A depolarizer as a possible precise sunstone for Viking navigation by polarized skylight

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Viking navigation from Norway to America in the northern latitudes remains a mystery for physicists, historians and archaeologists. Polarimetric methods using absorbing dichroic crystals as polarizers to detect a hidden Sun direction using the polarized skylight have led to controversies. Indeed, these techniques may lack in sensitivity, especially when the degree of polarization is low. Here, we demonstrate theoretically and experimentally that using the transparent common Iceland spar as a depolarizer, the Vikings could have performed a precise navigation under different conditions. Indeed, when simply rotated, such a birefringent crystal can completely depolarize, at the so-called isotropy point, any partially polarized state of light, allowing us to guess the direction of the Sun. By equalizing the intensities of the ordinary and extraordinary beams at the isotropy point, we show that the Sun direction can be determined easily, thanks to a simple sensitive differential two-image observation. A precision of a few degrees could be reached even under dark crepuscular conditions. The exciting recent discovery of such an Iceland spar in the Alderney Elizabethan ship that sank two centuries before the introduction of the polarization of light in optics may support the use of the calcite crystal for navigation purposes.

Keywords: Viking navigation; sky polarization; Iceland spar

1. Introduction

Although the first observation of the polarization of the skylight was performed only in 1809 by Arago (Arago 1811), earlier uses of the phenomenon have been suggested and discussed, namely in the Viking navigation nearly 1000 years before (Ramskou 1967; Können 1985; Mills 1992). Following the Icelandic sagas,

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the Vikings sailed thousands of kilometres of open water between Norway and America, without magnetic compass (Thirslund 2007). At these latitudes of about 61° N in the near-polar regions, they could use the Sun, the stars and also the direction of the wind, the waves and swell. However, when the Sun was hidden by clouds or fog or during the long twilights in these polar summers, Ramskou (1967) suggests that the Vikings could also have used polarimetry for finding their way. An enigmatic crystal called a sunstone could have been used then as a polarizer of overhead skylight for finding the location of the obscured Sun. As the Rayleigh scattering of the light provides us with E-vector patterns over the entire celestial hemisphere (Coulson 1988; Berry *et al.* 2004; Horváth & Varjú 2004; Hoeppe 2007; Smith 2007; Horváth *et al.* 2011), a small patch of blue sky at the zenith is sufficient to perform polarimetry. The polarimetric method could have been used by the Vikings, using dichroic analyzers such as cordierite, for example. However, the weakness of these polarimetric methods for the Viking navigation was emphasized by Roslund & Beckman (1994). They consider that when the Sun is hidden behind clouds, its location can often be found with the naked eye from the bright lining of the cloud tops and crepuscular rays emanating from the Sun. However, such counterarguments are not supported by psychophysical studies (Barta *et al.* 2005). The invisible Sun cannot be quite accurately located from only the celestial brightness and colour patterns under partly cloudy or twilight conditions. Recently, full-sky imaging polarimetry (Hegedüs *et al.* 2007) has shown that the atmospheric optical prerequisites for polarimetric navigation are fulfilled under certain sky conditions, but for the moment physics cannot rule it out one way or another. All the polarimetric methods require some dichroic analyzer, so as to align its transmission axis parallel to the E-vector of the scattered celestial polarized light, but absorbing the orthogonal component. However, such methods may lack in sensitivity and precision, namely when the sky is cloudy or foggy and the degrees of polarization of the light are weak (Roslund & Beckman 1994). By contrast, one may wonder if the Vikings were able to determine with precision, the direction of the hidden Sun by depolarizing any partially polarized skylight through a transparent crystal like an Iceland spar for a peculiar orientation of the crystal, corresponding to the isotropy point defined below, so as to use sensitive differential observations avoiding any absorption. The recent amazing discovery of an Iceland spar in an Elizabethan ship sunk in 1592 (Bound & Monaghan 2001), which we had the opportunity to test (figure 1*a*), reinforces the potential of the peculiar calcite crystal that is even currently used to build optical cloaks (Chen *et al.* 2010; Zhang *et al.* 2011).

