

A glance beyond the quantum model

BY MIGUEL NAVASCUÉS^{1,2,*} AND HARALD WUNDERLICH^{1,2,3}

¹*Institute for Mathematical Sciences, Imperial College London,
London SW7 2PG, UK*

²*Quantum Optics and Laser Science Group, The Blackett Laboratory,
Imperial College London, London SW7 2BW, UK*

³*Fachbereich Physik, Universität Siegen, Siegen 57068, Germany*

One of the most important problems in physics is to reconcile quantum mechanics with general relativity, and some authors have suggested that this may be realized at the expense of having to drop the quantum formalism in favour of a more general theory. Here, we propose a mechanism to make general claims on the microscopic structure of the Universe by postulating that any post-quantum theory should recover classical physics in the macroscopic limit. We use this mechanism to bound the strength of correlations between distant observers in any physical theory. Although several quantum limits are recovered, such as the set of two-point quantum correlators, our results suggest that there exist plausible microscopic theories of Nature that predict correlations impossible to reproduce in any quantum mechanical system.

Keywords: non-local theories; quantum correlations; foundational physics

1. Introduction

At the beginning of the twenty-first century, one of the main goals of theoretical physics is to come up with a theory that reconciles quantum mechanics and general relativity. Currently, there are several approaches in this direction, such as string theory (Polchinski 1998) or loop quantum gravity (Thiemann 2003; Rovelli 2004; Smolin 2004). What most of these approaches have in common is that they take the mathematical structure of quantum mechanics for granted and then try to find a suitable dynamics, such that the resulting theory approaches general relativity in some limit. The problem at stake is, thus, how to ‘quantize gravity’. Such an approach may be doomed to fail, since it could very well be that quantum mechanics is not a fundamental theory of Nature, but an effective model, only valid within a specific range of energies. Indeed, some considerations about black-hole evaporation suggest that certain axioms of quantum theory should be re-examined (Page 1994).

*Author for correspondence (m.navascues@imperial.ac.uk).

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However, as the particle experiments that we can perform nowadays are energetically very far from the Planck mass $m_P \approx 1.2 \times 10^{19} \text{ GeV } c^{-2}$, trying to formulate a new candidate beyond the quantum theory would be premature (although cosmological observations could also provide some insight). To address this issue, different authors have suggested different alternatives. One approach, followed by Popescu and others (e.g. Rohrlich & Popescu 1994; Masanes *et al.* 2006; Barnum *et al.* 2007), is to derive general results that should apply to any physical theory fulfilling a set of reasonable axioms.

It is generally agreed that the next candidate theory should prevent instantaneous communication between distant parties. If we take this as an axiom (the no-signalling principle), we can derive a set of restrictions to be satisfied by any physical theory compatible with it. Many properties of such theories are now known: any non-local theory inside that set does not allow to replicate an unknown state (Barnum *et al.* 2007), and it has to respect a specific set of uncertainty relations (Masanes *et al.* 2006). If the theories are non-local enough (in particular, quantum theory), we can even have secret communication between distant parties (Masanes 2009).

Nevertheless, it seems that the no-signalling principle alone is not enough to prevent the existence of very ‘unphysical’ theories. For that reason, several authors have considered that we could further restrict the set of allowed theories by adding more reasonable axioms to the no-signalling principle. That way, we have partial characterizations of the set of correlations allowed between distant parties in no-signalling worlds where the communication complexity is not trivial (Brassard *et al.* 2006), or where the efficiency of random-access coding is limited by the number of bits we are allowed to communicate classically (Allcock *et al.* 2009; Pawłowski *et al.* 2009).

In this paper, we propose a new mechanism to discard unphysical models of the world by demanding consistency with classical physics at the macroscopic level. More explicitly, we postulate that, whenever a microscopic experiment is brought to the macroscopic scale, the resulting macroscopic observables should be subject to classical laws.

