Particle absorption by black holes and the generalized second law of thermodynamics

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The change in entropy, ΔS, associated with the quasi-static absorption of a particle of energy ε by a Schwarzschild black hole (ScBH) is approximately $(\varepsilon/T) - s$, where T is the Hawking temperature of the black hole and s is the entropy of the particle. Motivated by the statistical interpretation of entropy, it is proposed here that the absorption should be suppressed, but not forbidden, when $\Delta S < 0$, which requires the absorption cross section to be sensitive to ΔS. A purely thermodynamic formulation of the probability for the absorption is obtained from the standard relationship between microstates and entropy. If $\Delta S \gg 1$ and $s \ll \varepsilon/T$, then the probability for the particle not to be absorbed is approximately $\exp[-\varepsilon/T]$, which is identical to the probability for quantum mechanical reflection by the horizon of an ScBH. The manifestation of quantum behaviours in the new probability function may intimate a fundamental physical unity between thermodynamics and quantum mechanics.

Keywords: black holes; thermodynamics; generalized second law

1. Introduction

Consider an isolated system $\boldsymbol{X}$, subject only to internal forces, that consists of a Schwarzschild black hole (ScBH) and a stable test particle that is initially approaching the black hole. Let the energy ε of the particle be very small in comparison to the total energy of the black hole. Note that ε is defined when the particle is, effectively, infinitely far from the black hole, and in a frame that is stationary relative to the black hole. According to the classical description, the Schwarzschild metric sufficiently characterizes the relevant properties of the black hole, and the incident particle is essentially point-like. The classical probability for the black hole to absorb the particle may differ substantially, therefore, from the probability determined according to any model that provides a field-theoretic description of the particle. The purpose of this present work is to demonstrate that the thermodynamic properties of the black hole also generate a departure from the classical description of absorption, independent of the quantum behaviours of the particle. A surprising consistency emerges between the quantum theory of particles and the thermodynamics of black holes.

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It is instructive to review the basic components of the classical and quantum mechanical models for the evolution of a system like X. Let the black hole have a mass M, and the magnitude v of the velocity of the incident particle be defined as $(1 - m^2/\varepsilon^2)^{1/2}$, where m is the mass of the particle. The units here are natural units in which the Newtonian gravitational coupling G, the Planck constant $\hbar$, the vacuum speed of light c and the Boltzmann constant k_B are equal to 1. The probability for the black hole to absorb the particle is, classically, characterized by the cross section (Bogorodski 1962; Unruh 1976)

$$\sigma_C = \pi \frac{M^2}{v^2} \left\{ \frac{512(1 - v^2)^3}{4(1 - 4v^2 + [1 + 8v^2]^{1/2})(3 - [1 + 8v^2]^{1/2})^2} \right\}. \qquad (1.1)$$

The slowly varying quantity in brackets is equal to 16 when $v = 0$, and approaches 27 in the limit as v approaches 1. The cross section (1.1) is therefore approximately proportional to $1/v^2$ for all $v < 1$, and diverges as v vanishes (Unruh 1976).

The first major revision of the classical model was, historically, the incorporation of a field-theoretic treatment of the particle, which was most notably presented by Unruh (1976). By solving the Klein–Gordon and Dirac equations in the Schwarzschild metric, it is possible to determine the absorption cross sections σ_S and σ_D for scalar and Dirac particles, respectively. Although σ_S and σ_D are both similar to σ_C in the high-energy regime $\varepsilon \gg 1$, significant deviations from the classical description occur at low energies, where (Unruh 1976)

$$\sigma_S \approx \frac{16\pi^2 M^2 (1 + v^2) 2M\varepsilon}{v^2 \left\{ 1 - \exp[(-2\pi Mm\varepsilon(1 + v^2))/v] \right\}}. \qquad (1.2)$$