2. Material and methods

(*a*) *Material*

To introduce the isotropy point of a calcite crystal, where any state of partially polarized light passing through the crystal can be completely depolarized, let us recall the double refraction of a birefringent crystal as first described by Bartholinus in the seventeenth century (Hecht 2002). Each light ray falling on such a crystal, which is not a polarizer (Bretenaker & Le Floch 1991), is split into an ordinary beam that behaves as if it merely passes through a plate of
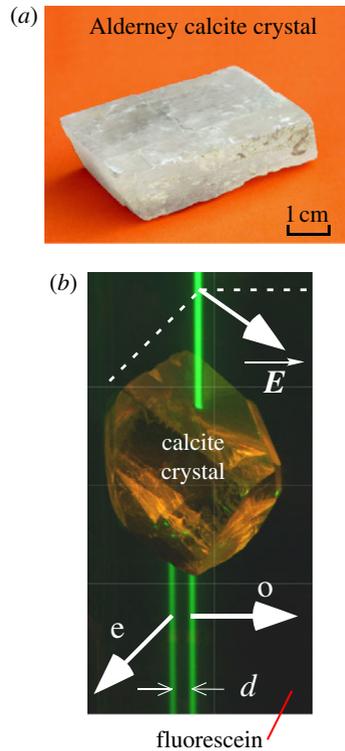
Figure 1. (*a*) Photograph of the Alderney calcite crystal. The rhombohedral geometry is in an excellent state of preservation with its 79° and 101° angles. (*b*) Typical response of a similar Iceland spar, observed in a fluorescein tank, using a 532 nm laser ray. Here, the walk-off distance d between the ordinary and extraordinary beams reaches 4 mm for a 40 mm crystal thickness. (Online version in colour.)

glass and an extraordinary ray that is deflected in the crystal and does not obey Snell's Law. Today, the ordinary and extraordinary indices associated with the ordinary and extraordinary axes of the crystal are well-known and their spatial separation, i.e. the walk-off distance d (figure 1*b*) is given by the expression $d = e$ $\tan \gamma$ (Hecht 2002), where e is the thickness of the crystal and γ is the deviation angle of the extraordinary beam in the crystal. For the crystal in figure 1*b*, the thickness is 40 mm, $\gamma \simeq 6°$ and the walk-off distance between the two orthogonally and totally linearly polarized beams is about 4 mm.

(*b*) Methods

(i) *Wide beam experiment at the isotropy point*

When the beam impinging on this crystal is wide compared with the walk-off distance, if one looks directly at the sky through the crystal, the two beams are no more spatially resolved as shown in figure 2*a*. For a random orientation of the crystal illuminated by a partially polarized light, there is, in general, no balance between the intensities of the ordinary and extraordinary beams as schematized in figure 2*a*. Let us call α the angle between the ordinary axis of the
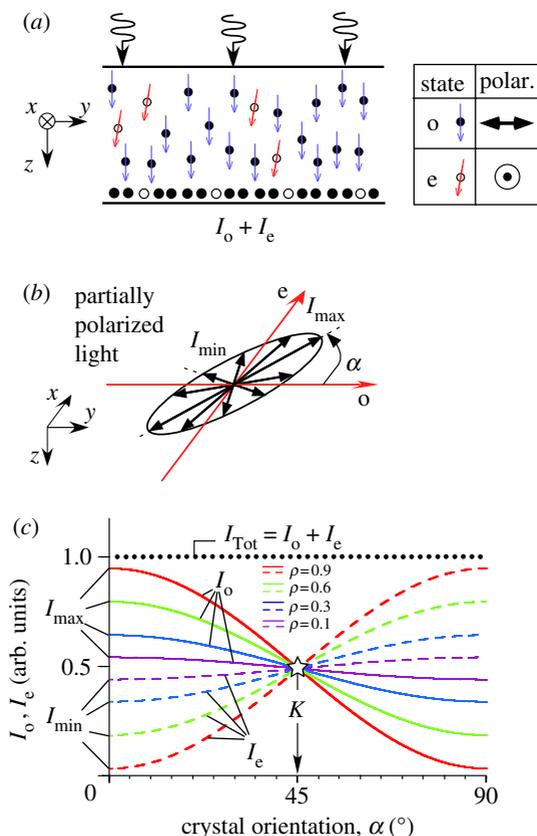
Figure 2. (*a*) Scheme of the propagation for a partially polarized wide beam impinging on an Iceland spar without any specific orientation. (*b*) Scheme of the polarization ellipse of the incident light. α represents the angle between the axis of maximum intensity of the partially polarized light and the ordinary axis of the crystal. (*c*) Respective theoretical intensities of the ordinary and extraordinary beams versus the crystal rotation for different degrees of polarization ρ corresponding to clear or cloudy skies. Note that the isotropy point K corresponds to a precise 45° rotation angle of the crystal. The total intensity represented by the dotted line always remains constant, as no dichroic absorption occurs. (Online version in colour.)