The structure of this article is as follows. First, we will apply our mechanism to limit the set of possible correlations between distant observers, arriving at the notion of *macroscopic locality*. Then, we will provide a complete characterization of the set correlations that emerge out of this axiom and the no-signalling principle and comment on its consistency. Finally, we will compare the set of macroscopically local correlations with the quantum set. We will see that, although very similar (the set of all accessible two-point correlators is the same, for instance), these two sets are not identical. If we accept macroscopic locality as a fundamental law, this implies that a deviation from quantum mechanics could, in principle, be detected via a Bell-type experiment.

2. Main problem and notation

Suppose that we have two space-like separated parties, say Alice and Bob, in regions \mathcal{A} and \mathcal{B} . In a *microscopic* experiment of non-locality (see figure 1), there is an event in some intermediate region that produces a pair of particles. One of the particles of the pair ends up in region \mathcal{A} , while the other one is received in

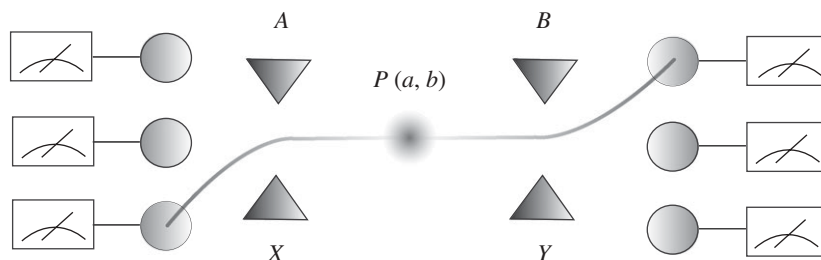


Figure 1. Microscopic experiment. If Alice and Bob repeat an experiment many times, each time applying random interactions X and Y , they can compare their measurement outcomes, make statistics and obtain a set of probabilities $P(a, b)$, the probability of measuring clicks in detectors $D(a), D(b)$ when Alice applies an interaction $X(a)$ and Bob applies an interaction $Y(b)$. From now on, we will assume that the no-signalling condition holds, i.e. that, for any X, X' with $X \neq X'$, $\sum_{a \in X} P(a, b) = \sum_{a \in X'} P(a, b) \equiv P(b)$, and, for any Y, Y' with $Y \neq Y'$, $\sum_{b \in Y} P(a, b) = \sum_{b \in Y'} P(a, b) \equiv P(a)$. These two conditions just assert that Alice's choice of measurement setting cannot affect Bob's statistics and vice versa. Also, the set of marginal probability distributions $P(a, b)$, in general, will not admit a *local hidden variable model*. This means that, in some cases, there will *not* exist a joint probability distribution for all $2s$ possible measurements $P(c_1, \dots, c_{2s})$, such that $P(a, b) = \sum_c P(\dots, c_{X-1}, a, c_{X+1}, \dots, c_{Y-1}, b, c_{Y+1}, \dots)$.

region \mathcal{B} . The moment the particles arrive at regions \mathcal{A} and \mathcal{B} , Alice and Bob will subject them to an interaction. In addition, the nature of this interaction is going to determine with which probabilities the particles will collide with the detectors in Alice's and Bob's laboratories.

We will identify the *measurement settings* of Alice and Bob with the particular interactions X and Y they subject their corresponding particles. If Alice and Bob can each perform one out of s possible interactions, the measurement settings of Alice will be numbered from 1 to s , while Bob's will go from $s + 1$ to $2s$.

The particular detectors that click after the experiment will determine the *measurement outcomes*. To avoid a hypercomplicated notation, we will associate each measurement outcome c to a pair of symbols (Z, D) , corresponding to the interaction Z performed during the measurement procedure and the detector D that received the particle. Outcomes corresponding to Alice's (Bob's) detectors will be denoted by a (b), and will belong to the set A (B) of Alice's (Bob's) possible outcomes. The application $D(c)$ will label the physical detector related to the outcome c , while $X(a)$ ($Y(b)$) will return the measurement setting X (Y) associated with that particular outcome. Finally, the expression $a \in X$ ($b \in Y$) will be shorthand notation for $X(a) = X$ ($Y(b) = Y$).