In all of the cases discussed here, $\sigma_S = 8\sigma_D$ (Unruh 1976). If the wavelength $\lambda = 1/\varepsilon$ of the particle is larger than the horizon $R = 2M$ of the black hole, then equation (1.2) is approximately

$$\sigma_S \approx \left(\frac{16\pi^2 M^2}{v^2} \right) 2Mm \qquad (1.3)$$

for $v < 2\pi Mm$, and is approximately

$$\sigma_S \approx \frac{16\pi M^2}{v} \qquad (1.4)$$

for $v > 2\pi Mm$ (Unruh 1976). The most important features distinguishing the low-energy quantum mechanical cross sections (1.3) and (1.4) from σ_C are the factor $2Mm$ in equation (1.3) and the $1/v$ dependence in equation (1.4). The term $2Mm = Rm$ in equation (1.3) is approximately equal to R/λ at low velocities where $\varepsilon \sim m$. As λ increases with respect to R, the factor $2Mm$ decreases and absorption is thus attenuated, which represents one of the basic wave-like behaviours of quantum theory. Note that equation (1.4), which characterizes the regime of high velocity and low energy, does not exhibit the characteristic quantum mechanical dependence on R/λ (Unruh 1976).

The seminal field-theoretic descriptions of particle absorption in the Schwarzschild metric have been expanded to model scenarios where the charge and angular momentum of the incident particle may be important, and where the Kerr, Reissner–Nordstrom (RN) or Kerr–Newman (KN) metric is necessary

to describe the black hole (Frolov & Novikov 1998). There exists, however, a latent singularity in the original classical models that may intimate an inadequacy in the standard quantum mechanical treatments. Although there are regular solutions for the trajectory of a point-like particle crossing the horizon of an ScBH, the classical action of the particle is singular on the horizon. Specifically, the radial component $A(r)$ of the action is proportional to $\varepsilon \ln[r - R]$ (Kuchiev 2003). The classical action is relevant to the quantum descriptions because the monochromatic wave function $\varphi(r)$ of the incident particle becomes asymptotically proportional to $\exp[-iA(r)]$ as r approaches R (Kuchiev 2003). The divergence of the classical action implies therefore that $\varphi(r)$ is undefined on the horizon. Historically, field-theoretic models have circumvented the singularity implicitly by defining solutions only outside the horizon, and by allowing only those solutions that correspond to incident waves. The amplitudes of all reflected waves are accordingly fixed to zero, which is equivalent to requiring that the black hole absorbs perfectly (Futterman *et al.* 1988). The assumption of perfect absorption is justified in the context of the classical understanding of black holes. Although such models are self-consistent, the singularity of the action on the horizon may signify the existence of additional behaviours that are lost when the usual boundary conditions are imposed (Kuchiev 2003).

An important revision to quantum treatments of particle absorption has emerged from a reconsideration of the problematic horizon action. According to the seminal arguments of Kuchiev, the irregularity of $\varphi(r)$ on the horizon constitutes a catastrophic departure from the classical paradigm, and the assumption of perfect absorption is therefore not well justified. Allowing the amplitudes of reflected waves to be non-vanishing, it is possible to identify consistent solutions of the associated field equations near the horizon and to determine the probability for reflection. In addition to the original derivation in Kuchiev (2003), Kuchiev and Flambaum (Kuchiev 2004*a,b*; Kuchiev & Flambaum 2004*a,b*; Flambaum 2004) have provided a variety of different theoretical motivations for allowing reflection at the horizon. All of the descriptions have led consistently to the same elegant reflection probability

$$P_r = \exp\left(\frac{-\varepsilon}{T}\right), \tag{1.5}$$

where $T = 1/(8\pi M)$ is the Hawking temperature of the black hole (Kuchiev 2003, 2004*a,b*; Kuchiev & Flambaum 2004*a,b*; Flambaum 2004). It is important to note that the appearance of T in equation (1.5) is merely a substitution for M and does not represent the consideration of thermodynamic behaviours of the black hole. The probability for absorption is consequently $1 - P_r$, and the associated absorption cross section σ_a is given by