crystal and the main axis of a given partially polarized light, which is defined by I_{max} and I_{min}, the intensities measured through a usual polarizer (figure 2*b*). The degree of polarization is defined as $\rho = (I_{max} - I_{min})/(I_{max} + I_{min})$. This parameter varies from 0 (unpolarized natural light) to 1 (totally linearly polarized light). According to Malus' Law applied simultaneously to the two beams, the ordinary and the extraordinary intensities passing through the birefringent crystal can be written as

$$I_o = \left(\frac{I_{max} + I_{min}}{2} \right) (1 + \rho \cos 2\alpha) \qquad (2.1)$$

and

$$I_e = \left(\frac{I_{max} + I_{min}}{2} \right) (1 - \rho \cos 2\alpha). \qquad (2.2)$$

The corresponding theoretical variations of I_o and I_e for different degrees of polarization are reported on figure $2c$, when the crystal is rotated. As no light is absorbed in a birefringent system, we note that for different orientations of the crystal, the sum $I_o + I_e = I_{Tot}$ remains constant through the crystal, but the difference $I_o - I_e$ determines the isotropy point K, where $I_o - I_e = 0$ (figure $2c$). So for any partially polarized light impinging on the crystal, there exists an orientation of the crystal for which the ordinary and extraordinary beams have strictly equal intensities. This isotropy point is reached when the bisector of the angle between the ordinary and extraordinary axes of the crystal is exactly along the main axis of the partially polarized light. Then for this precise rotation of 45° of the crystal, it behaves like a perfect depolarizer for any partially linearly polarized light. As seen below, if a patch of a blue sky is available, the isotropy point K offers a first detection of a hidden Sun, through the Iceland spar using the so-called Haidinger's brushes, i.e. the polarization sense in human vision (Haidinger 1844; Le Floch *et al.* 2010).

(ii) *Narrow beam experiment at the isotropy point*

Narrow beam observations from the skylight impinging on the surface of the Iceland spar, for the same isotropy point K, bring as shown below a more powerful method, based on the same equations (2.1) and (2.2), to locate the hidden Sun. Let us now look through a small square hole in an opaque screen covering the Iceland spar as shown in figure $3a$, where the side a of the square hole is about the value of the walk-off distance d between the ordinary and the extraordinary beams. When the crystal is rotated, so as to reach the isotropy point of figure $2c$, the two beams give two equal half-intensity images on a completely dark background as shown in figure $3a$. Note that, at the isotropy point, the two image irradiances are rigorously equal, whatever the degree of polarization of the partially polarized light impinging on the crystal. Moreover, the human eye is well known to be highly sensitive to low contrasts (Hubel 1988). This sensitivity of the naked eye to contrasts has also recently made it possible to detect a single layer of atoms, which exhibits a very small 2.3 per cent opacity (Nair *et al.* 2008). From the theoretical curves of figure $2c$ representing the variations of the irradiances I_o and I_e of the two images, we can deduce the differential irradiances $|I_o - I_e|$, i.e. the contrasts between the two images as shown in figure 4 for different values of the degree of polarization of the incoming light. We observe that on both sides of the isotropy point at 45°, the theoretical contrasts between the two images increase sharply, the ordinary and extraordinary images exchanging their respective irradiances. Thanks to the Babinet principle, similar curves could be obtained for the complementary screen shown in figure $3b$. However, if instead of a hole in an opaque screen, we have an opaque spot on the incoming face of the crystal, we will observe two half-intensity shadows. Note that in this case, the ordinary and extraordinary beams are interchanged as shown in figure $3b$.

