

## In the light of time

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The concept of time is examined using the second law of thermodynamics that was recently formulated as an equation of motion. According to the statistical notion of increasing entropy, flows of energy diminish differences between energy densities that form space. The flow of energy is identified with the flow of time. The non-Euclidean energy landscape, i.e. the curved space–time, is in evolution when energy is flowing down along gradients and levelling the density differences. The flows along the steepest descents, i.e. geodesics are obtained from the principle of least action for mechanics, electrodynamics and quantum mechanics. The arrow of time, associated with the expansion of the Universe, identifies with grand dispersal of energy when high-energy densities transform by various mechanisms to lower densities in energy and eventually to ever-diluting electromagnetic radiation. Likewise, time in a quantum system takes an increment forwards in the detection-associated dissipative transformation when the stationary-state system begins to evolve pictured as the wave function collapse. The energy dispersal is understood to underlie causality so that an energy gradient is a cause and the resulting energy flow is an effect. The account on causality by the concepts of physics does not imply determinism; on the contrary, evolution of space–time as a causal chain of events is non-deterministic.

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### 1. Introduction

*Where does the arrow of time come from?* Is the question that phrases succinctly (Eddington 1928) the puzzle about the origin of irreversibility. The direction of time materializes in many macroscopic phenomena, yet it is thought that physical processes at the microscopic level are symmetrical under time reversal. The flow of time is inherent in the second law of thermodynamics (Carnot 1824). The statistical notion of increasing entropy (Boltzmann 1905) appears to also underlie radiative processes and cosmic expansion. A quantum system takes a step forward in time at detection when the wave function collapses. Moreover, evolutionary courses of diverse natural processes, inanimate as well as animate, display causal relationships between past and future.

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Time in its many manifestations would deserve a through inspection (Savitt 1995; Zeh 2007) but the present study is limited to examining the basic question, *What is time?*, using the second law given as a recently derived equation of motion (Sharma & Annala 2007). It connects the principle of increasing entropy to the decreasing free energy explicitly. Various energy gradients are equalized by diverse transport and transformation processes, e.g. diffusion, heat flows, electric currents, chemical reactions (Kondepudi & Prigogine 1998). These flows of energy propel systems towards more probable states eventually reaching stationary states, i.e. free energy minima in their respective surroundings.

The second law can also be given by the principle of least action (Kaila & Annala 2008). During evolution diverse paths of energy dispersal are explored (Feynman 1948), e.g. by random variation, and those that lead to faster entropy increase, equivalent to more rapid decrease in free energy, become naturally selected (Darwin 1859; Sharma & Annala 2007; Kaila & Annala 2008). In other words, a curved non-Euclidean landscape of energy densities flattens by the energy dispersal processes that direct down along the steepest descents, equivalent to the shortest paths of space–time. The formalism has given insight into various evolutionary courses that emerge with functional structures, hierarchical organizations and characteristically skewed natural distributions (Grönholm & Annala 2007; Annala & Annala 2008; Annala & Kuismanen in press; Jaakkola et al. 2008a,b; Karnani & Annala 2008; Würtz & Annala 2008; Sharma et al. 2009). Although the old law, as such, does not provide profoundly new findings, it yields more meaning to fundamental relationships and communicates a gratifyingly holistic view of nature. In the following sections, we will first introduce the second law as the energy dispersal process and then clarify the notions of entropy and disorder. Thereafter, we use the second law and the principle of least action to analyse how the flow of time manifests itself in the basic physical processes.

## 2. Evolution as probable motion

When the energy density of a system is unequal to its surroundings, there is a gradient in energy, i.e. a motive force that tends to abolish the difference (Carnot 1824). The resulting motion, in the quest for a stationary state, manifests as flows of energy. These natural motions are referred to as probable. The statistical law of increasing probability  $P$ , related to entropy  $S = k_B \ln P$  via Boltzmann's constant, was recently given as a differential equation of motion (Sharma & Annala 2007; Kaila & Annala 2008),

$$\frac{dP}{dt} = LP \geq 0, \quad (2.1)$$

where the propagator,

$$L = \frac{-1}{k_B T} \sum_{j,k} \frac{dx_j}{dt} \left( \frac{\partial U_k}{\partial x_j} - \frac{\partial Q_k}{\partial x_j} \right), \quad (2.2)$$

generates transport, e.g. diffusion or current, at a velocity  $v_k = dx_k/dt = -\sum dx_j/dt = -\sum v_j$  between coordinates  $x_k$  and  $x_j$  by draining the potential energy gradient  $\partial U_k/\partial x_j$  between the potential  $U_k$  at  $x_k$  and  $U_j$  at  $x_j$ . Also the surroundings, denoted by the field  $\partial Q_k/\partial x_j$ , couples to the  $jk$ -transformation by either providing or accepting quanta (figure 1). The average energy per particle

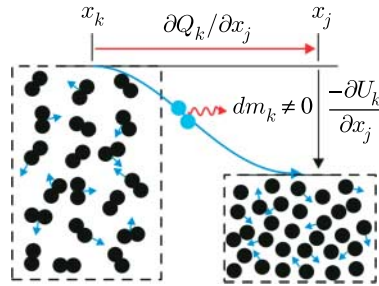


Figure 1. The potential energy gradient  $\partial U_k/\partial x_j$  between a high-energy density at  $x_k$  and a low-energy density at  $x_j$  (enclosed by dashed contours) drives a flow of energy. As a result,  $U_k$  at  $x_k$  is decreasing and  $U_j$  at  $x_j$  is increasing. The kinetic energy  $2K$  flow (blue arrow) corresponds to the flow of entities (circles) that remain invariant during the transition, i.e. retain a constant mass  $m_k$ . Dissipation (red arrow) is inherent in the change of state accounted by the mass change  $dm_k \neq 0$ . Dissipation stems from the changes in interactions between the entities at  $x_k$  and  $x_j$ . In the absence of net flows,  $U$  matches exactly  $2K$  (small blue arrows in each reservoir).

$k_B T$  implies that the system, defined here as an ensemble of interacting energy densities, is sufficiently statistic (Kullback 1959) to absorb or emit quanta without a marked change in its total energy content. The frequent and effective mutual interactions ensure that the gained or lost energy is rapidly distributed among the energy densities of the system. The equation of motion given by directional derivatives  $D_j = \mathbf{v}_j \cdot \nabla_j$  pictures an evolving energy landscape, an affine manifold of densities, spanned by time-dependent tangent vectors (Lee 2003; Kaila & Annala 2008). The mutually orthogonal gradients  $-\mathbf{v}_j \cdot \nabla_j U_k$  and  $\mathbf{v}_j \cdot \nabla_j Q_k$  project a change in kinetic energy, i.e.  $d(TS)/dt$ .

The inequality in the principle of increasing entropy  $dS/dt = k_B d \ln P/dt = k_B L \geq 0$  (equation (2.1)) implies a net flow from the system to its surroundings or vice versa. When the system is high (low) in relation to its surroundings,  $\partial U_k/\partial x_j - \partial Q_k/\partial x_j > 0$  ( $< 0$ ) and the transfer  $dx_k/dt > 0$  ( $< 0$ ) directs from (to) the system to (from) the surroundings to diminish the gradient. Thus, superior surroundings direct evolution of a system. Obviously, it is somewhat of a subjective decision how one labels some densities as the system and others as its surroundings. Regardless of the viewpoint, entropy of the system and entropy of its surroundings are increasing when the free energy is decreasing by the flows down along gradients. It is conceivable, albeit statistically unlikely, that energy would flow upwards along the gradients and entropy both of a system and its surroundings would be decreasing. However, it is impossible that the system's entropy would be decreasing at the expense of entropy increase in its surroundings because such a process would violate the conservation of energy in the flows.

In general, when  $d(TS)/dt \neq 0$ , flows do affect driving forces  $\partial U_k/\partial x_j - \partial Q_k/\partial x_j$ , which in turn redirect the flows  $v_j$  (figure 1). Owing to the interdependent energy densities, the motion is without invariants (equation (2.1)) and in general non-integrable (Sharma & Annala 2007). The probabilities keep changing owing to the influx to or efflux from the evolving system. Non-conserved trajectories cannot be traced in detail; only symmetrical cases (Noether 1918) have analytical solutions (Schwarzschild 1916). Even a small early change in a

coordinate may propagate to a substantial coordinate change later, i.e. evolution is per definition chaotic (Poincaré 1890; Strogatz 2000). Future paths cannot be predicted in detail although the ultimate equilibrium, where all density differences have vanished, is in sight. Likewise, information of past trajectories is lost when its physical representations (Landauer 1961; Karnani *et al.* submitted) as deviations from  $k_B T$  have become consumed in the subsequent transformations. Without motional invariants, no coordinate transformation can be found to make evolution time independent.