$$\sigma_a = \frac{4\pi C^2 \tanh(\pi \varepsilon)}{v(1 + v\varepsilon^2 C^2 \tanh(\pi \varepsilon))^2}, \tag{1.6}$$

where $C^2 = 2\pi w/(1 - \exp[-2\pi w])$ and $w = v\varepsilon(1 + 1/v^2)/2$ (Kuchiev & Flambaum 2004*a,b*). In the low-energy regime $\varepsilon \ll 1$, equation (1.6) is approximately

$$\sigma_a \approx \frac{64\pi^3 M^4 m^2}{v^2} \tag{1.7}$$

for $v \ll \varepsilon$, and is approximately

$$\sigma_{\mathrm{a}} \approx 32 M^3 \pi^2 \varepsilon \qquad (1.8)$$

for $v \gg \varepsilon$. By comparing equation (1.6) with (1.2), it is evident that the introduction of reflection reduces the low-energy absorption cross sections by a factor $2\pi M \varepsilon$. This modification is particularly significant in the high-velocity regime as ε approaches zero, in which case equation (1.8) vanishes but equation (1.4) remains finite (Kuchiev & Flambaum 2004*a*,*b*).

The non-classical behaviours represented in equations (1.2) and (1.6) emerge naturally from a field-theoretic treatment of particles in a classical Schwarzschild metric. Reflection at the horizon (RH) is particularly significant because it constitutes essentially an alternative demonstration of the phenomenon of black hole radiation. Allowing an incident particle to be reflected by the horizon of an ScBH is equivalent to allowing a particle to escape from an ScBH, across the event horizon, and into the external environment (Kuchiev 2003). The probability function (1.5), which characterizes both reflection and escape, has the form of a conventional Boltzmann factor, and ensures that the flux of escaping particles is in equilibrium with the thermal spectrum of a black body with temperature T.

A fundamental connection between quantum theory and thermodynamics is also evident in this present work. The purpose here is to explore the non-classical behaviours that emerge from a straightforward thermodynamic treatment of a system like **X**. The analysis in §2 identifies a certain regime in parameter space where particle absorption would decrease the total entropy of the system. According to the interpretation developed in §3, absorption is allowed in that regime, but must be statistically suppressed. It follows that the probability for absorption must be sensitive to the change in the entropy of the system, and that the standard quantum mechanical description of particle absorption requires thermodynamic corrections. A thermodynamically consistent probability for absorption is then derived from the conventional relationship between entropy and microstates. The probability function exhibits a wave-like behaviour, and the corresponding probability for the particle not to be absorbed features a compelling degree of consistency with the reflection probability (1.5). Section 4 provides a discussion of some implications of the conclusions, and the calculations are presented in §§2 and 3.

2. The thermodynamics of particle absorption

The revolutionary identification of the thermodynamic properties of black holes has led to the specification of behaviours that may be important for any model of the interactions between particles and black holes (Bardeen *et al.* 1973). This section presents a basic thermodynamic description of particle absorption in a system like **X**. The most important feature of the present description is the attribution of thermodynamic entropy to the black hole. Let the initial mass and horizon of the black hole be M and $R = 2M$, respectively. The entropy S_{bh} of the black hole is accordingly (Bekenstein 1973; Hawking 1976*a*)

$$S_{\mathrm{bh}} = \pi R^2. \qquad (2.1)$$

Analogous to conventional thermodynamic entropy, the entropy of a given black hole is a measure of the associated number of internal microstates (Bekenstein 2008). Among the possible physical interpretations of the microstates measured by S_{bh} are the internal states of matter and gravity (Frolov & Novikov 1993; Frolov & Page 1993; Mukhanov 2003), the entanglement between internal and external degrees of freedom (Bombelli *et al.* 1986; Srednicki 1993) and the thermodynamic microstates of the atmosphere above the horizon (Thorne & Zurek 1985; 't Hooft 1985, 1996). According to the generalized second law of thermodynamics (GSLT), the total entropy of a closed system containing a black hole should not decrease (Hawking 1971; Bekenstein 1973; Misner *et al.* 1973). The requirements of the GSLT are not imposed in the following description, but are addressed at the end of this section and in the interpretation presented in §3.