3. Results

The first navigational experiment using the Haidinger's brushes is performed at 48°07′ N, 1°41′ W, at twilight with a clear sky. First in absence of the Iceland spar, the degree of polarization of the skylight can be measured by rotating a
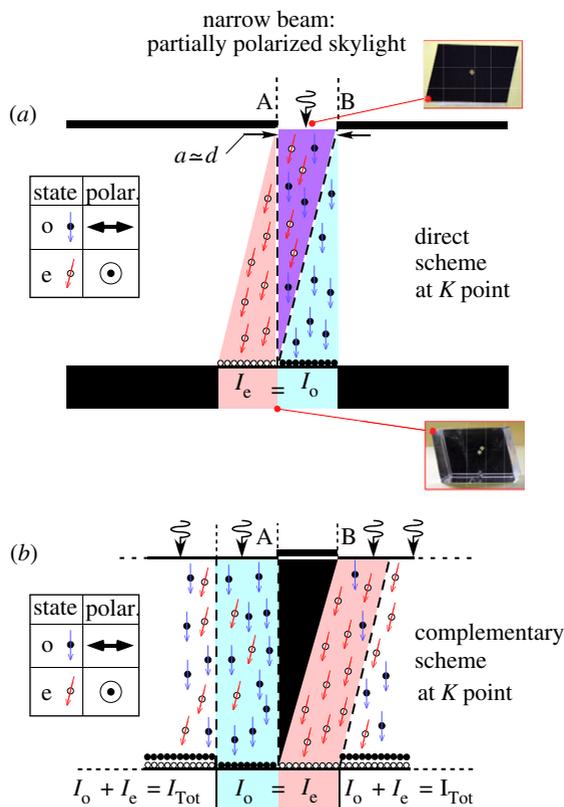
Figure 3. Narrow beam experiment in the presence of an additional screen in front of a calcite crystal illuminated by a partially polarized skylight. (*a*) For a narrow incident beam, scheme of the ordinary and extraordinary beams, at the isotropy point ($\alpha = 45°$), through a square hole represented by AB in the opaque screen (see the photo of the upper inset). AB = a is here about the walk-off distance d. At the output of the calcite crystal, the two images (see the photo in the low inset) have equal irradiances. (*b*) In the complementary situation with a small square opaque screen schematized by AB, the two shadows appearing here on a bright background have also equal irradiances, but the ordinary and extraordinary beams are interchanged. (Online version in colour.)

commercial polarizer P in front of a detector D (figure 5*a*). From the varying intensity (right of figure 5*a*), we obtain $\rho = 0.6$. Second, interposing the Iceland spar shown in the inset of figure 5*b* in front of the polarizer P (figure 5*b*), when the crystal is rotated at 45°, i.e. at the isotropy point, the transmitted intensity no more varies as shown on the right of figure 5*b*. For this peculiar position of the crystal, we reach a complete depolarization, i.e. $\rho = 0$. We can now try to guess the direction of the hidden Sun by observing successively the polarization patterns on the fovea, when using an Iceland spar. Let us recall that for the natural unpolarized light falling on the scatterers (figure 6*a*), i.e. the air molecules of the atmosphere, the direction of the polarization of the skylight is perpendicular to the plane of scattering determined by the Sun, the observer and the point observed. Then for an observation at the zenith, only the Rayleigh component polarized along the x-axis can reach the observer (figure 6*a*). Haidinger has earlier
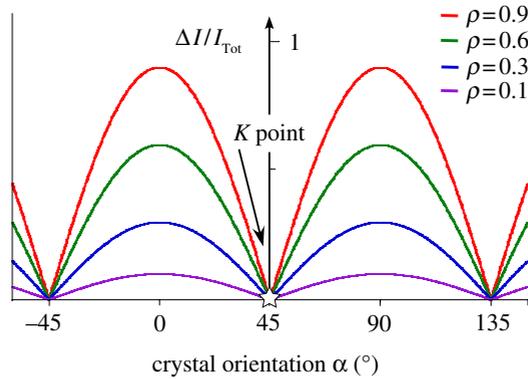
Figure 4. Theoretical variations of the relative irradiance $\Delta I/I_{\text{Tot}} = |I_{\text{o}} - I_{\text{e}}|/I_{\text{Tot}}$ of the two images for different values of the degree of polarization of the incident light versus the crystal rotation. (Online version in colour.)
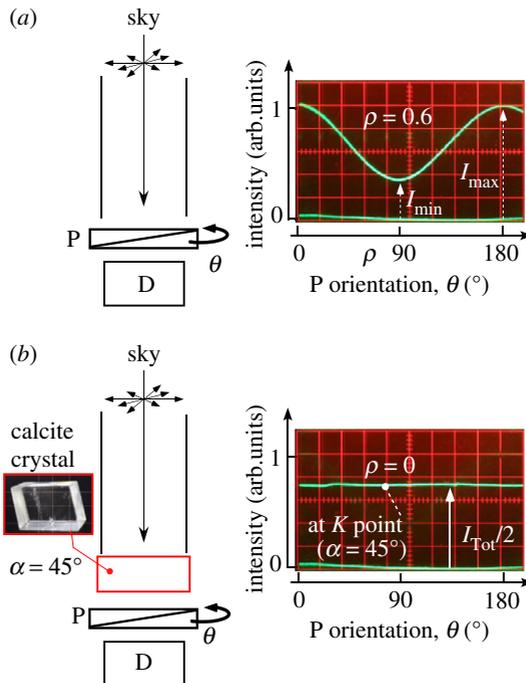