Specifically, when  $d(TS)/dt=0$ , the system is at a steady state. There is no change in kinetic energy and the parallel transport  $D_j U_k=0$  does not alter the densities indexed by  $j$  and  $k$ . Then any sporadic density difference is abolished by frequent interactions and no mutual time differences will develop. The conserved dynamics without net changes are referred to as reversible and at least in principle, it can be transformed to a time-independent frame.

Based on the above-mentioned common characteristics of evolutionary processes, we propose that the flow of time is the flow of energy. The net energy flow is the basis of irreversibility. The conjecture as such is by no means new. Specifically, it has been stated in the context of causation (Hume 1739; Ducasse 1926; Weyl 1949; Quine 1973; Fair 1979). In the following sections, the standpoint is strengthened using the universal principle of energy dispersal to describe various time-dependent phenomena.

### 3. On the entropy concept

In contrast to the prevalent, yet counter-intuitive, stance that the principle of increasing entropy would be limited to isolated systems, it is emphasized that a closed system cannot evolve from one coordinate (state)  $x_k$  to another  $x_j$ , i.e.  $dS=0$ . The conserved system, as was understood already early on (Poincaré 1890), does not advance in time but may reoccur arbitrarily close to its initial configuration. It seems that the confusion about entropy stem from the definition of a system. Here, a system in a state is defined in energetic terms. The system is not merely an arbitrarily chosen enclosure but an ensemble of interacting energy densities, i.e. particles and fields. It is emphasized that the description is holistic and hierarchical (Salthe 2007). Increasingly stronger interactions at increasingly shorter length scales structure energy densities that are systems themselves, to a nested hierarchy of systems within surrounding systems (Annala & Kuismanen in press).

The commanding role of gradients in evolution is easily missed when the equilibrium partition is determined by mathematical methods that implicitly constrain the system to a stationary state (Alonso & Finn 1983). Lagrange multipliers associate with conserved quantities, typically a fixed total number of particles and total energy that de facto impose a stationary state, i.e. a time-independent system. In addition, when an entropy formula is derived from a near-equilibrium series expansion of a state function (Mandl 1988), *primus motor* of evolution remains obscure. Presumably for these reasons, the one-to-one correspondence between increasing entropy and decreasing free energy, the basic maxim of chemical thermodynamics, was not firmly established earlier. Consequently, there has been ground for a provisional proposition that entropy

of a system could possibly decrease at the expense of entropy increase at its exterior. Such a gratuitous stance, as will become evident below, does in fact violate the conservation of energy.

The equation of evolution (equation (2.1)) does not associate increasing entropy with increasing disorder (Schrödinger 1948) but with energy dispersal (Atkins & de Paula 2006). This means that densities in energy (Gibbs 1993–1994; states)  $k$  and  $j$  are distinguished from each other in energetic terms, i.e.  $k \neq j$  when  $U_k \neq U_j$  and  $\partial U_k/\partial x_j \neq \partial U_j/\partial x_k$ . In addition, a mere measurement may impose a sufficient energy gradient, i.e. a field to distinguish energetically equivalent configurations, e.g. interconverting conformational isomers (Louhivuori *et al.* 2006) from each other by recording differences in dissipation  $\partial Q_k/\partial x_j \neq \partial Q_j/\partial x_k$  (Brillouin 1963). Thus a state change is inherent in dissipation. This is evident in nuclear transforms from one isotopic identity to another. Also chemical reactions are either endergonic or exergonic transformations of molecular identities. Many transport processes take place at length scales where the energy landscape is nearly flat (planar) to give almost unnoticeable, yet conceptually important, net dissipation when moving from one spatial coordinate  $x_k$  to another  $x_j$ .

The concept of increasing disorder is restricted here to the coherence loss due to exchange of quanta with incoherent surroundings. Sporadic exchange without net flow leads to isergonic phase dispersal. It is conceivable that the order in a system may increase at the expense of decreasing order in its surroundings. In any case, the probability  $P = \int \psi^* \psi dx$  of a state is the same in the coherent and decoherent configurations. The wave function's  $\psi \propto \exp(-i\varphi)$  phase factor  $\varphi$  accounts for the density's intrinsic but unresolved conserved dynamics. The microscopic system, just as a coherent macroscopic ensemble, will engage in energy transfer with its surroundings depending on the phase of its motions. Therefore, observables do depend on  $\varphi$  but  $P$ , and hence also  $S = k_B \ln P$ , is the phase-independent statistical status measure that increases in energy dispersal.

#### 4. Forces and flows by least action

The principle of increasing entropy as an equation of motion (equation (2.1)) describes flows of energy preferentially down along the steepest gradients. These shortest paths of space–time are known as geodesics. This identifies the second law with the principle of least action (Kaila & Annala 2008). Calculus of least variations of Moreau de Maupertuis' (1746) action

$$S = \mathbf{p}\mathbf{x} = \int L(x, t)dt, \quad (4.1)$$

given by the momentum  $\mathbf{p}$  and coordinate  $\mathbf{x}$ , serves to deduce from the Lagrangian  $\mathcal{L}$  the optimal path ( $\partial^2 \delta S / \partial t^2 = 0$ ) for the energy–momentum transfer (figure 2). The simple notation, given as a geometric product (Hestenes & Sobczyk 1984), accounts for directional flows, i.e.  $\mathbf{p}\mathbf{x} \neq \mathbf{x}\mathbf{p}$ , along directed arcs. Below the variational principle (Hanc & Taylor 2004) is used to reproduce the basic formulae of mechanics to reveal the flow of time in the changes of states.

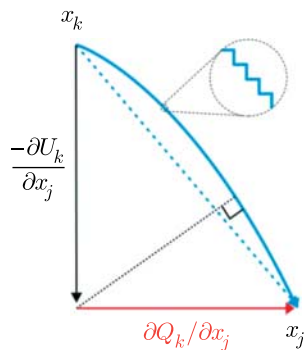


Figure 2. Flow of energy directs from the high potential  $U_k$  at  $x_k$  down along the geodesic of space-time towards the low potential  $U_j$  at  $x_j$ . The gradient  $-\partial U_k/\partial x_j$  (black vertical arrow) translates invariant mass  $m_k$  into acceleration  $a_k$  along the steepest gradient (blue arrow). The directed arc  $\mathbf{s}$  is curved owing to the concurrent dissipation  $\partial Q_k/\partial x_j$  (red horizontal arrow) that stems from the differences in interactions that distinguishes the state  $k$  from  $j$ . The curvilinear motion adds up (equations (4.3) and (4.4)) from the potential gradient and dissipative projections. The chord (dashed line) is a good approximation of  $\mathbf{s}$  in a locality that does not curve much. The expanded microscopic path forms from quantized actions.

The principle of least variations gives the balance of energy in its three forms (see appendix A)

$$\delta \mathcal{S} = \int \left( \mathbf{v} \cdot \nabla - \frac{\partial}{\partial t} \right) \mathbf{L} dt = 0 \Rightarrow mv^2 - \max - v \frac{dm}{dt} x = 0 \Leftrightarrow 2K + U = Q, \quad (4.2)$$

where the terms are labelled as the kinetic  $2K = mv^2$  and potential  $U = -\max$  energy and dissipation  $dQ/dt = v^2 dm/dt = (v^2/c^2)dE/dt$ . The emitted energy  $\beta^2 dE/dt$  at the speed of light  $c$  stems from changes in interactions. These are accounted by the mass change  $dm$  that is often omitted as being insignificant. However, the mass change is very important to understand time-dependent phenomena. Since the states along an evolutionary path distinguish from each other in energetic terms, the change in mass signals for a change of state. The change in mass, e.g. expelled as heat, is apparent in rearrangements of nuclear interactions that transform a nuclear state to another. Electronic restructuring energies in chemical transformations from a molecular state to another compound are much smaller but readily observable. During many transport processes, energy differences between adjacent coordinates (states) are extremely small, i.e.  $dm \approx 0$ . Although dissipation that is inherent in gradient reduction is almost negligible, it is importantly non-zero. No change of state is possible without dissipation and there is no evolution without a mass change.

A stationary system  $2K + U = Q$  has an invariant mass ( $dm = 0$ ). Its integrable motions are along closed conserved paths. Trajectories are straight lines in the time-independent frame of reference obtained from a transformation that delivers invariants over the period (cycle) of vanishing net flows  $\langle \partial Q/\partial t \rangle = 0$ . At the dynamic stationary state, there are to-and-fro flows. These reversible flows, e.g. as photon-mediated interactions, total an interaction energy reservoir (Provotorov 1962; Oja *et al.* 1988), denoted by  $Q$ . A stationary spectrum of radiation with its characteristic  $k_B T$  means that there are no gradients between the system and its surroundings.