A microscopic experiment is completely characterized by the set of probabilities $P(a, b)$ that Alice and Bob can estimate through statistical inference (see figure 1). As stated before, the main goal of this article is to limit this set of probabilities by demanding consistency with classical physics when we bring the experiment to the macroscopic scale. We will find that only certain microscopic correlations can give rise to a classical model, and so all correlations outside this set should be regarded as unphysical.

But first, let us discuss what we mean by 'bringing the experiment to the macroscopic scale'.

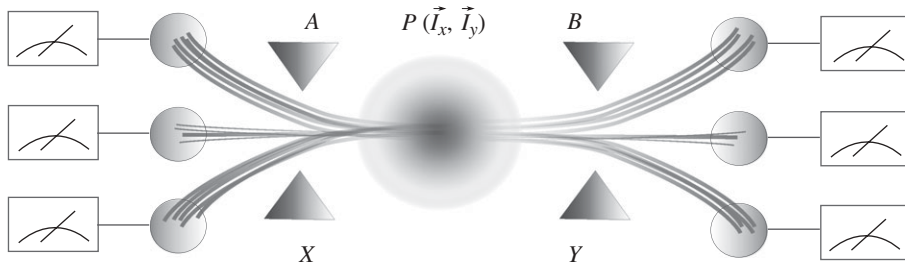


Figure 2. Macroscopic experiment. Alice and Bob receive two particle beams, interact with them collectively and then take a record of the intensities measured on each detector. A macroscopic experiment implies restrictions on the physical states to be measured (N identical and independent copies of a microscopic state), on the possible interactions to be performed (identical microscopic interactions over all the particles of each beam) and on the resolution of the detectors used (able to measure intensity fluctuations of order \sqrt{N} , but unable to resolve individual particles).

3. Macroscopic picture

A microscopic experiment of non-locality usually starts with an event that produces a pair of particles. Correspondingly, a *macroscopic* experiment will start with a *macroscopic event* producing, not a pair of particles, but N independent pairs, with $N \gg 1$. This time, therefore, Alice (Bob) will not receive a single particle at a time, but a *beam* of them. As before, Alice and Bob will be able to interact with these particles. However, they will not be able to address them individually, so, whatever microscopic interaction they intend to use, it will be applied to *all* the particles of the beam at the same time. As a consequence of those interactions, the initial beam will be divided into several beams of different intensity that will collide with Alice's (Bob's) detectors, as shown in figure 2; a similar scenario was proposed by Bancal *et al.* (2008).

This time we cannot associate measurement outcomes with clicks on a detector, since all detectors will click in each instance of the experiment. Thus, in this scenario, the measurement outcomes will be the distribution of intensities measured in each experiment. Now, in order to establish a connection with classical physics, Alice and Bob should not regard these intensities as fluxes of discrete particles, but rather as continuous fields. And this leads us to make a further assumption on the resolution of Alice's and Bob's detectors.

If the precision of their detectors is not very good, Alice and Bob will always observe the same distribution of intensities, the intensity registered at detector $D(a)$ being equal to $NP(a)$, and so we would end up in a very poor scenario. However, if Alice and Bob have a resolution that allows them to measure changes in intensity values of the order \sqrt{N} , each time they repeat the experiment, they will observe fluctuations around this mean value $NP(a)$. This is also the minimum precision Alice and Bob require in order to observe intensity fluctuations, and so this is the resolution we will assume Alice and Bob are working with.