Figure 5. Observations through the calcite crystal at the isotropy point. (*a*) Scheme of the apparatus for the measurement of the degree of polarization ρ of the sky light. A commercial polarizer P (HN22 polaroid) is rotated in front of the detector D. On the right, typical result performed at $48°07'$ N, $1°41'$ W, just before twilight, for a clear sky corresponding to $\rho = 0.6$. (*b*) Scheme of the apparatus for isolating the isotropy point. The calcite crystal can be rotated with an angle α. On the right, when the Iceland spar is interposed at $\alpha = 45°$, i.e. at the isotropy point K, the intensity remains constant. The depolarization of the light is complete ($\rho = 0$). (Online version in colour.)

shown (Haidinger 1844) that when a polarized beam of white light falls on the foveal cone mosaic, a blue-yellow propeller-like pattern is observed with the naked eye, subtending a visual angle of 3° centred on the fovea, the polarization of the light being parallel to the blue axis and perpendicular to the yellow axis. Moreover, recently, a short 0.1 s reading and erasing time associated with the chromophore response has been isolated (Le Floch *et al.* 2010), enabling us to rapidly compare clearly two different successive patterns, namely for a polarized and an unpolarized state of light. If we interpose the crystal at the isotropy point, the x polarized light is completely depolarized and then no pattern is observable in the plane of the eye fovea (figure 6a). Now, if we remove the crystal (figure 6b), the polarized electric field E of the skylight, which is aligned along the x-axis induces Haidinger's brushes on the fovea, as shown at the bottom of figure 6b. Thanks to the short 0.1 s reading and erasing time, one may alternatively interpose the depolarizing crystal in the line of gaze (figure 6a) and rapidly remove it (figure 6b). Then we successively easily observe the no-pattern image and the pattern image on the fovea as shown at the bottom of figure 6a,b, respectively. Note that the yellow brush of figure 6b is perpendicular to the scattered electric field E of the skylight, but directly gives the direction of the Sun. To estimate the precision that can be obtained by such an observation, we have compared the bearings made by 20 observers, using a sun locator device built in our laboratory (see the inset in figure 6). The observations were essentially made at sunset, at 48°07′ N, 1°41′ W. We have tested 10 Iceland spars of our collection, but most of the tests have been performed using the Iceland spar shown in the inset of figure 8a. As a typical measurement of the Sun bearing lasts 1 min, a set of measurements takes about 1 h. Such measurements have been repeated over several months. Of course during the set of measurements, we have taken account of the Sun azimuth variations, which is about $11°\,h^{-1}$, at our location. With the stability of our sun locator device, the Sun direction can be determined with a precision of about ±5°. Aboard a boat, the precision can be reduced, but could have provided the Vikings with a sufficient estimation of the boat course.