The time derivative of least action  $\partial^2 \delta S / \partial t^2 = 0$  gives the second law, i.e. the balance for the energy flows

$$\frac{\partial}{\partial t} 2K = -\mathbf{v} \cdot \nabla U + i\mathbf{v} \cdot \nabla Q = -\mathbf{v} \cdot \nabla V. \quad (4.3)$$

The identity  $\partial/\partial t = \mathbf{v} \cdot \nabla$  between temporal and spatial gradients follows from the conservation of energy. The diminishing potential energy is dispersed exactly by flows of kinetic energy and dissipation. The gradient  $\nabla V$  is composed of  $\nabla U$  and  $i\nabla Q$  to denote explicitly that dissipation is orthogonal to the irrotational part (figures 1 and 2). Technically speaking, the direction of time does not commute with the directions of space.

The familiar counterpart of the energy continuity equation is the Newton's (1687) second law for balancing forces

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m\mathbf{a} + \mathbf{v} \frac{dm}{dt} = -\nabla U + i\nabla Q = -\nabla V, \quad (4.4)$$

where  $\mathbf{a}$  is acceleration. The dissipative part  $\mathbf{v} dm/dt$  forces a rotation that transforms some density at  $\mathbf{x}$  to an outflow along the orthogonal direction  $t$ . At the stationary state,  $dm=0$  and  $\nabla U + d\mathbf{p}/dt=0$ . For example, in an irrotational central potential  $\nabla \times \nabla U(r)=0$  angular velocities  $\omega^2 = r^{-3}$  are time-independent whereas when  $\partial Q/\partial t \neq 0$ ,  $\nabla \times \nabla V \neq 0$  and  $\omega(t)$ .

For a statistical ensemble of densities, the law of forces reads in the component form as

$$\sum_k m_k a_k + \sum_k v_k \frac{dm_k}{dt} = -\sum_{j,k} \frac{\partial V_k}{\partial x_j}, \quad (4.5)$$

where the dissipative force  $v_k dm_k/dt = \partial Q_k/\partial x_j$  displaces the flow from  $x_k$  to  $x_j$  orthogonal to  $\partial U_k/\partial x_j$  (figures 1 and 2) along a directed curved path  $\mathbf{s}$  (figure 2; Carroll 2004).

The concise operator for a curvilinear motion is  $\partial/\partial s = \partial/\partial x - i\partial/c\partial t$  where the  $(-)$  sign signifies conservation and the imaginary part means that the spatial and temporal gradients are orthogonal. When the net dissipation is negligible ( $dm \approx 0$ ), the directed path  $s$  does not curve much and the straight chord  $ds \approx v dt$ , the familiar result of Isaac Barrow, the first in the Lucasian chair, is an excellent approximation of  $dx = v dt$ . However, truly rectilinear motions are characteristics of a stationary system ( $dm=0$ ) where the phase may advance from one configuration to another but there is no change of state. The principle of least action serves to formulate evolution involving also rotational motions but will not be digressed into.

## 5. Evolution driven by light

The flow of time as the flow of energy from one coordinate to another is inherent in dissipation. The efflux (or influx) as electromagnetic radiation emerging from (or integrating into) the system's emissive (or absorptive) transitions is given, as equation (4.1), by the action

$$\mathbf{S} = \rho_e \mathbf{A} \mathbf{x} = \int \mathbf{L}(x, t) dt, \quad (5.1)$$

where  $\mathbf{p} = m\mathbf{v}$  per volume has been substituted for the momentum density  $\rho_e \mathbf{A}$  due to the charge density  $\rho_e$  and vector potential  $\mathbf{A}$ . The least variations ( $\partial\delta S/\partial t=0$ ) gives the balance between the kinetic  $2k$  and potential energy  $u$  densities and dissipation  $q$  (cf. equation (4.2))

$$\rho_e \mathbf{A} \cdot \mathbf{v} = \rho_e \mathbf{x} \cdot \frac{\partial \mathbf{A}}{\partial t} + \mathbf{x} \cdot \mathbf{A} \frac{\partial \rho_e}{\partial t} \Leftrightarrow 2k = -u + q, \quad (5.2)$$

and the geodesic equation ( $\partial^2 \delta S / \partial t^2 = 0$ ) gives the balancing flows in analogy to equation (4.3)

$$\mathbf{v} \cdot \frac{\partial}{\partial t} \rho_e \mathbf{A} = \rho_e \mathbf{v} \cdot \frac{\partial \mathbf{A}}{\partial t} + \mathbf{v} \cdot \mathbf{A} \frac{\partial \rho_e}{\partial t} \Leftrightarrow \frac{\partial}{\partial t} 2k = -\mathbf{v} \cdot \nabla u + i\mathbf{v} \cdot \nabla q, \quad (5.3)$$

where the change in state is driven, as before, by the two orthogonal gradients.

The form of the second law in equation (5.3) is unfamiliar but when the definition of electric field  $\mathbf{E}$

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}, \quad (5.4)$$

where  $\nabla\phi$  is the irrotational scalar and  $\partial\mathbf{A}/\partial t$  non-conserved part is used as well as the identity  $\partial/\partial t = c\nabla \cdot$ , equation (5.3) turns out to be Poynting's theorem (Poynting 1920; Griffiths 1999; see appendix A)

$$\begin{aligned} \mathbf{J} \cdot \mathbf{E} &= -\rho_e \mathbf{v} \cdot \nabla\phi - \mathbf{v} \cdot \frac{\partial}{\partial t} \rho_e \mathbf{A} + \mathbf{v} \cdot \mathbf{A} \frac{\partial \rho_e}{\partial t} = -\rho_e \frac{\partial \phi}{\partial t} - \rho_e \mathbf{v} \cdot \frac{\partial \mathbf{A}}{\partial t} \\ &= -\frac{\partial u}{\partial t} - \varepsilon c^2 \nabla \cdot (\mathbf{E} \times \mathbf{B}), \end{aligned} \quad (5.5)$$

where the flow of kinetic energy density  $2k$ , as the current density  $\mathbf{J} = \rho_e \mathbf{v}$  in  $\mathbf{E}$  is balanced exactly by the changing potential density  $\partial u/\partial t$  and light, the orthogonally dissipated density  $\partial q/\partial t = -\varepsilon c^2 \nabla \cdot (\mathbf{E} \times \mathbf{B})$  as electric and magnetic fields (figure 3). The initial and final spatial separation of the charges, i.e. the potential energy difference between the initial  $x_k$  and final  $x_j$  states, is bridged by the dissipated quanta that span exactly the time of charges in motion. It takes its time, i.e. dissipation to change from a state to another. Emission generates radiation that escapes from the system to the surroundings whereas absorption integrates radiation from the surroundings into the system. The flow of light, as the carrier of energy from the evolving system to its surroundings (or vice versa), manifests as the flow of time.

The equation of continuity  $\nabla V_e + \partial \rho_e \mathbf{A} / \partial t = 0$  (cf. equation (4.4)) or  $\partial V_e / \partial t + v^2 \nabla \cdot \rho_e \mathbf{A} = 0$  is equivalent to the law of forces

$$\frac{\partial}{\partial t} \rho_e \mathbf{A} = \rho_e \frac{\partial \mathbf{A}}{\partial t} + \frac{\partial \rho_e}{\partial t} \mathbf{A} = -\nabla u + i\nabla q = -\nabla V_e, \quad (5.6)$$

where  $\nabla V_e = \nabla u + i\nabla q$ . The unfamiliar form is rearranged, using equation (5.4) and  $\nabla \times \mathbf{A} = \mathbf{B}$  to a force density (see appendix A)

$$\rho_e \mathbf{E} = -\rho_e \nabla\phi - \rho_e \frac{\partial \mathbf{A}}{\partial t} = -\rho_e \nabla\phi + \mathbf{J} \times \mathbf{B}. \quad (5.7)$$



When all densities in material forms have transformed to radiation, i.e.  $\rho \rightarrow 0$  and  $v^2 \rightarrow c^2$  (or, e.g. by annihilation), equation (5.5) will reduce to the familiar Lorenz gauge (Lorenz 1867)

$$\frac{\partial \phi}{\partial t} + c^2 \nabla \cdot \mathbf{A} = 0. \quad (5.8)$$

The steady-state continuity, equivalent to  $\nabla \phi + \partial \mathbf{A} / \partial t = 0$  (cf. equation (5.4)), is the geodesic for light in the flat space. The straight propagation results from repeatedly reversing rotation driven by the recurrent sign reversal of oscillating but vanishing  $\langle \nabla \phi \rangle = 0$  (figure 3). In the perspective of light, the gauge is equivalent to Minkowski's norm  $\tau = 0$ . Then  $\phi = c|\mathbf{A}|$  and equations for  $\phi$  and  $\mathbf{A}$  are symmetrical under time reversal, in fact, time independent. Time does not elapse for light that retains constant frequency. Conversely, light shifts its frequency in response to a density change, just as particle velocities respond to changes in potentials.