Let the boundary of $\boldsymbol{X}$ be a sphere, concentric with the black hole, whose radius is large enough, in comparison to R, to ensure that the system is effectively closed. The total energy of the Hawking radiation emitted during the interaction between the particle and the black hole is assumed to be negligible with respect to M, ensuring that the black hole is approximately in equilibrium with its thermal aura. As ε is comparatively miniscule, the black hole is quasi-static with respect to the interaction with the particle, remaining near equilibrium. The total entropy S_γ of the Hawking radiation within the system is assumed here to change only trivially during the interaction. Let S_{out} represent the entropy of the region at the outside of the black hole, including the Hawking photons. The total entropy S of the system is therefore initially given by $S = S_{\mathrm{bh}} + S_{\mathrm{out}}$.

Suppose that the particle is absorbed by the black hole, the mass of the black hole increases accordingly by ε, and the associated event horizon is

$$R_2 = R + 2\varepsilon. \tag{2.2}$$

Absorption of the particle therefore causes the entropy of the black hole to increase by an amount $\Delta S_{\mathrm{bh}} = \pi R_2^2 - \pi R^2$. It follows from equations (2.1) and (2.2) that

$$\Delta S_{\mathrm{bh}} = \pi R^2 \left(1 + \frac{\Delta R}{R}\right)^2 - \pi R^2, \tag{2.3}$$

where $\Delta R \equiv R_2 - R$. Note that $\Delta R = 2\varepsilon = R\varepsilon/M$, and is therefore very small with respect to R as $\varepsilon \ll M$. The binomial expansion of $(1 + \Delta R/R)^2$ in terms of $\Delta R/R$ allows equation (2.3) to be expressed as

$$\Delta S_{\mathrm{bh}} = 2\pi R \Delta R + \delta, \tag{2.4}$$

where δ represents the higher order terms in the expansion. As $(\Delta R/R) \ll 1$, δ is negligible with respect to the first-order term given explicitly in equation (2.4), and may be therefore ignored. For the sake of clarity, δ is omitted in the remainder of this work. The Hawking temperature T of the black hole is $1/(8\pi M)$, and equation (2.4) may be expressed as

$$\Delta S_{\mathrm{bh}} = \frac{\varepsilon}{T}. \tag{2.5}$$

Although it was derived in the context of particle absorption, equation (2.5) is valid generally, giving the increase in the entropy of an ScBH resulting from the absorption of any quantity of energy ε, provided that the black hole is

quasi-stationary (Mendoza *et al.* 2009). Relationship (2.5) is also noteworthy since it is analogous to the conventional thermodynamic expression q/τ giving the change in the entropy of a closed system, with temperature τ, whose total energy changes isovolumetrically by some amount q.

If the particle is absorbed, then it is effectively removed from the region at the outside of the black hole. The entropy of the region at the outside of the black hole should change consequently by some amount ΔS_{out} that is presumably non-positive. The absolute value of ΔS_{out} may be accordingly identified as the entropy s of the particle, defined at infinity. Although the present analysis allows s to be treated as a variable, it is instructive to consider a typical scale of particle entropy. According to the statistical definition, the entropy of a system, characterized by a given macrostate, is defined as the logarithm of the associated number W of microstates, or *complexions*. If the energies and coordinates of all of the constituent particles are well specified, then there is only one possible microsate, being the macrostate itself, and the corresponding entropy is thus $\ln(1) = 0$. The entropy of a macrostate having only two microstates is $\ln(2)$. It is also meaningful to interpret the entropy of a system as a measure of the number of bits of unavailable information registered by the system. A bit of information has, by definition, only two possible states, and $\ln(2)$ is therefore the entropy of one bit (Lloyd 2002, 2006). In the context of quantum theory, the number of microstates of a thermal particle, freely moving within some region of radius r, is determined primarily by the associated phase-space volume $(pr)^3$, where p is the momentum of the particle (Phillies 2000). A typical molecule within an ideal gas, at standard temperature and pressure, registers the order of 10^1 bits.