Although the isotropy point permits a direct observation of the Sun direction, thanks to our polarization sense based on Haidinger's brushes, the theoretical curves of $|I_o - I_e|$ near the isotropy point suggest a more powerful differential observation, allowing the Sun detection for weaker degrees of polarization of the light. Figure 7a shows typical experimental results obtained with a narrow beam like in figure 3a, when the crystal is rotated back and forth around the isotropy point, for a degree of polarization of 0.6 corresponding to a clear sky. Here, the entrance face of the crystal is simply covered with an opaque sheet with a small $4.8 \times 4.8\,mm^2$ hole. The simultaneous differential observations were also made at 48°07′ N, 1°41′ W, when the zenith was rather clear and the Sun below the horizon. At the isotropy point ($\alpha = 45°$), the irradiances of the two images are equal (figure 7a). For $\alpha = 35°$, the ordinary image irradiance is greater than the extraordinary one, but the result is reversed for $\alpha = 55°$ in agreement with the theoretical curves. The corresponding density profiles shown in figure 7b confirm that the irradiances of the two images are easily discriminated in this case where the degree of polarization is measured to be $\rho = 0.6$ (clear sky). Here, these density profiles show that a contrast of 18 per cent can be obtained by a rotation of the calcite crystal of ±10°. Hence a precision of ±1° can be reached as the sensitivity of the human eye to contrast can be as low as 1 per cent (Hubel 1988). This
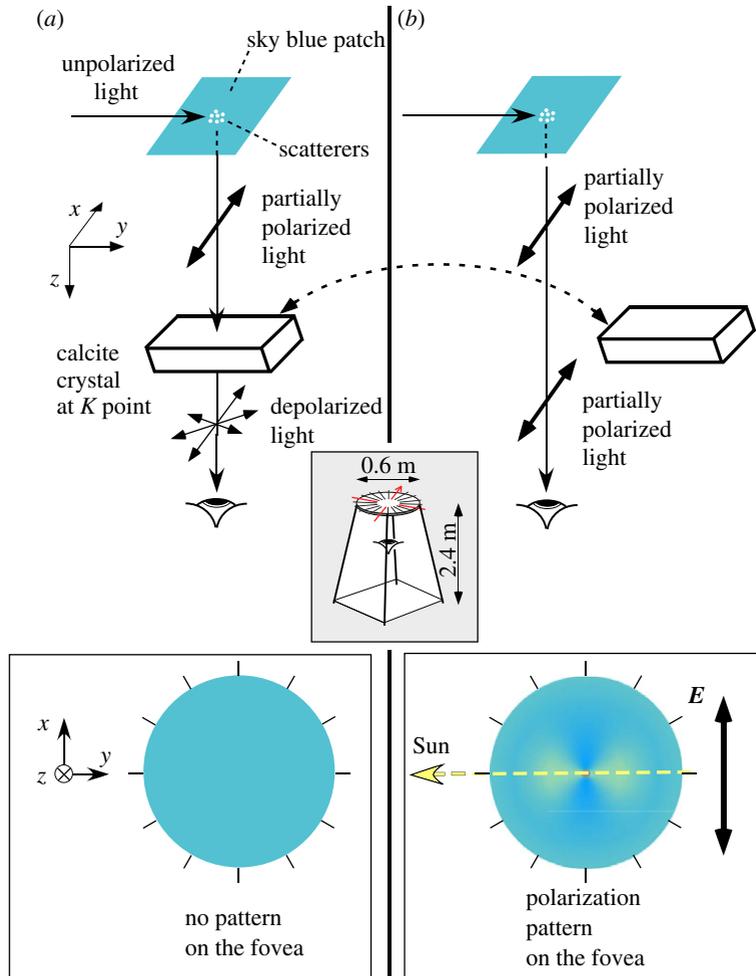
Figure 6. (*a*) Gazing at a blue patch of the sky at the zenith, simply through a calcite crystal oriented at the isotropy point *K* acting then as a perfect depolarizer. No pattern is observed on the fovea (see left bottom). (*b*) When the crystal is rapidly removed, a Haidinger's polarization pattern appears on the fovea (see right bottom). The yellow brush directly gives the Sun direction. As the entoptic pattern on the fovea cannot be photographed, the scheme here is simulated (Le Floch *et al.* 2010). A scheme of the sun locator device used is shown in the inset. (Online version in colour.)

precision has been directly verified with the eye in the same conditions as for the Haidinger's pattern tests. This high precision is confirmed by equations (2.1) and (2.2), which give a 1 per cent contrast between the two images for a rotation of the calcite crystal of only 0.5°. By contrast, for a dichroic polarizer, in similar conditions, reaching a variation value of the intensity of 1 per cent requires a rotation of 5° according to the Malus' Law. Moreover, note that on the one hand as discussed in textbooks and by Mills (1992), the human eye is very well adapted to matching the irradiances of two adjacent images, such as the ordinary and extraordinary ones (photometric matching). On the other hand, the eye/brain system is not good at remembering and mentally comparing varying
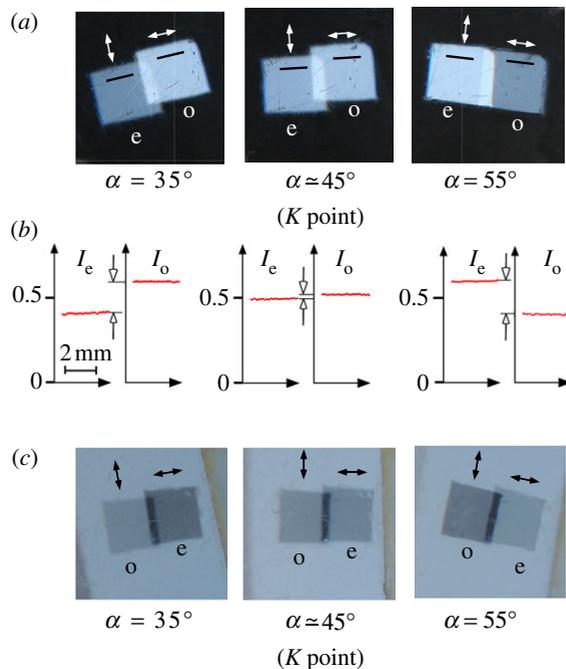
Figure 7. High sensitivity to contrasts around the isotropy point. (*a*) Experimental two-image photos for a narrow beam passing through a square hole in a screen in front of an Iceland spar, at the isotropy point ($\alpha = 45°$) and on both sides of K, at 35° and 55°. Here, the side of the square is $a = 1.2d = 4.8\,\text{mm}$ and the sky is clear ($\rho = 0.6$). (*b*) Measured irradiances of the two experimental images along the black segments shown on the different images of figure 7*a*. Note that a precision of $\pm 1°$ on the Sun bearing can be reached as the physiological sensitivity of the human eye to contrasts is known to be 1% (Hubel 1988). (*c*) Two experimental complementary half-intensity shadows predicted in figure 3*b* can be obtained in the same conditions, but on a bright background. (Online version in colour.)