The retarded time  $t' = t - s/c$  that is characteristic of the four-potential solutions of inhomogeneous equations (Griffiths 1999) is a mere consequence of the conservation in dispersal. The alternative sign (+) would mean violation. These insights provided by the principle of least action into the radiative arrow of time follow from the universal law for energy dispersal along the geodesics, the natural paths.

## 6. Dissipation at detection

The time flow as the energy flow holds also for microscopic systems. However, seemingly strange phenomena are encountered when detection puts a small system in evolution. The microscopic system is easily driven by the amount of energy transformed in the detection to an unrecognizable coordination. The observation may purge the microscopic system altogether. Peculiar phenomena are also encountered when the surroundings displays coherent density variations that tug and pull the system (Sudarshan & Misra 1977; Schieve *et al.* 1989; Erez *et al.* 2008).

The detection is expressed by the basic formalism of quantum mechanics as an expectation value of an operator  $\langle A \rangle = \langle \psi | A | \psi \rangle$ . However the recipe, as well understood (Griffiths 1995), is not quite adequate since the observation perturbs the steady state represented by the wave function  $\psi$ . Especially, when the detection-driven flow is large in relation to the system's repositories of energy, the probabilities will evolve substantially. A single quantum is a giant leap for a microscopic system whereas it is a small step for a macroscopic system. The trajectory of sudden changes is difficult to follow and the fate of the microscopic system is hard to comprehend using the conventional conserved formalism. If  $\bar{P}$  is fallaciously normalized to unity, it is 'frozen' against the natural motions that are inherent in the detection. Here, the evolving system is described without norms because its energy is changing.

In attempts to account for evolution using the traditional closed formalism, an external perturbing field is often included ad hoc, e.g. in a series expansion (Zubarev 1974), Lindblad's (1976) form or in non-Markovian-generalized master equation (Paz & Zurek 1999). However, such a description of evolution remains approximate and reminds us of Bertrand Russell's quote that *all exact science is dominated by the idea of approximation*. Calculations may yield even negative

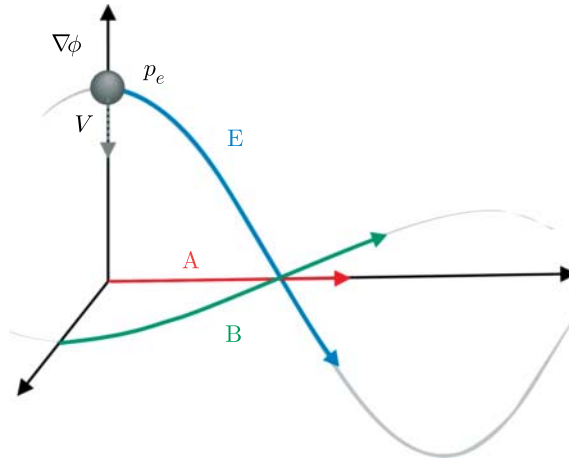


Figure 3. A charged density  $\rho_e$  when moving at velocity  $\mathbf{v}$  generates a current  $\mathbf{J}$  in an electric field  $\mathbf{E}$ . The flow of kinetic energy is balanced exactly by the changing electromagnetic field scalar potential  $\rho_e \partial\phi/\partial t$  and the perpendicularly dissipated light  $\epsilon_0 c^2 \nabla \cdot (\mathbf{E} \times \mathbf{B})$  (equation (5.5)). Subsequently, the nascent light propagates through a homogeneous medium without sources  $\nabla \cdot \mathbf{E} = 0$ . In the flat space, the oscillating  $\partial\phi/\partial t$  balances exactly (equation (5.8)) the divergence of vector potential  $c^2 \nabla \cdot \mathbf{A}$  and the flight of light is straight on the average  $\langle \phi \rangle = 0$ .

probabilities that are although recognized as unphysical (Piilo *et al.* 2008). The account on flows by equation (2.1), i.e.  $\langle \partial Q/\partial t \rangle \neq 0$ , allows for the necessary changes in  $P$ . Since the open formalism is without norms, elegant mathematical methods devised for conserved currents (Noether 1918) are disabled. This is inconvenient but best to bear with to understand irreversibility.

The equation of motion for the probable quantized flows of energy is obtained, as before, from the quantized action

$$\hat{S} = \hat{p}\hat{x} = \int \hat{\mathbf{L}} dt, \quad (6.1)$$

now given by the momentum  $\hat{p} = -i\hbar\nabla$  and coordinate  $\hat{x}$  operators. The emitted quanta result from the transit from the state  $k$  to a state  $j$ . The reverse path  $\hbar\omega t$  from  $j$  to  $k$  absorbs quanta from the surroundings to power the transition. The curved path at the microscopic resolution is a staircase of quantized steps (figure 2). The least variations implies the commutator relationship  $a[\hat{p}, \hat{x}] = -i\hbar$  (Griffiths 1995). The balancing flows are obtained, as above, from (see appendix A)

$$\frac{\partial}{\partial t} \left\langle \left( \mathbf{v} \cdot \nabla - \frac{\partial}{\partial t} \right) \hat{\mathbf{L}} \right\rangle = 0 \Rightarrow \frac{\partial}{\partial t} 2K + \mathbf{v} \cdot \nabla U - \frac{\partial Q}{\partial t} = 0, \quad (6.2)$$

as expectation values for the kinetic energy  $2K = \langle vp \rangle$  and for  $V$ , which total from the spatial  $U = -\langle x(dp/dt) \rangle$  and temporal  $Q = -i\hbar \langle \partial/\partial t \rangle$  components.

The equation of motion for  $\psi(x, t)$  is obtained, as usual (Sakurai 1994), from the least action

$$\begin{aligned} \frac{\partial}{\partial t} \left\langle x \left| \frac{\partial}{\partial t} \delta \hat{S} \right| \psi \right\rangle &= 0 \Leftrightarrow \frac{\partial \psi}{\partial t} \\ &= -\frac{1}{2K} \left[ \left( \frac{\partial}{\partial t} 2K + \mathbf{v} \cdot \nabla U - \frac{\partial Q}{\partial t} \right) \psi + (U - Q) \frac{\partial \psi}{\partial t} \right]. \end{aligned} \quad (6.3)$$

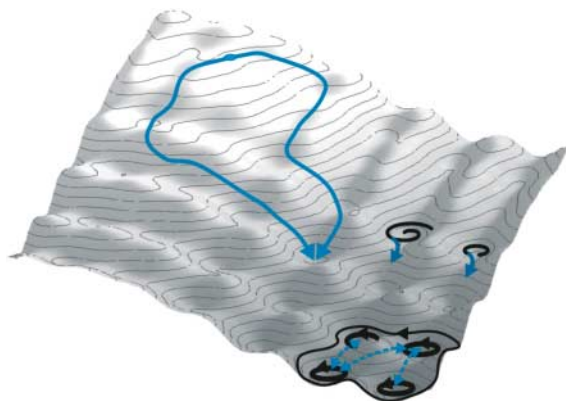


Figure 4. Energy landscape is levelling when high-energy densities (upper left corner) disperse by flows of energy that may direct along multiple paths (blue arrows) towards low-energy densities (lower right corner). When the flows recombine, relative phases acquired along paths through differing potentials give rise to interference. Stationary systems, depicted as closed cyclic paths (black), reside in attractive basins. They may also structure to nested hierarchical systems when communicating with each other by flows (dashed blue double-ended arrows). The landscape evolves via the flows. Thus a previously confined steady system, accommodating at least one integral circulation  $\hbar$ , may open up (semicircles) and emit a quantum to settle anew to another state or purge itself altogether.

For an open evolving system, the dissipative term denotes gradients that drive flows of energy between the system and its surroundings. A particular field  $\partial Q/\partial t$  can be given by ladder operators and spectral densities. Since the open evolving system has no norm, equation (6.3) cannot in general be solved by integration.

The description of a levelling energy density landscape using time-dependent state vectors  $\psi(x, t)$  can, of course, be transformed to a view of time-dependent operators. In the Heisenberg picture, the state does not evolve but forces, obtained by Ehrenfest's theorem (Griffiths 1995; cf. the Newton's second law equation (4.4)), are fading away with time. The two alternative views on evolution correspond, in mathematical terms, to the tangent and cotangent bundles of the manifold.

The probability flow as a directional integral is calculated, as usual (Griffiths 1995), from (see appendix A)

$$\frac{dP}{dt} = \int \frac{\partial}{\partial t} \psi^* \psi \, dx = -\frac{1}{k_B T} \left( \frac{dx}{dt} \frac{\partial U}{\partial x} - \frac{\partial Q}{\partial t} \right) P - \frac{d}{dt} \Delta\varphi, \quad (6.4)$$

where the change in geometric phase  $d\Delta\varphi/dt$  accumulates from the net flow  $\langle \psi^* | \partial\psi/\partial t \rangle - \langle \partial\psi^*/\partial t | \psi \rangle$  when the path opens up and the system spirals from a state towards another (figure 4). When flows funnel via multiple paths through different densities, relative phase differences may develop. The interference effect, that is well known from the Aharonov–Bohm (1959) experiment, shows up at the common destination, e.g. at the detector (figure 4). When constituents of a sufficiently statistic and incoherent ensemble are index by  $j$  and  $k$ , the average phase and its net precession vanish and the resulting equation is identical with equation (2.1).