4. Macroscopic locality

If we denote the intensities measured by Alice's (Bob's) d detectors as \mathbf{I}_X (\mathbf{I}_Y), then Alice and Bob, through classical communication, will be able to estimate marginal probability densities $P(\mathbf{I}_X, \mathbf{I}_Y)d\mathbf{I}_Xd\mathbf{I}_Y$ for each pair of measurements X, Y . Consistency with classical physics implies that, in the limit $N \gg 1$, this set of marginal distributions should admit a classical description, but which one? We have not said a word about the nature of the particles involved in the experiment. Indeed, those could be pairs of photons, electrons, kaons, etc. and, for every type of particle, we would have to force our intensities to follow a different classical law. Nevertheless, since classical physics is a local theory, any such law should give rise to a local hidden variable model for the intensities. That is, there should exist a *global* probability density $P(\mathbf{I}_1, \mathbf{I}_2, \dots, \mathbf{I}_{2s})$ such that, for any X, Y ,

$$P(\mathbf{I}_X, \mathbf{I}_Y) = \int \left(\prod_{Z \neq X, Y} d\mathbf{I}_Z \right) P(\mathbf{I}_1, \mathbf{I}_2, \dots, \mathbf{I}_{2s}). \quad (4.1)$$

Whenever this is the case, we will say that Alice and Bob's shared system exhibits *macroscopic locality*.

It is straightforward to generalize the notion of bipartite macroscopic locality to the multi-partite case, i.e. to the case where more than two separate observers participate in the experiment. It is also clear that all classical (local) microscopic correlations remain local when driven to the thermodynamical limit.

(a) Characterization of macroscopic locality

As shown in the electronic supplementary material, it can be proven that the set of bipartite microscopic correlations that give rise to local distributions at the macroscopic level is equal to the set of correlations Q^1 , introduced by Navascués *et al.* (2008) as a first approximation to the set Q of quantum correlations. We will therefore call it Q^1 in the rest of this article. The set Q^1 admits a very simple numerical characterization via semidefinite programming (Vandenberghé & Boyd 1996; Navascués *et al.* 2008).

It is easy to infer that all quantum correlations are macroscopically local in the bipartite case (i.e. $Q \subset Q^1$), as well as in the multi-partite case (see the electronic supplementary information). However, the characterization of all possible microscopic correlations (quantum or not) compatible with macroscopic locality in the case of more than two observers seems a bit more complicated. For instance, in the case of three parties, we would not only have to impose macroscopic locality over the tripartite correlations $P(a, b, c)$, but also over the *conditional* bipartite microscopic correlations $P(a, b|c)$ that would result if party \mathcal{C} announced its measurement outcome.

(b) Closure under wiring

Brunner & Skrzypczyk (2009) showed that there exist some sets of correlations $P(a, b)$ very close to local from which extremely non-local correlations can be derived, provided that both parties have access to many copies of the given physical system. The key is to generate a new *effective* set of correlations by measuring the n systems sequentially, each time applying a measurement setting