intensities such as those produced through the dichroic polarizers. On a ship, the precision can also be reduced, but the Vikings could have been able to benefit from an efficient tool with a calcite crystal at the isotropy point to locate a hidden Sun. Using the Babinet principle, two complementary half-intensity shadows can be obtained (figure 7*c*) in the same conditions as earlier qualitatively observed (Karlsen 2003), but on a bright background.

To demonstrate the high efficiency of the differential method with a hole in an opaque screen for quite low degrees of polarization, we compare in figure 8*a* that the theoretical signals for the orientations of the crystal with the corresponding experimental measurements. Here $\rho = 0.08$ corresponds to a cloudy sky. The agreement remains quite good even when the back and forth rotations of the crystal are reduced around the isotropy point at 45°. Note that for rotations of $\pm 10°$ around 45°, the contrast remains as high as 3 per cent, i.e. clearly above the limit fixed by the 1 per cent sensitivity of the human eye to a contrast (Hubel 1988). So to have a bearing of the Sun direction remains possible for low degrees of polarization. The inset of figure 8*a* shows the Iceland spar used in this experiment, similar to the one the Vikings could have employed as a
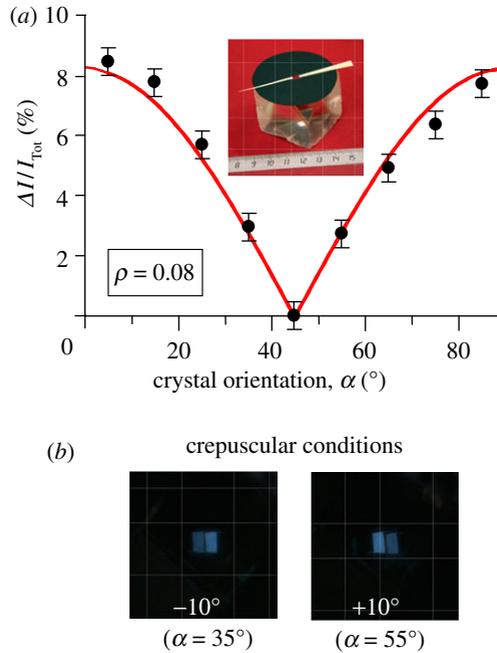
Figure 8. (*a*) Theoretical curve (solid line) and experimental points (filled circles) for the relative variations $\Delta I/I_{\text{Tot}} = |I_{\text{o}} - I_{\text{e}}|/I_{\text{Tot}}$ of the respective irradiances of the two images, in a case of a low degree of the polarization ρ. Here, $\rho = 0.08$ corresponds to a rather cloudy sky. The error bars correspond to the fluctuations of the irradiance measurements for back and forth rotations of the crystal on both sides of the isotropy point. The inset shows the Iceland spar used. (*b*) Typical observations through the Iceland spar (at $\pm 10°$ from the isotropy point) performed a half hour after the sunset, 10 min before the stars appear, on 31 July 2011, at $48°07'$ N, $1°41'$ W. Here, the sky irradiance is still reduced by a factor 50, compared with the irradiance of the sky half an hour before the sunset at 21.45 h. (Online version in colour.)

sunstone, the arrow beside the hole indicates the true bearing of the Sun, when the crystal is rotated at exactly the isotropy point. The calibration can be performed by rotating the crystal at the isotropy point, i.e. by equalizing the two-image irradiances, when looking at the zenith a day when the sky is clear and the Sun not hidden. The arrow is then fixed towards the Sun. To choose the course of the ship, the Vikings could, for instance, go to the bow of the ship and align a horizon board with the bearing of the Sun given by the sunstone.