In the absence of gradients, there can be no dissipation, no evolution, no probability flow (Griffiths 1995) and no flow of time either. The stationary-state dynamics along the closed paths are obtained in the same way as equation (6.3) from the conserved variations

$$\begin{aligned} \frac{\partial}{\partial t} \langle x | \delta \hat{S} | \psi \rangle &= 0 \Leftrightarrow i\hbar \frac{\partial \psi}{\partial t} \\ &= -\frac{\partial}{\partial t} (xp - px + i\hbar) \psi - (xp - px) \frac{\partial \psi}{\partial t} \\ \Leftrightarrow i\hbar \frac{\partial \psi}{\partial t} &= (2K + U - Q) \psi - \hbar \varphi \frac{\partial \psi}{\partial t}. \end{aligned} \quad (6.5)$$

Obviously, in the time-independent frame  $2K + U = Q$  there is no evolution, accordingly equations (6.3) and (6.5) yield  $\partial \psi / \partial t = 0$ . The stationary motion is pictured (figure 4) as a cyclic path (loop) in an attractive basin (Strogatz 2000). Its circulation in multiples of  $\hbar$  totals the geometric phase  $\varphi$  (Berry 1984). These reversible motions are, however, represented versus  $t$  when the system and detector frames are not identical. Then to-and-fro fluxes report from the conserved dynamics and equation (6.5) reduces to the familiar form  $i\hbar \partial \psi / \partial t = H\psi$  governed by Hamiltonian  $H = K + U$ . The detected dynamics exposes  $U$  by solutions  $\psi \propto \exp(-i\omega t)$  that match  $H$  by an invariant rate  $\omega = d\varphi/dt$  of isergonic phase precession. The condition  $dP/dt = 0$  is the same as for light propagating in a constant density (equation (5.8)).

Customarily quantum mechanics is formulated for an isolated or stationary system ( $dP/dt = 0$ ). Without energy transfer, no information can be obtained from such a system (Brillouin 1963; Karnani *et al.* submitted). Thus, a measurement appears problematic and its analysis may yield even counter-intuitive or confusing conclusions when using the closed formalism. However, time-dependent surrounding fields are inherently included in the open formalism. Hence a microscopic system, just as a macroscopic ensemble, will evolve ( $dP/dt > 0$ ) from a state to another by a series of transforming steps until a steady state, the stationary system, has been reached.

The probabilistic nature of detection, pictured as a collapsing wave function (Griffiths 1995), follows from unresolved intrinsic dynamics embedded in  $\varphi$ . An observable may acquire different values depending on the phase of motion. For example, the system may be in a phase relative to the detector to accept or in another phase to eject a quantum, or eventually in a phase to remain inert against energy transfer. Usually this is described somewhat confusingly so that the system prior to the observation would be in a superposition of states. On the contrary, the unobserved system *is a* stationary state having conserved currents along closed paths. The detection as an energy transfer process drives evolution by opening a geodesic from one state to another, which one, depends on the phase of motion. The detection of a coherent macroscopic ensemble is, of course, also probabilistic. Phase-dependent distributions are symmetric because there are no mutual energy differences. By contrast, energy-dependent distributions resulting from diverse evolutionary processes are skewed, nearly lognormal natural distributions without closed form integrals (Grönholm & Annala 2007; Würtz & Annala 2008).

*Observations*, by the words of Pascual Jordan, *not only disturb what is to be measured, they produce it ...* (Mermin 1985). The energy gradient, i.e. the field imposed on the system by the observer influences strongly by extracting or

incorporating quanta, what will emerge from the detection and determines what can be distinguished. The resolution in action, as it seems, is ultimately limited to the very least action  $\hbar$ , the uncertainty in defining a state in relation to other energy densities. Apparently, when the smallest integral circulation opens up, it drains altogether. Thus, time is a quantized flow that steps in increments of  $i\hbar\partial/\partial t$ . These insights into evolution and decoherence (Schlosshauer 2007) of quantum systems remind us of Richard Feynman's remark (Taylor & Wheeler 1992). *What may look like a paradox is often only a conflict between reality and your feeling of what reality ought to be.*

## 7. Evolution by expansion

The time flow as the energy flow is often perceived as irreversible when the system cannot command its superior surroundings. When the surrounding density rises above that of the system (or drops below), efflux will turn to influx (or vice versa). It is emphasized that there is no principle of macroscopic irreversibility but both influx and efflux, when levelling differences, are probable motions (equation (2.1)). Likewise, as was reasoned above, there is no principle of microscopic reversibility. Irreversibility is exclusively based on reasons of probability (Ritz & Einstein 1909). Ability to record fluxes from deep space has made us aware of the grand dispersal. The cosmic expansion as an energy dispersal process seems irreversible, more precisely irrevocable (Georgescu-Roegen 1967), since there is no evidence of greater densities that could possibly reverse the universal arrow of time.

The ubiquitous imperative to disperse energy is austere but its mechanisms are diverse. Stars, black holes, galaxies, blazers, etc., are, according to the second law, mechanisms that have evolved to disperse energy. Although the equation of evolution (equation (2.1)) indexes numerous constituents, only overall kinematics is discussed here. The cosmological principle (Taylor & Wheeler 2000) of large-scale homogeneity follows from the natural selection for the most rapid dispersal. Decreasing density  $\rho$  in material forms disperses at increasing velocity  $v \rightarrow c$  to balance produced and diluting radiation density.

In general, flow rates change with changing gradients but over a short space-time interval  $ds = dx - ic dt$ , the short arc  $ds$  can be approximated by a straight path as if the source and sink were inert despite the directional energy transfer. As Gauss understood, a local patch of the landscape can be regarded as flat, i.e. Euclidean (Weinberg 1972). When the density change  $\partial u/\partial t = -\partial(\rho ax)/\partial t \approx -(\partial\rho/\partial t)(dx/dt)^2$  is rewritten as  $\partial u/\partial t = -(\partial\rho/\partial t)(d\sigma/dt)^2$  using the proper length  $d\sigma$  or the proper time  $d\tau = id\sigma/c$  (Taylor & Wheeler 1992; Foster & Nightingale 1994; Berry 2001; figure 5), the balance of flows yields the famous invariant of motion (Einstein 1905)

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial q}{\partial t} - \frac{\partial}{\partial t} 2k \Leftrightarrow c^2 \frac{\partial \rho}{\partial t} \left( \frac{d\tau}{dt} \right)^2 = c^2 \frac{\partial \rho}{\partial t} \left( 1 - \frac{v^2}{c^2} \right) \Leftrightarrow d\tau^2 \\ &= dt^2 - c^2 ds^2 \Rightarrow d\tau/dt = \sqrt{1 - v^2/c^2}, \end{aligned} \quad (7.1)$$

where the flow  $\partial(2k)/\partial t = v^2 \partial\rho/\partial t$  relates to the potentials that are dispersing with velocity  $v$ . Ultimately all potentials will transform to the radiation density  $\partial q/\partial t = c^2 \partial\rho/\partial t$ .

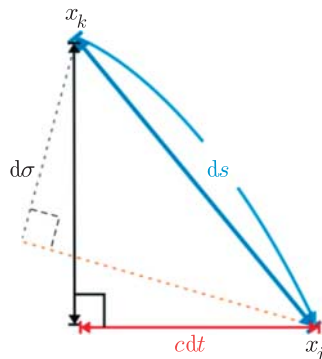


Figure 5. Energy flows from the high-potential  $U_k$  at  $x_k$  down along the shortening directed arc towards the low-potential  $U_j$  at  $x_j$ . When the short chord  $ds$  (blue line) equals approximately the short curved path, the balance of energy density  $u=2k-q$  corresponds, according to Pythagoras' theorem, to the local Lorentz invariant metric (equation (7.1)). It is covariant with respect to a change in the frame of reference (dotted).

During the cosmic expansion, energy is released from high densities  $u$  by diverse mechanisms and dispersed as kinetic energy density  $2k$  and dissipated radiation density  $q$ . On the scale of the Universe,  $u$  remains in balance with  $q$  (equation (7.1)) when the diminishing sources are displacing from each other with an increasing average velocity approaching the speed of light ( $\beta^2 = v^2/c^2 \rightarrow 1$ ). Likewise on a galactic scale, radiation of a star in a gravitational potential is balanced by an additional kinetic contribution to its velocity. On all scales, the Lorentzian manifold, with its opposite signs for the temporal and spatial coordinates, is a mere consequence of  $u$  being, on the average, in balance with  $q$  and  $2k$  (Penzias & Wilson 1965).