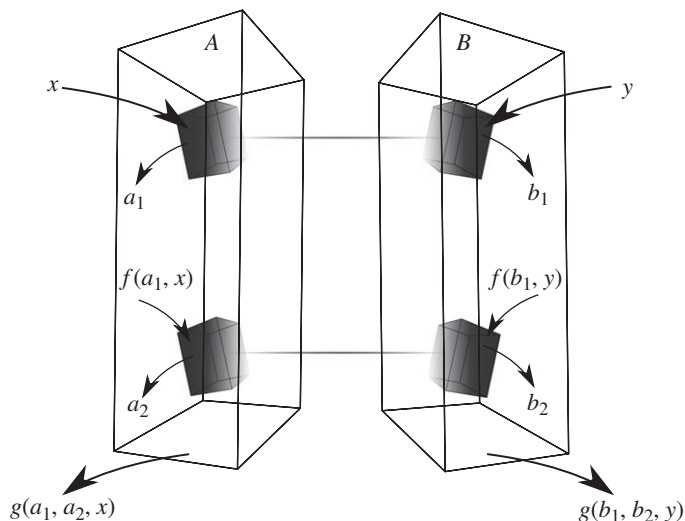


Figure 3. Wiring. Example of deterministic wiring between two physical systems: the effective measurement X (Y) of Alice's (Bob's) is applied over the first system, giving an outcome a_1 (b_1), while the interaction to be applied over the second subsystem is a function of both X and a_1 (Y and b_1). Labelling the second outcome by a_2 (b_2), the effective outcome of the whole scheme is a function of a_1, a_2, X (b_1, b_2, Y). Note that, by definition, wiring is local, i.e. it does not require communication between Alice and Bob.

dependent on the (local) outcomes of the previously measured subsystems and our effective (local) measurement setting. Once all subsystems have been measured, the effective outcome of our virtual subsystem is then taken to be a function of all (local) measurement results. This mechanism of classically coupling a set of independently correlated systems in order to generate a new single effective system of microscopic correlations is known as *wiring* (see figure 3), and we will denote it by \mathcal{W} .

Since it can be used to increase non-locality violations, wiring poses a problem to our previous characterization of the set of macroscopically local correlations. In principle, it would be possible that Alice and Bob shared two systems of microscopic correlations $P(a, b), P'(a, b) \in Q^1$, that, through some clever wiring \mathcal{W} , allowed to generate a new set of correlations $R(a, b) = \mathcal{W}(P, P') \notin Q^1$. Any consistent theory compatible with macroscopic locality could therefore not admit both sets of correlations, and a detailed classification of such theories would be very complicated. Or even worse, it could also happen that $P(a, b) \in Q^1$, but $\mathcal{W}(P^{\otimes n})(a, b) \notin Q^1$, for some n . This would imply that the set of *macroscopically local* correlations is bigger than the set of microscopic correlations *compatible* with macroscopic locality, and then we would have to deal with the problem of characterizing the latter. These two possibilities are ruled out by the next result.

The set Q^1 is closed under wiring. That is, let $\{P_i\}_{i=1}^n$ be any set of n microscopic correlations, such that $P_i \in Q^1$ for all $i=1, \dots, n$, and let $\mathcal{W}(P_1, P_2, \dots)$ denote the effective set of correlations that results after some wiring of such behaviours. Then, $\mathcal{W}(P_1, P_2, \dots) \in Q^1$. For the proof, see the electronic supplementary material.

A consequence of closure under wiring is that the underlying microscopic non-locality of an otherwise macroscopically local process $P(a, b)$ cannot be revealed through the use of *beam splitters*. A beam splitter is an experimental device that divides a beam of particles into several parts, the intensity of each part i being equal to the intensity of the initial beam times the *transmitivity* $T_i \geq 0$ of the beam splitter (note that intensity conservation imposes that $\sum_i T_i = 1$). Suppose then that Alice (Bob), instead of applying the same microscopic interaction over all her (his) beam of particles, splits such a beam into s parts of relative intensities $\{T_i^A\}_{i=1}^s$ ($\{T_i^B\}_{i=s+1}^{2s}$) and then applies a different interaction i to each part. Since all available microscopic interactions are present, in this scenario, the measurement settings of Alice and Bob will correspond to the values of the set of transmitivities $\{T_i^A\}, \{T_j^B\}$ of their beam splitters, that we will assume they can vary at will. To see that this experiment will not violate locality, notice that the previous set-up can be microscopically modelled as a wiring between $P(a, b)$ and an uncorrelated (and thus, macroscopically local) bipartite system $P'_A \otimes P'_B$. Here, the action of P'_A (P'_B) is to send a particle to the set of detectors corresponding to the interaction $i \in \{1, 2, \dots, s\}$ ($i \in \{s+1, s+2, \dots, 2s\}$) with probability T_i^A (T_i^B) when Alice (Bob) sets her (his) measurement setting to $\{T_i^A\}_{i=1}^s$ ($\{T_i^B\}_{i=s+1}^{2s}$). The resulting set of microscopic correlations $\mathcal{W}(P, P'_A \otimes P'_B)$ thus belongs to Q^1 and so will give rise to a local model in the macroscopic limit.