With $\Delta S_{out} = -s$, and ΔS_{bh} given by equation (2.5), the change in the total entropy of a system like $\boldsymbol{X}$, where an ScBH absorbs a particle quasi-statically, is given by $\Delta S = \Delta S_{bh} + \Delta S_{out}$, and is thus

$$\Delta S = \frac{\varepsilon}{T} - s. \tag{2.6}$$

Note that the entropy S_γ of the thermal photons is eliminated since it is assumed to be effectively constant. It follows from equation (2.6) that particle absorption would decrease the total entropy of the system if ε/T is less than s.

Consider a system like $\boldsymbol{X}$ in which a particle with energy ε and entropy s_1 approaches an ScBH whose temperature is T. Let the given parameters be such that $\Delta S > 0$ if the particle were absorbed. Consider, in comparison, a scenario that is identical in all respects, except that the entropy of the particle is $s_2 > s_1$, and s_2 is such that ΔS would be negative if absorption occurred. The current descriptions of particle absorption provide no distinction between the two cases, and the predicted absorption probability would be identical for both. Furthermore, the Hawking radiance of an ScBH is a function only of the mass of the black hole, which is not related to the entropy of the incident particle. The change in the Hawking radiance would therefore be identical in both instances, and could not introduce additional entropy when $\Delta S < 0$. Within the current understanding, the interactions between particles and black holes are not manifestly consistent with the GSLT. According to the primary thesis of this

present work, presented in the following section, a proper description of particle absorption requires thermodynamic corrections that may be inherently linked to quantum theory.

3. Thermodynamically consistent absorption and quantum theory

The second law of thermodynamics (SLT) represents essentially a probabilistic argument. It is technically not *impossible* for the entropy of a particular closed system to decrease, but only *improbable*. Consider a closed system, near equilibrium, that is characterized at an initial time by a thermodynamic macrostate Σ_x in which the total entropy of the system is S_x. Suppose that, as a consequence of an ordinary thermodynamic transformation or fluctuation, the macrostate and the corresponding entropy of the system change over time. In general, the probability P_j for finding a system, near equilibrium, in some macrostate Σ_j is proportional to the number W_j of complexions encompassed by Σ_j (Phillies 2000; Kojevnikov 2002). For any j, W_j is directly related to the associated total entropy S_j according to $W_j = \exp(S_j)$. It is thus possible to find the system, at some later time, in a macrostate whose entropy is smaller than S_x. It is, however, exponentially more likely to find the system in a macrostate in which the entropy is greater than S_x or equal to S_x. In that manner, the SLT is preserved.

As in the case of ordinary systems, any closed system containing a black hole may be subject to thermodynamic transformations or fluctuations that change the macrostate and entropy of the system. If the system is near equilibrium, then the probability for finding the system in a given macrostate should be proportional to the associated number of complexions. It follows that there may be certain instances in which an internal transformation decreases the entropy of a closed system containing a black hole. It also follows, however, that transformations, which increase the total entropy of the system, should be exponentially more likely. There are two important consequences of this interpretation: (i) although statistically suppressed, it is nonetheless possible for absorption to occur when $\Delta S < 0$ and (ii) the probability (or cross section) for absorption must be sensitive to ΔS. If those conclusions are correct, then the current quantum mechanical descriptions of particle absorption require certain thermodynamic revisions.