The rate of expansion  $H=1/dt=\sqrt{(1-\beta^2)}/d\tau$  (Hubble 1929) due to the energy dispersal during the radiation dominated era does not, according to the second law, allow for additional parameters, e.g. deceleration but the unbound Universe is natural (figure 6). Consistently, the expansion has been going on for the time  $1/H$  (Berry 2001). Time is approaching its end when potentials are dispersing and transforming to kinetic energy and radiation which encompass a larger and larger portion of the total energy of the Universe.

The energy balance (equation (7.1)) can also be given by Euler equations where the energy density gradient  $\nabla u$  is expressed as a pressure gradient  $\nabla \pi$  and the orthogonality between the negative driving pressure and the radiation is implicit, just as it is in the Poynting theorem (equation (5.5)). The differentials in equation (7.1) are often factored further to Navier–Stokes equations.

The second law for the flows of energy is the system's subjective view of its surroundings. In fact no other (objective) view is conceivable because all observations are dissipative transfer processes between subjects (Brillouin 1963). A local flow of energy identifies with a local flow of time experienced by a subject. According to Hermann Weyl, the subjective non-Euclidean world, i.e. a curved landscape *happens* [evolves] whereas the objective Euclidean world, i.e. a flat manifold simply *is* [stationary] (Hume 1739).

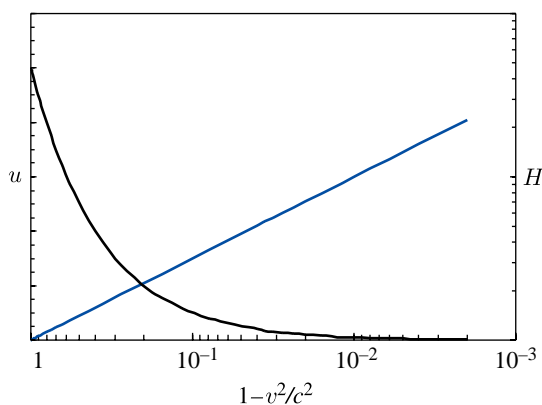


Figure 6. Energy dispersal by the expansion of the Universe is plotted versus  $1 - v^2/c^2 \rightarrow 0$  on a log scale. The potential energy density  $u$ , on a linear left scale, is devoured in transformations to the radiation density  $q$  that is diluting at the speed of light  $c$ . To maintain balance between  $u$  and  $q$ , sources are moving apart from each other at increasing velocity  $v \rightarrow c$  when  $u \rightarrow 0$ . Thus, the expansion takes place at the increasing rate  $H$ , shown on a log scale.

## 8. Evolution in a bound system

In the expanding Universe, there are no everlasting stationary states, nevertheless some systems such as Earth may enjoy rather steady influx over aeons. Evolution in the flux can also be described by the second law or by the principle of least action. Over the aeons, matter on Earth has evolved in transduction in the quest for a stationary state in the high-energy influx from hot Sun and thermal efflux to cold space as has been understood by the maximum entropy production principle (Ulanowicz & Hannon 1987; Brooks & Wiley 1988; Lovelock 1988; Salthe 1993; Schneider & Kay 1994; Chaisson 2001; Lorenz 2002; Dewar 2003; Lineweaver 2005). Increasingly, more effective transduction mechanisms, i.e. species have emerged to transform energy in radiation to increasing chemical potential. Densities associated with photosynthetic primary mechanisms are dispersed further down by other species within system's secondary repositories of energy. Biotic species form the transduction network customarily referred to as the food chain. The ultimate waste, low-energy radiation, is dissipated to the surrounding cold space.

During evolution, new mechanistic species may emerge merely from random variation in the synthesizing flows. Those mechanisms that channel energy along ever shorter paths equivalent to even steeper descents are naturally selected (Darwin 1859; Sharma & Annala 2007; Kaila & Annala 2008). Novel mechanisms may drive some old ones to extinction by depriving them from vital sources of free energy. Access to new resources by more economical means will trigger rapid flows that will punctuate the prior steady state. Influx directs in raising increasing populations to absorb more from the high-energy surroundings. This is also known as the constructal law (Bejan 1997). The autocatalytic proliferation is not exclusive to biota and economies but familiar also from elementary physical processes, e.g. from stimulated emission (Einstein 1916). Later, when the free energy narrows, development settles towards a stasis, the

stable operational mode. Intricate and even chaotic courses of punctuated evolution (Eldredge & Gould 1972) cannot be predicted but an overall sigmoid course can be simulated because flows are proportional to the gradients (Lavenda 1985; Grönholm & Annila 2007; Jaakkola *et al.* 2008*a,b*; Würtz & Annila 2008; Sharma *et al.* 2009).

At the dynamic steady state  $2K + U = Q$ , diverse pools of energy convert to one and another without net dissipation  $\langle \partial Q / \partial t \rangle = 0$ . The maximum-entropy state is, in accordance with the Le Chatelier's principle, maintained by steady through flow. The irrotational free energy minimum is, according to Lyapunov (Kondepudi & Prigogine 1998; Strogatz 2000), stable against internal fluctuations, i.e. variations in its densities and kinetics but must adapt to changes in the surrounding densities. The quest for  $S_{\max}$  is the physical rationale of unconscious self-regulation, the invisible hand (Smith 1776), articulated also by the Gaia theory to account for the global homeostasis (Lovelock 1988; Karnani & Annila 2008).

A system that is subject to a periodically varying influx, such as the annual rhythm on Earth, may attain a stationary state. Then generations follow generations but there is no net evolution. The concept of time is not meaningless but perceived as cyclic. However, a mere revival of surrounding densities in energy may not suffice to reverse trajectories because the very mechanisms as repositories of energy may have become consumed in other processes prior to the flux reversal. These mechanistic obstacles of reversibility are, e.g. characteristics of protein folding (Onuchic *et al.* 1995; Sharma *et al.* 2009).

It is time to remind, the common conjecture that entropy of a living system could possibly decrease at the expense of increasing entropy in its abiotic surroundings does not satisfy the conservation of energy because the energy influx to the system must match exactly the outflow from its surroundings or vice versa. It is although possible but statistically unlikely that energy would be flowing up along gradients and entropy both of a system and its surroundings would be decreasing. Thus, to put in a caveat against the second law signals deep-rooted illusions that life would be an unnatural process. There is no demarcation line between animate and inanimate but undoubtedly suitable settings for the processes referred to as biological are rare, considering simply the appropriate energy range that spans only a minute band of the cosmic background spectrum (Annila & Annila 2008).

## 9. Causation as energy flow

According to the second law a force is a cause and a flow is an effect. The causal connection being the energy–momentum transfer relationship (Hume 1739; Quine 1973) does not undermine intricacy of causal relationships. Causality is a systemic notion. A particular, singled-out causal relationship is a mere part of the entirety. Evolutionary courses among interdependent densities are intractable in detail, i.e. non-integrable (Sharma & Annila 2007) because flows affect forces that, in turn, alter the flows. Neither the effect nor the cause is static. The effect changes the cause as well. The inherent inability to predict precisely dispersal clarifies why it is difficult to track a chain of events to reveal its all and ultimate causes. When flows level gradients, so to speak effects abolish



causes, the energetic identity of the cause that existed at an earlier point in the space–time alters. Also information of the past, as represented by deviations from the average energy  $k_B T$ , degrades during evolution.

Causal relationships are embedded in the properties of time-oriented Lorentzian manifold (Weyl 1949) whose evolution is given by the second law and the principle of least action. When events  $k$  and  $j$  are separated by a time-like interval  $d\tau^2 = dt^2 - ds^2/c^2 > 0$ , energy may flow from  $x_k$  to  $x_j$  along the connecting arc  $\mathbf{s}$ . There is enough time for the cause–effect relationship to exist between the two events. Since the operators  $\hat{p}$  and  $\hat{x}$  do not commute,  $\mathbf{s}$  is future-directed down along the gradient. On the other hand when  $x_k$  to  $x_j$  are separated by a space-like interval  $\tau^2 < 0$ , not even light is a fast enough means to transfer over the distance in between. Since there is no action, the commutator vanishes. There is not enough time for the causal relationship to exist between the two events. Finally,  $x_k$  and  $x_j$  may be separated exactly by the time needed for light to connect them, i.e.  $\tau^2 = 0$ . However, there is no action because light cannot quite absorb to exert an effect. There is no force acting on light. The space, given by the gauge (equation (5.8)), is flat and time does not advance.

It is emphasized that the principle of increasing entropy is a meaningful imperative for a causal system. Since densities are connected only by flows, it is impossible to identify a cause without experiencing any effect. The interacting densities form the affine (free energy) manifold that evolves according to the Lorentz invariant or generally covariant equation of motion (figure 5). By contrast, manifolds without affine connections, i.e. transforming paths of free energy, make only a collection of non-communicating systems without mutual causal relationships. They share no common standard of time and evolve independently from each other.

