many partly cloudy skies.

Note that at a given latitude, with some experience, the approximate position of the Sun's disc occluded by clouds can be guessed on the basis of the time of day. Unfortunately, it is not known how accurately Vikings could have estimated the time of day to compensate for the apparent movement of the Sun. However, if the polarization of light from foggy or overcast skies was too weak ($p < p^*$), nothing was left to an experienced Viking navigator but estimating the solar position from the approximate time of day.

(*b*) *Validity of assumption (v) and satisfaction of atmospheric optical prerequisite (2)*

Then, let us consider the validity of assumption (v). In this work, we obtained that 65.8–70.7% (equal to s_{clear}) and 41.4–50.0% (equal to s_{foggy}) of the measured α-pattern of the clear and the sunlit foggy sky, respectively, corresponds with the

theory. Hence, at a given Sun position, the α-pattern of a sunlit foggy sky is similar to that of the clear sky. This means that in the case of sunlit foggy skies, the atmospheric optical prerequisite (ii) of polarimetric Viking navigation mentioned in the introduction is satisfied. The three major quantitative differences in skylight polarization between sunlit foggy (figure 1*a*,*d*,*g*; table 2) and clear (figure 1*b*,*e*,*h*; table 3) skies are the following: depending on the wavelength, as well as on the density and thickness of the sunlit fog layer, (a) the average degrees of linear polarization $p_{\text{foggy}}=4$–15% of light from foggy skies are much lower than the averages $p_{\text{clear}}=16$–34% of light from clear skies, (b) the noisiness $n_{\text{foggy}}=5$–45% of the α-pattern of sunlit foggy skies is usually much larger than the noisiness $n_{\text{clear}}=3$–6% of clear skies, and (c) the α-pattern of sunlit foggy skies deviates from the theory much more ($d_{\text{foggy}}=47.6$–55.9%) than that of the clear sky ($d_{\text{clear}}=22.9$–25.6%).

Recently, Hegedüs *et al.* (submitted) found that depending on the optical thickness of the cloud layer, the α-pattern characteristic to the clear sky is more or less transmitted through the ice or water clouds of heavy overcast, while the degrees of linear polarization of light from overcast skies remain rather low ($p \leq 16\%$). These findings and the results presented in this work show that the celestial distribution of the direction of polarization is a very robust pattern being qualitatively always the same under almost all possible sky conditions. This is of great importance for the orientation of polarization-sensitive animals based on sky polarization under conditions when the Sun is not visible. In principle, Viking navigators could also have exhausted this robust celestial feature, if p of skylight was high enough.

5. Conclusions

Summarizing our results, in sunlit fog, the α-pattern of the foggy sky is similar to that of the corresponding clear sky. Consequently, atmospheric optical prerequisite (2) of the polarimetric Viking navigation is fulfilled under sunlit foggy conditions. This means that condition (v) is satisfied for sunlit foggy skies. However, the degree of linear polarization p of light from foggy skies is so low that it might not be high enough to fulfil atmospheric optical prerequisite (1) of the polarimetric navigation. From this, it follows that the satisfaction of condition (ii) is doubtful for foggy skies. Vikings could have navigated under sunlit foggy conditions on the basis of skylight polarization only, if p was sufficiently high. On the other hand, under partly cloudy conditions, usually both prerequisites (1) and (2) of the polarimetric navigation are fulfilled, which means the satisfaction of conditions (ii) and (v) too. Finally, we would like to emphasize that on the basis of our celestial polarization data measured in the Arctic regions only the validity of atmospheric optical prerequisites (1) and (2) as well as conditions (ii) and (v) of the alleged polarimetric Viking navigation can be investigated. Further research should study the validity of assumptions (i), (iii) and (iv) mentioned at the beginning of the discussion.

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References

Barfod, J. H. P. 1967 Navigation. *Kulturhistoriskt Lexikon Nordisk Medeltid* **12**, 260–263.

Barta, A., Horváth, G. & Meyer-Rochow, V. B. 2005 Psychophysical study of the visual sun location in pictures of cloudy and twilight skies inspired by Viking navigation. *J. Opt. Soc. Am. A* **22**, 1023–1034. (doi:10.1364/JOSAA.22.001023)

Berry, M. V., Dennis, M. R. & Lee Jr, R. L. 2004 Polarization singularities in the clear sky. *New J. Phys.* **6**, 162. (doi:10.1088/1367-2630/6/1/162)

Binns, A. L. 1971 Sun navigation in the Viking age, and the Canterbury portable sundial. *Acta Archaeologica* **42**, 23–34.

Britton, W. 1972 The Britton Viking sun-stone expedition. *Nutr. Today* May/June, 14–23. (doi:10.1097/00017285-197205000-00003)

Coulson, K. L. 1988 *Polarization and intensity of light in the atmosphere.* Hampton, VA: A. Deepak Publishing.

Gál, J., Horváth, G., Meyer-Rochow, V. B. & Wehner, R. 2001 Polarization patterns of the summer sky and its neutral points measured by full-sky imaging polarimetry in Finnish Lapland north of the Arctic Circle. *Proc. R. Soc. A* **457**, 1385–1399. (doi:10.1098/rspa.2000.0726)

Hannay, J. H. 2004 Polarization of sky light from a canopy atmosphere. *New J. Phys.* **6**, 197. (doi:10.1088/1367-2630/6/1/197)

Hegedüs, R., Åkesson, S. & Horváth, G. Submitted. Polarization of overcast skies and implications for animal orientation: radiative transfer of the clear-sky E-vector pattern through thick ice and water clouds.

Können, G. P. 1985 *Polarized light in nature.* Cambridge, UK: Cambridge University Press.

Kreithen, M. L. & Keeton, W. T. 1974 Detection of polarized light by the homing pigeon, *Columba livia*. *J. Comp. Physiol.* **89**, 83–92. (doi:10.1007/BF00696165)

LaFay, H. 1970 The Vikings. *Natl Geogr.* **137**, 492–541.

McGrath, W. H. 1991 The stars look down. *Navig. News* **3** (May/June), 14–15.

Nussbaum, A. & Phillips, R. A. 1982 *Contemporary optics for scientist and engineers.* Englewood Cliffs, NJ: Prentice Hall.

Ramskou, T. 1967 Solstenen. *Skalk* **2**, 16–17.

Ramskou, T. 1969 *Solstenen—Primitiv Navigation I Norden for Kompasset.* Kobenhavn, Denmark: Rhodos.

Roslund, C. & Beckman, C. 1994 Disputing Viking navigation by polarized skylight. *Appl. Opt.* **33**, 4754–4755.

Schaefer, B. E. 1997 Vikings and polarization sundials. *Sky Telescope* **93**, 91–94.

Schnall, U. 1975 Navigation der Wikinger. *Schriften des Deutschen Schiffahrtsmuseums* **6**, 92–115.

Shashar, N., Cronin, T. W., Wolff, L. B. & Condon, M. A. 1998 The polarization of light in a tropical rain forest. *Biotropica* **30**, 275–285. (doi:10.1111/j.1744-7429.1998.tb00061.x)

Thirslund, S. 2001 *Viking navigation: sun-compass guided Norsemen first to America.* Humlebaek, Denmark: Gullanders Bogtrykkeri a-s, Skjern.

Walker, J. 1978 More about polarizers and how to use them, particularly for studying polarized sky light. *Sci. Am.* **238** (January), 132–136.

Wehner, R. 1976 Polarized-light navigation by insects. *Sci. Am.* **235** (July), 106–115.