In the absence of a rigorous model incorporating thermodynamics, quantum theory and general relativity, it is appropriate to consider the predictions of a strictly thermodynamic treatment of particle absorption by a black hole. In a system like X, the black hole is quasi-static and near equilibrium with respect to the interaction with the particle. It is therefore possible, in principle, to quantify the probabilistic behaviours according to the standard statistical prescription. Let Σ_a represent the macrostate in which the black hole has effectively absorbed the particle, and let Σ_s represent the macrostate in which the particle was not absorbed, or 'scattered'. The total entropy S_a of Σ_a is the sum of the photon entropy S_γ, which is effectively constant, and the entropy $S_{bh} + \varepsilon/T$ of the black hole after absorbing the particle. The number W_a of complexions available to Σ_a is thus given by

$$W_a = \exp\left(S_{bh} + \frac{\varepsilon}{T} + S_\gamma\right). \tag{3.1}$$

The entropy S_s of Σ_s is identical to the initial entropy $S_{bh} + s + S_\gamma$ of the system. The number W_s of complexions of Σ_s is accordingly

$$W_s = \exp(S_{bh} + s + S_\gamma). \tag{3.2}$$

The probability P_a for the system to be found in the macrostate Σ_a is proportional to W_a, and the probability P_s for finding the system in the macrostate Σ_s is proportional to W_s. If Σ_a and Σ_s encompass all possible configurations, then $P_a/P_s = W_a/W_s$ and $P_a + P_s = 1$. It follows readily from the associated algebra that

$$P_a = \frac{\exp(\Delta S)}{1 + \exp(\Delta S)} \tag{3.3}$$

and

$$P_s = \frac{1}{1 + \exp(\Delta S)}, \tag{3.4}$$

where ΔS is given by equation (2.6). In equation (3.4), ΔS represents the change in entropy that would have occurred if the black hole had absorbed the particle. Note that $P_a = P_s = 1/2$ when $\Delta S = 0$, and that P_a (P_s) increases with increasing (decreasing) ΔS. Probabilities (3.3) and (3.4), therefore, satisfy the most general requirement for thermodynamic consistency, which is that scattering must be more likely than absorption for all $\Delta S < 0$.

Although the statistical treatment in the preceding paragraph may require certain corrections following from a more complete description, it is plausible that equations (3.3) and (3.4) are, nonetheless, good representations of nature. The statistical thermodynamic description of a conventional system, near equilibrium, is generally insensitive to the details of interactions among the constituent particles and radiation. Similarly, an accurate statistical thermodynamic description of the interaction between a particle and a black hole, near equilibrium, may be insensitive to the details of the interaction. If equations (3.3) and (3.4) are meaningful, then they should be consistent with the quantum mechanical descriptions in the regime where entropy is not important. Thermodynamic suppression of absorption must become stronger as ΔS decreases, and must be very significant when $\Delta S \ll -1$. Conversely, suppression must decrease as ΔS increases, and must be negligible when $\Delta S \gg 1$. In the regime, $\Delta S \gg 1$ where suppression is not important, the scattering probability (3.4) is approximately

$$P_s \approx \exp\left(\frac{-\varepsilon}{T}\right)\exp(s). \tag{3.5}$$

Additionally, the entropy of the particle would be unimportant if $\varepsilon/T \gg s$. If $\Delta S \gg 1$ and $\varepsilon/T \gg s$, then it follows from equation (3.5) that $P_s \approx \exp[-\varepsilon/T]$, which is identical to the probability (1.5) for RH. Note also that, according to equation (1.5), reflection at the horizon is more likely than absorption when $\varepsilon/T < \ln(2)$. Although P_r is not identical to equation (3.4), RH is nonetheless consistent with the basic requirement of thermodynamic suppression for particles having one bit of entropy. The consistency between RH and the present statistical model may be physically significant and is addressed, among other implications of this work, in the following section.