Action at a distance, i.e. an effect without energy transfer is perceived by the second law impossible. Specifically, there is no common probability  $P$  for non-interacting systems, e.g. for a correlated pair of photons that emerge from a radiative decay process. The correlation will remain until one of the two flows impinges on surroundings. The correlation, denoted by  $\varphi$ , is a mere measure of order that does not relate to  $S$  and hence not with  $P$  either. Accordingly, nothing (certain) can be said about a system that is not being interacted with or whose fluxes are not being monitored. This is, of course, in contrast to the stance of realism. Extent and frequency of interactions with surroundings define the subject’s realm of realism.

## 10. Discussion

*The flow of time is the flow of energy* is a simple statement but one that places time on the same footing with space. It takes time to distinguish a spatial loci from another, i.e. to move energy from one distinct density to another. The evolving space–time is energy–momentum transfer process. Its Lorentzian metric satisfies the conservation of energy in flows at a local patch of the universally non-Euclidean landscape and the covariance means that a path, i.e. physics is the same irrespective of the viewpoint that is in line with operational ideals of the physical sciences (Bridgman 1936).

Time is without its arrow in the standard formalism for microscopic systems because the flows from the system to its surroundings (and vice versa) have not been denoted explicitly—for good reasons. A microscopic system cannot tolerate much of flux as it would soon extinguish altogether or become transformed into something unrecognizable. However, there is nothing intrinsically reversible at the microscopic level. Likewise, there is nothing inherently irreversible at the macroscopic level either. At every level of hierarchy, the flows of energy funnel along the steepest gradients from the system to its surroundings or vice versa. The flows will transform a macroscopic ensemble, just as the microscopic system, but a single quantum will not do all that much to a big statistic system but it may wash out a small system altogether.

Perhaps the view of time as a flow of energy has also been obscured since one is easily enticed by powerful mathematical methods that a closed, stationary system offers to investigate its deterministic dynamics. By contrast, evolution of an open system that is without norms may at first sight appear as an unattractive study because the non-conserved motions are in general non-integrable and trajectories are intractable in detail. Driving forces change with flows that in turn affect the forces hence *ceteris paribus* does not hold (Gould 2002). On the other hand, the stationary system may not be that fascinating after all because its conserved motions can be transformed to a time-independent frame. Per definition, a deterministic system has no future to be predicted.

In this study, elementary notation was used to emphasize basic concepts but admittedly at places it does not match the rigour of mathematical thermodynamics (Owen 1984; Lucia 1995, 2008). Our simple notation is also clumsy, in particular, when keeping track of orthogonal directions. However, geometric (Clifford's) algebra (Hestenes & Sobczyk 1984) offers particularly elegant ways to denote densities in energy and to perform operations corresponding to the energy transfer. For example, quaternions comply intrinsically with the Lorentzian metric and operations are directional by conjugation. Geometric product denotes a space–time path by inner and outer products that, in turn, as anticommutator and commutator relationships correspond to changes in densities and dissipation. The physical meaning of these mathematical operations appears also in the classification of particles as being either spatially mutually exclusive fermions or space-spanning force-carrier bosons. Using geometric algebra, the geodesics, involving rotational motion, can be related to more widely known forms of curvature. No wonder that William R. Hamilton was enthused when understanding the meaning of non-commutative operations to write (O'Conner & Robertson 1805). *And here there dawned on me the notion that we must admit, in some sense, a fourth dimension of space [time] for the purpose of calculating with triples ...*

It is perhaps surprising that the principle characteristics of diverse evolutionary courses were reasoned from the least action without any reference to particular potentials or motional mechanisms. However, the mere conservation of energy in flows down along geodesics is regarded by the second law as a powerful enough imperative to impose regularities on any motion irrespective of forms of densities and fields or means of dispersal. Nevertheless, the statistical law does not undermine their roles. On the contrary, new sources and mechanisms are naturally selected to disperse faster and further (Darwin 1859;

Smolin 1997; Sharma & Annala 2007; Kaila & Annala 2008). Certainly, only analyses of specific evolutionary settings will first reveal how the dispersal actually takes place.

It is perhaps strange that no remarks were made on the problem of clock (Bridgman 1936). A running clock, just as any other natural process, is a dissipative one. Eddington pointedly observed, *the better the clock is, the less it shows the passage of time* (Eddington 1928). In other words, the smaller quanta are dissipated, e.g. by clocks based on atomic oscillations, the finer are the counted steps of time. Yet, for time to elapse, even the very finest clocks must dissipate some quanta.

Characteristics of energy–momentum transfer by the second law can be seen to parallel Aristotle’s four forms of causation whereas much of the theorizing in the natural sciences focuses explicitly or implicitly on the mechanistic notion of causation. Contemporary attempts, just as earlier ones, to give a philosophically satisfactory analysis of causation in physical settings (Castañeda 1980; Bigelow *et al.* 1988; Heathcote 1989; Dowe 2000) rely on the conserved formalism that implies deterministic causal courses. The present study follows the same mechanistic tenet by stating that energy gradients are causes and energy flows are effects but perhaps paradoxically provides causation in the form of non-deterministic equation of motion. Often a causal analysis aims at revealing a past chain of events or predicting a future sequence. This assignment, however, seems impossible to carry out into details owing to one’s own subjective setting amid interdependent nature.

Admittedly at first sight, the holistic view of natural processes as ubiquitous energy dispersal phenomena may appear as an odd idea; however the review above on the foundations of physics, in the light of time, did not bring about inconsistencies but the familiar formula were recovered from the old principle of least action. Considering the contemporary consistency in physics, answers to yet open questions could hardly be other than a revised viewpoint to remain compatible with the largely successful and verified description of reality. The simple style used here in communicating the minimal message about the inexorable energy dispersal honours the prophecy phrased by John Wheeler (Taylor & Wheeler 2000). *How can physics live up to its true greatness except by a new revolution in outlook which dwarfs all its past revolutions? And when it comes, will we not say to each other, ‘Oh, how beautiful and simple it all is! How could we ever have missed it so long!’.*

We are grateful to Szabolcs Galambosi, Ville Kaila, Tommi Markkanen and Mika Torkkeli for valuable corrections and instructive comments.

## Appendix A

### (a) Mechanics

Evolution as an energy–momentum transfer process from a state to another (figure 2) is pictured as a directed arc  $\mathbf{s}$  spanned by momentum  $\mathbf{p}$  and coordinate  $\mathbf{x}$  to yield the action

$$\mathbf{S} = \mathbf{p}\mathbf{x} = \int p \, dx + \int x \, dp = \int d\mathbf{L}(p, x) = \int \mathbf{L}(x, t) dt. \quad (\text{A } 1)$$

The open evolutionary path  $\mathbf{s}$  is the shortest when time-dependent variations in the Lagrangian  $\mathcal{L}$  are at minimum. For example, in terms of geometric calculus, the circumference of an ellipse, defined by a fixed sum of semi-axes (conservation), is at the shortest for a circle. (For a stationary system the shortest path, a straight line, is available from Euler–Lagrange equation.) The least variation is obtained by expanding the integrand  $\mathcal{L}$  to the first order in  $p$  and  $x$ . When  $t$  is a preferred variable over  $p(t)$  in  $\mathcal{L}$ , then

$$\delta S = \int \frac{\partial \mathbf{L}}{\partial x} dx - \int \frac{\partial \mathbf{L}}{\partial t} dt = \int \left( \frac{dx}{dt} \frac{\partial \mathbf{L}}{\partial x} - \frac{\partial \mathbf{L}}{\partial t} \right) dt = 0 \Rightarrow \frac{\partial}{\partial t} \delta S = \left( \mathbf{v} \cdot \nabla - \frac{\partial}{\partial t} \right) \mathbf{L} = 0, \quad (\text{A } 2)$$

where  $dx = v dt$ . The identity  $\mathbf{v} \cdot \nabla = \partial / \partial t$  between spatial and temporal gradients follows from the conservation. It is not possible to change from one to another spatially distinct density coordinate without dissipation, i.e. without the flow of time.

The least variation yields the balance in energy (equation (4.2)), flows (equation (4.3)) and forces (equation (4.4)) that are expressed by the second law of thermodynamics and the Newton's second law. These relationships are familiar, e.g. from celestial mechanics. The balance  $2K + r dU/dr = 0$  yields for a planet of mass  $m$  orbiting a star of mass  $M_0$  at a distance  $r$  an orbital velocity  $v^2 = GM_0/r$  where  $G$  is the gravitational constant. For a star of luminosity  $dQ/dt$  at a distance  $R$  from the galactic centre of a (reduced) mass  $M$ , the balance  $2K + R dU/dR = t dQ/dt$  gives  $v^2 = GM/R + (1/M_0)(dQ/dt)(R/v)$ . The energy landscape has an overall slant towards the galactic centre but in addition it peaks at the dissipating star. The additional slant towards the centre is balanced by an increased  $v$  of the dissipative structure relative to non-dissipative structures. The effect displays itself, e.g. as relative motions during star formation. In accordance with the equivalence principle, also dissipated flows of matter generate forces that mould the energy landscape  $d(2K)/dt = d(vMv)/dt = -v\nabla U + dQ/dt - \partial U/\partial t$ .