(c) *The limits of macroscopic locality*

For any measurement Z , we can always define an *observable* O_Z by associating each possible measurement outcome c of Z with a real number $O_Z(c)$. Thus, if X, Y are two measurements by Alice and Bob, respectively, then the *two-point correlator* of the observables O_X, O_Y will be given by

$$E_{XY} \equiv \langle O_X O_Y \rangle = \sum_{a,b \in X, Y} P(a, b) O_X(a) O_Y(b). \quad (4.2)$$

Suppose now that we are in a scenario where $k = d = 2$ (with the measurement settings ordered as $\overbrace{12}^A \overbrace{34}^B$), and consider the set of observables $\{O_Z\}_{Z=1}^4$, with spectrum $\{1, -1\}$, that is, such that $O_Z : c \rightarrow \{1, -1\}$, for all $c \in Z$. Then, the Clauser–Horne–Shimony–Holt (CHSH) parameter (Clauser *et al.* 1969) can be written as

$$S \equiv E_{13} + E_{23} + E_{14} - E_{24}. \quad (4.3)$$

It is well known that, for any set of local correlations, the value of S satisfies $|S| \leq 2$, whereas in the quantum case $|S| \leq 2\sqrt{2}$, the famous Tsirelson bound (Cirel'son 1980). Moreover, both inequalities can be saturated.

The maximum value of $|S|$ in Q^1 can also be proven to be $2\sqrt{2}$. Moreover, it can be shown that any set of correlators $\{E_{13}, E_{23}, E_{14}, E_{24}\}$ arising from a macroscopically local theory admits a quantum representation (Navascués *et al.* 2007, 2008), and therefore has to satisfy the Tsirelson–Landau–Masanes inequalities (Tsirelson 1987; Landau 1988; Masanes 2003)

$$\left| \sum_{X,Y} \arcsin(E_{XY}) - 2 \arcsin(E_{X'Y'}) \right| \leq \pi, \quad \forall X', Y'. \quad (4.4)$$

The previous results can be easily extended. It can be shown that, for $d = 2$ and an arbitrary number of measurement settings s , *any* set of two-point correlators $\{E_{XY}\}_{X=1,\dots,s, Y=s+1,\dots,2s}$ arising from correlations exhibiting macroscopic locality can also be simulated in a quantum system. See the electronic supplementary material for a proof.

However, not even when $s = d = 2$ does the set of two-point correlators contain all the information about the distribution $P(a, b)$. A little thought will convince us that, in order to recover the whole set of probabilities $P(a, b)$, we also need to know the value of the *one-point correlators* $E_Z \equiv \langle O_Z \rangle$. In general, one would have to resort to numerical methods to characterize the set of one-point and two-point correlators compatible with macroscopic locality. For the case $s = d = 2$, though, such a set is completely specified by the constraints¹

$$\left| \sum_{X,Y} \arcsin(\tilde{E}_{XY}) - 2 \arcsin(\tilde{E}_{X'Y'}) \right| \leq \pi, \quad \forall X', Y', \quad (4.5)$$

where $\tilde{E}_{XY} = (E_{XY} - E_X E_Y) / \sqrt{(1 - E_X^2)(1 - E_Y^2)}$ (Navascués *et al.* 2007, 2008).

From equation (4.5), we can see that, although $2\sqrt{2}$ is the maximum violation of the CHSH inequality, there exist macroscopically local distributions that attain this value, but nevertheless are slightly biased (i.e. $E_Z \neq 0$, for some Z). Such distributions are thus not compatible with a quantum theory of Nature (Werner & Wolf 2001). So even in this simple scenario, we can see that the inclusion $Q \subset Q^1$ is strict, although in this case, both sets are extremely close.