4. Discussion

The arguments presented in §3 emerged from a straightforward application of statistical thermodynamics to the interactions between particles and black holes. The most general conclusion is that particle absorption is allowed when $\Delta S < 0$, but is suppressed naturally by the dearth of associated microstates. Allowing the entropy of a closed system to decrease does not violate the SLT, provided that such excursions occur proportionately with their associated microstates. Similarly, allowing particle absorption to decrease the total entropy of a closed system containing a black hole does not necessarily violate the GSLT. In fact, it is presumably consistent with the spirit of the GSLT to treat systems containing black holes and conventional systems in a similar manner.

If particle absorption is allowed, even rarely, when $\Delta S < 0$, then information would be effectively destroyed. Note that the destruction of entropy in the present scenario is very different from the information loss paradox (ILP) (Hawking 1976*b*). The ILP addresses the contradiction that would occur if a black hole evaporates completely, converting the presumably pure initial state into a mixed thermal state (Preskill 1992). As discussed at the end of §2, the destruction of information by particle absorption in a system like **X** could not be related to the associated Hawking radiation.

The conclusions of §3 may be relevant to the weak cosmic censorship conjecture (WCCC). According to the WCCC, every singularity must be enclosed within a finite event horizon—except, perhaps, for the singularity associated with the Big Bang (Hawking & Penrose 1970). The observation of a 'naked singularity' could pose a fundamental problem for the notion of causality. The formation of an extreme KN (XKN) black hole could violate the WCCC as the associated event horizon would vanish (Matsas & da Silva 2007; Hod 2008*a*,*b*; Matsas *et al.* 2009). A KN black hole is characterized by a mass M, charge Q and rotational angular momentum J. A given KN black hole would be extreme if $\mu \equiv (M^2 - Q^2 - a^2)^{1/2} = 0$, where $a \equiv J/M$ (Bekenstein 2008). Although the event horizon of an XKN black hole is zero, the associated entropy S_{KN} remains finite and is given by (Bekenstein 2008)

$$S_{KN} = \pi(2M^2 + 2M\mu - Q^2) \equiv \pi B. \tag{4.1}$$

It follows from equation (4.1) that the entropy of an XKN black hole would be $\pi(2M^2 - Q^2)$. Note also that equation (4.1) is identical to the entropy (2.1) of an ScBH if $\mu = M$. According to some investigations of quantum particles in the KN and RN metrics, it is possible for interactions with certain incident particles to cause a nearly extreme black hole to become extreme (Matsas & da Silva 2007; Matsas *et al.* 2009). Other investigations have advanced the contrary conclusion by demonstrating inherent details of the interactions that ultimately prohibit the elimination of the event horizon (Hod 2008*a*,*b*). If the statistical interpretation in §3 is generally valid for black holes described by the RN and KN metrics, then it may have some relevance to the problem of the extreme black hole in the context of the WCCC. Suppose that any given change in the state of black hole is statistically suppressed if the associated change in entropy is negative. In principle, it would be statistically favourable for a black hole to evolve towards the extreme state, characterized by $\mu = 0$, if the entropy S_{KN} of

the black hole increased with decreasing μ. It follows from $\mathrm{d}\mu/\mathrm{d}M = M/\mu$ and $\mathrm{d}\mu/\mathrm{d}Q = -Q/\mu$ that

$$\frac{\mathrm{d}B}{\mathrm{d}\mu} = 2M + 6\mu + \frac{2\mu^2}{M}. \tag{4.2}$$

If $\mathrm{d}B/\mathrm{d}\mu < 0$, then the entropy would increase as μ decreased, thus favouring the evolutionary path towards an XKN black hole. It follows, however, from equation (4.2) that there are no positive M and μ for which $\mathrm{d}B/\mathrm{d}\mu < 0$. Every interaction that reduces μ must therefore decrease the entropy of the black hole, and should be statistically suppressed. Unless the black hole is 'born' with μ near 0, its evolution towards the extreme state would require a significant decrease in entropy, and may be prohibitively improbable.