### (b) *Electrodynamics*

The principle of least variations for the action electrodynamics  $\rho_e \mathbf{A} \mathbf{x}$  gives the conservation in equation (5.3) in the same way as  $\rho \mathbf{v} \mathbf{x}$  in the continuum mechanics. The Poynting theorem can also be approached by placing the charge density  $\rho_e$  into the electric field  $\mathbf{E}$  to give the force density

$$\mathbf{f} \equiv \rho_e \mathbf{E} = -\rho_e \nabla \phi - \rho_e \frac{\partial \mathbf{A}}{\partial t} = -\rho_e \nabla \phi - \rho_e \mathbf{v} \nabla \cdot \mathbf{A} = -\nabla u + \mathbf{J} \times \mathbf{B}, \quad (\text{A } 3)$$

where the energy density  $u = \rho_e \phi$  is written proportional to the scalar potential  $\phi$ . A moving charge radiates orthogonally to  $\mathbf{v}$ , hence in the orthonormal basis  $\partial \mathbf{A} / \partial t = \mathbf{v} \nabla \cdot \mathbf{A} = -\mathbf{v} \times (\nabla \times \mathbf{A})$  (figure 3). Thus equation (A 3) contains the Lorentz force by the definitions of magnetic field  $\mathbf{B} = \nabla \times \mathbf{A}$  and current density  $\mathbf{J} = \rho_e \mathbf{v}$ . To obtain the expression for the energy flow,  $\mathbf{f}$  is multiplied with  $\mathbf{v}$

$$\mathbf{v} \cdot \mathbf{f} \equiv \rho_e \mathbf{v} \cdot \mathbf{E} = -\rho_e \frac{\partial \phi}{\partial t} - \rho_e v^2 \nabla \cdot \mathbf{A}, \quad (\text{A } 4)$$

where the identity  $\mathbf{v} \cdot \nabla = \partial/\partial t$  has been used. When the Gauss laws  $\rho_e = \varepsilon \nabla \cdot \mathbf{E}$  and  $\nabla \cdot \mathbf{B} = 0$  are used, the Poynting theorem is recovered

$$\mathbf{v} \cdot \mathbf{f} \equiv \mathbf{J} \cdot \mathbf{E} = -\frac{\partial u}{\partial t} - v\varepsilon(\nabla \cdot \mathbf{E})(\mathbf{v} \times \mathbf{B}) \Leftrightarrow \mathbf{J} \cdot \mathbf{E} = -\frac{\partial u}{\partial t} - \varepsilon_0 c^2 \nabla \cdot (\mathbf{E} \times \mathbf{B}), \quad (\text{A } 5)$$

where  $\varepsilon = \varepsilon_0 c^2/v^2$  is the permittivity given in relation to the speed of light  $c$  in vacuum.

(c) *Quantum mechanics*

The calculation of expectation values for the least variations of quantized action

$$\begin{aligned} \langle \delta \hat{\mathcal{S}} \rangle &= \left\langle \int \left( \frac{d\hat{x}}{dt} \frac{\partial \hat{\mathcal{L}}}{\partial x} - \frac{\partial \hat{\mathcal{L}}}{\partial t} \right) dt \right\rangle = 0 \\ &\Leftrightarrow \iint \psi^* \left( \frac{dx}{dt} \frac{\partial}{\partial x} \frac{\hbar}{i} \frac{\partial}{\partial x} x - \frac{\partial}{\partial t} \frac{\hbar}{i} \frac{\partial}{\partial x} x \right) \psi \, dx \, dt = 0 \\ &\Leftrightarrow \int \left( vp - \frac{dp}{dt} x - \frac{\hbar}{i} \frac{\partial}{\partial t} \right) dt = 0 \\ &\Leftrightarrow xp - px - \frac{\hbar}{i} = 0 \Rightarrow [\hat{x}, \hat{p}] = i\hbar, \end{aligned} \quad (\text{A } 6)$$

implies by the integrand the commutator relationship where  $i\hbar$  is associated with the integrated dissipation. The vanishing integrand for expectation values yields the balance for the flows

$$vp + x \frac{\partial U}{\partial x} - \frac{\hbar}{i} \frac{\partial}{\partial t} = 0 \Rightarrow \frac{\partial}{\partial t} 2K + \mathbf{v} \cdot \nabla U - \frac{\partial Q}{\partial t} = 0, \quad (\text{A } 7)$$

where  $i\hbar \langle \partial/\partial t \rangle = -Q$  and  $\langle \partial p/\partial t \rangle = -\partial U/\partial x$  using the Ehrenfest theorem or equivalently  $2K + \mathbf{x} \cdot \nabla U = 0$  using the virial theorem.

The microscopic system described by the wave function  $\psi$  will change when energy flows. The time-dependent equation of motion for  $\psi$  is obtained, similarly as equations (A 2) and (A 6) and (A 7), from

$$\frac{\partial}{\partial t} \left\langle x \left| \frac{\partial}{\partial t} \delta \hat{\mathcal{S}} \right| \psi \right\rangle = 0 \Leftrightarrow (2K + U - Q) \frac{\partial \psi}{\partial t} = - \left( \frac{\partial}{\partial t} 2K + \frac{dx}{dt} \frac{\partial U}{\partial x} - \frac{\partial Q}{\partial t} \right) \psi, \quad (\text{A } 8)$$

where the energy gradients drive evolution and the constant energy terms gives rise to the geometric phase  $\varphi$  in multiples of  $\hbar$ .

The probability current (Griffiths 1995) at  $x$ , in the middle of the transition from the state  $x_k$  to  $x_j$  (figure 7), is obtained from equation (A 8) using the fundamental theorem of calculus

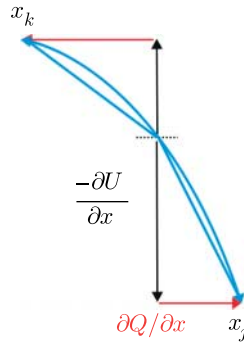


Figure 7. The landscape is curved (blue) about the point  $x$  between  $x_k$  and  $x_j$  when the opposite spatial (black) and temporal gradients (red) are of unequal lengths. The driving force of evolution generates a tangential flow of energy. For material forms of energy densities, the evolving landscape is convex (Lavenda 1985) whereas when a stationary state has been attained, the landscape is a plane slanted by the ratio of  $\partial U/\partial x$  to  $\partial Q/\partial x$ .

$$\begin{aligned}
 \frac{dP}{dt} &= \frac{\partial}{\partial t} \int \psi^* \psi \, dx = \int \left( \frac{\partial \psi^*}{\partial t} \psi + \psi^* \frac{\partial \psi}{\partial t} \right) dx \\
 &= \int \frac{1}{2K} \left[ \left( \frac{\partial}{\partial t} 2K - \frac{dx}{dt} \frac{\partial U}{\partial x} + \frac{\partial Q}{\partial t} \right) \psi^* \psi + (U - Q) \frac{\partial \psi^*}{\partial t} \psi \right. \\
 &\quad \left. - \left( \frac{\partial}{\partial t} 2K + \frac{dx}{dt} \frac{\partial U}{\partial x} - \frac{\partial Q}{\partial t} \right) \psi^* \psi - (U - Q) \psi^* \frac{\partial \psi}{\partial t} \right] dx \\
 &= - \int \left[ \frac{2}{2K} \left( \frac{dx}{dt} \frac{\partial U}{\partial x} - \frac{\partial Q}{\partial t} \right) \psi^* \psi + \left( \psi^* \frac{\partial \psi}{\partial t} - \frac{\partial \psi^*}{\partial t} \psi \right) \right] dx \\
 &= - \frac{1}{k_B T} \left( \frac{dx}{dt} \frac{\partial U}{\partial x} - \frac{\partial Q}{\partial t} \right) P - \frac{d}{dt} \Delta \varphi. \tag{A 9}
 \end{aligned}$$

The kinetic energy, quadratic in  $\mathbf{v}$ , vanishes but the flow, linear in  $\mathbf{v}$ , along the net gradient remains as much as the landscape is curved, i.e. non-Euclidean (figure 7). The last term denotes the change in the geometric phase when the path is lengthening or shortening but it vanishes in integral numbers of  $\hbar$ . In a sufficiently statistic system  $2K$  is in balance with  $U$  and  $Q$ , and hence  $k_B T$  as the expectation value of the average energy serves for normalization of  $P$ . However, it should be noted that each event of emission or absorption does change also  $k_B T$  but only little.

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