The similarities between Q^1 and Q decrease as we increase the number of available detectors d , as shown in figure 4. This opens the possibility of disproving quantum mechanics in the future via a Bell-type experiment.

5. Discussion

In this article, we have proposed a new mechanism to study the microscopic structure of our Universe from macroscopic models of reality. We have shown that, when applied to restrict the strength of the correlations between distant parties, this mechanism leads to the notion of macroscopic locality. We have identified the set Q^1 of bipartite correlations that can arise in theories limited by macroscopic locality and the no-signalling principle, and commented on its differences and similarities with standard quantum mechanics.

Nothing has been said, however, about the dynamics of such post-quantum theories. Following our approach, one could address this problem by studying the set of microscopic theories whose macroscopic behaviour approximates a

¹Here, we are also implicitly assuming that $\{E_Z, E_{ZT}\}$ arise from a no-signalling set of correlations $P(a, b)$. Note that condition (4.5) alone does not guarantee that $P(a, b) \geq 0$, for all a, b .

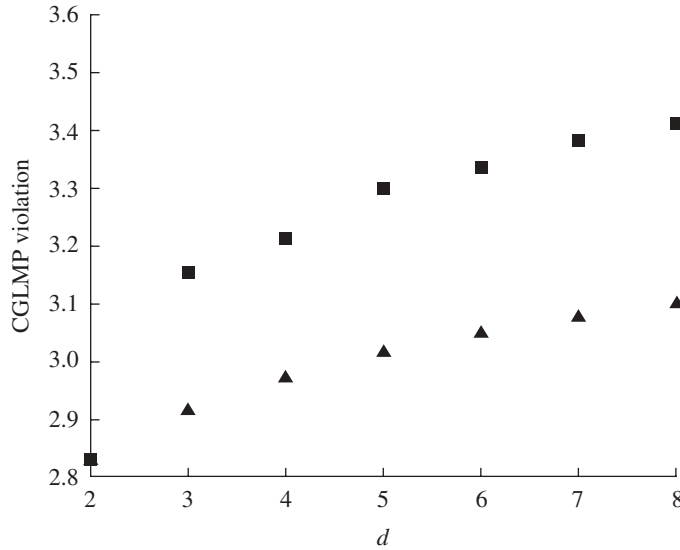


Figure 4. Departure from quantum mechanics. As for the CHSH parameter, the Collins–Gisin–Linden–Massard–Popescu (CGLMP) parameters (Collins *et al.* 2002) are linear functions S_d on the probabilities $P(a, b)$, to be applied over scenarios with $s = 2$ and arbitrary d , and, as for the CHSH parameter, the maximum possible value of these functions under the assumption of locality is 2. The plot above (filled squares, macroscopic locality; filled triangles, quantum mechanics) shows the difference between the maximal value of S_d inside Q , as well as inside Q^1 , as a function of d (Navascués *et al.* 2008). At first glance, there seems to be room for a wide variety of macroscopically local theories other than quantum mechanics. This suggests that, if our Universe happened to be described by one of these, we might be able to experimentally falsify quantum mechanics with a sufficient number of detectors.

well-known classical dynamical model. If that model were, say, general relativity, the corresponding research program would thus try to ‘relativize quantum theory’ rather than ‘quantize gravity’. This could lead to model-independent astrophysical predictions, based more on general axioms confirmed by observation than on current theoretical fashions.

Finally, it also remains to know how our World would behave if it *did* admit correlations slightly beyond the set Q^1 . Would this have negligible experimental consequences, or on the contrary, could we ‘distill’ those correlations somehow in order to obtain arbitrary violations of macroscopic locality? And, even if we could not, would the mere existence of deviations from macroscopic locality lead to a drastic change in our understanding of the Universe?

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