In addition to indicating that particle absorption is allowed, although rarely, when $\Delta S < 0$, the interpretation in §3 suggests that the absorption cross section must be a function of ΔS. The modern, quantum mechanical descriptions of particle absorption would thus require certain modifications in order to be consistent with the thermodynamics of black holes. The absorption probability (3.3) and the corresponding probability (3.4) for scattering feature wave-like behaviours. Furthermore, the scattering probability (3.4) is consistent with the probability (1.5) for RH in the regime where thermodynamic suppression is insignificant and the entropy of the particle is negligible. The quantum mechanical properties of equations (3.3) and (3.4) are remarkable given that the only explicitly non-classical component of their parent model is the attribution of entropy to the black hole. In the context of quantum theory, ε/T is proportional to the ratio of the horizon of the black hole to the wavelength of the particle. It is therefore natural to expect, in general, that the probability for absorption should be attenuated when the wavelength of the particle becomes increasingly larger than the horizon. An alternative physical interpretation of the variation of the absorption probability with respect to ε/T emerges from the attribution of thermodynamic entropy to a black hole. The total change in entropy ΔS associated with quasi-static particle absorption is, according to equation (2.6), given by $(\varepsilon/T) - s$. It follows from the basic thermodynamic principles that the probability for absorption must decrease as ΔS decreases. If T and s are constant, then ΔS always decreases with decreasing ε. The probability for absorption must accordingly decrease with decreasing ε. In that manner, the variation of equation (3.3) with ε/T, for a fixed s, is also a natural consequence of statistical thermodynamics.

The consistency between the wave-like behaviour associated inherently with quantum particles and the behaviour that follows from the thermodynamics of black holes is noteworthy. It is plausible that the attribution of entropy to a black hole implicitly constitutes the attribution of wave-like behaviours to the incident particle. It is also suggestive that one of the earliest intimations of the wave–particle duality is found in Einstein's formula giving the probability for a fluctuation of electromagnetic radiation within a system (Einstein 1909; Kojevnikov 2002; Varro 2006). Einstein obtained the fluctuation formula by considering the variation in the number W of microscopic complexions in terms of variations in the entropy S associated with a given fluctuation, where $W = \exp[S]$ (Einstein 1909). Probabilities (3.3) and (3.4) were obtained from the same basic principles. It is reasonable to consider that the manifestation of wave-like

behaviour in the thermodynamically determined probabilities represents the same physical unity implied by the wave-like behaviour in Einstein's fluctuation formula.

According to the analysis of Kuchiev & Flambaum (2004*a*,*b*), the probability for a particle to escape from a black hole is identical to the probability P_r for reflection, thereby generating a thermal spectrum of escaping particles. In the present interpretation, the probability for reflection must depend on ΔS in order to be thermodynamically consistent, and may be of the form (3.4). If the probability for escape depends on the entropy of the escaping particle, then the description of the emergent spectrum may require modifications that introduce some deviation from the blackbody spectrum. Such modifications, if necessary, would require a detailed consideration that is beyond the scope of this present work, particularly as the entropy of the escaping particle may not be well specified.

It may be important to address a final point concerning entropy and information. In the foregoing analysis, entropy was treated as a continuous variable. The entropy of a black hole may correspond necessarily to an integer number of bits, or some other unit of entropy (Bekenstein & Mukhanov 1995; Bekenstein 1998; Corichi *et al.* 2007). There may exist situations in which the quantization of black hole entropy is important. For instance, if $0 < |\Delta S| < \ln(2)$, then the behaviour of a system like X may deviate from models in which entropy is treated continuously. Furthermore, if any given particle must register at least one bit of information, then the lower bound on the entropy of a particle is $\ln(2)$, and s could not, therefore, vanish.

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