4. Discussion and conclusions

As its absorption is negligible, the crystal also works quite well for very low luminosities of the sky, namely when the Sun is largely below the horizon, i.e. in crepuscular conditions. Typical experimental results are shown in figure 8*b*, 10 min before the stars appear. This is another characteristic of the method, which is important from the Viking point of view. We have verified that the human eye can reliably guess clearly the Sun direction in dark twilights, even until the stars become observable. In this situation, the Iceland spar with its
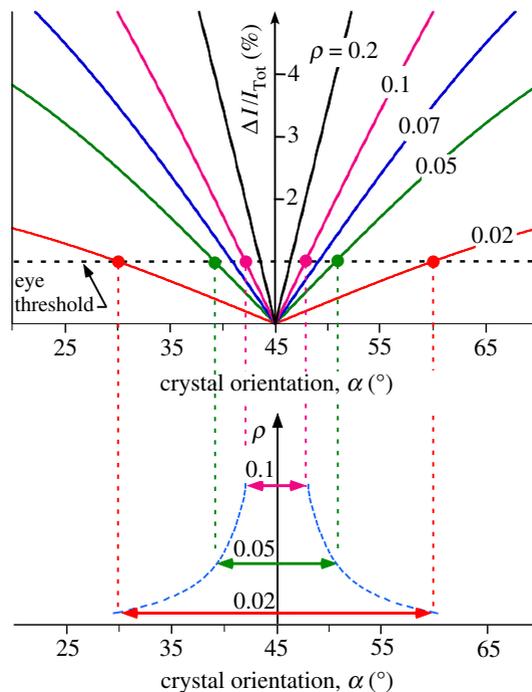
Figure 9. Theoretical signals of the differential two-image method, for quite low degrees of polarization of the incident light corresponding to rather cloudy or foggy skies ($0.02 < \rho < 0.2$). The sensitivity threshold to contrasts of the human eye chosen at 1% (Hubel 1988) determines the necessary minimum angle for back and forth rotations of the Iceland spar for each value of the degree of polarization. (Online version in colour.)

differential signatures at the isotropy point still behaves like an efficient sunstone. To appreciate the ultimate limits, we have plotted in figure 9 the theoretical contrasts for quite low degrees of polarization from $\rho = 0.02$ to $\rho = 0.2$. Two points are important to have a precise bearing of the hidden Sun. First for each value of ρ, we have to take into account the threshold imposed by the human eye, i.e. about 1 per cent (Hubel 1988). Another parameter is the minimum amplitude of the back and forth rotations, so as to reach the 1 per cent limit. Guessing the Sun direction is of course easier if the required rotations are smaller around the isotropy point, i.e. for larger ρ values.

We may note that 90° ambiguities could exist with the two-image comparison for the true position of the Sun, as the signals are the same for the ±90° directions of the true position as shown in figure 4. Other clues such as the swell or the wind could have helped the Vikings to resolve these ambiguities, but it is worthwhile noting that, when a patch of blue sky is still available, the yellow brush of the Haidinger's pattern can clearly resolve these ambiguities as it directly gives the true direction of the Sun. Hence combining the yellow brush and the two-image observations, for the same isotropy point of the Iceland spar, the Vikings could have been able to remove such ambiguities and to localize clearly the hidden Sun. Further tests and psychophysical experiments aboard a boat could be performed in the future, at about 61° N in various weather conditions.

Without any knowledge of the polarization nature of the light, by looking through a calcite crystal commonly found in Iceland and Scandinavia, the Vikings could have exploited the high sensitivity of the human eye to small contrasts. As the magnetic compass was introduced in Europe around the thirteenth century, the recent amazing discovery of a calcite crystal (figure 1*a*) on board a sixteenth century Elizabethan ship may seem useless. However, we have verified in Alderney that even only one of the cannons excavated from the ship is able to perturb a magnetic compass orientation by 90°. So, to avoid navigation errors when the Sun is hidden, the use of an optical compass could be crucial even at this epoch, more than four centuries after the Viking time. The Alderney discovery opens new possibilities as it looks very promising to find Iceland spars in other ancient shipwrecks, or in archaeological sites located on the seaside such as the Viking settlement with ship repair recently discovered in Ireland (Clinton 2010).

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