The competition between folding and faulting in the upper crust based on the maximum strength theorem

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It is proposed to complement the numerous geometrical constructions of fault-related folds relevant to fold-and-thrust belts by the introduction of mechanical equilibrium and of the rock limited strength to discriminate between various deformation scenarios. The theory used to support this statement is the maximum strength theorem that is related to the kinematic approach of limit analysis known in soil mechanics. The classical geometrical construction of the fault-propagation fold (FPF) is proposed for illustration of our claim. The FPF is composed of a kink fold with migrating axial surfaces ahead of the region where the ramp propagates. These surfaces are assigned frictional properties and their friction angle is found to be small compared with the usual bulk friction angle to ensure the full development of the FPF, a first scenario. For larger values of the axial surface friction angle, this development during overall shortening is arrested by the onset of fault breaking through the front limb, a second scenario. The amount of shortening at the transition from folding to break-through faulting is established.

Keywords: fault-propagation fold; faulting; maximum strength; limit analysis; fold-and-thrust belts; optimization

1. Introduction

Some fault-propagation folds (FPFs) become detached, and their growth arrested, by the development of underlying break-through faults (BTFs), while others do not. The difference between these two scenarios observed in nature is used to interpret the frictional properties at the time of deformation of sedimentary rocks. The method considered to compare these scenarios relies on the maximum strength theorem combined with the geometrical construction proposed by Suppe & Medwedeff (1984). Such coupling with mechanics could be applied to numerous geometrical constructions of fault-related folds found in the literature.

The historical development of the subject of fault-related folds from the geological point of view (Buil 2002) can be traced back to Willis (1893) and to the idea that faulting occurs at the latest stage of deformation of folds, resulting...
from bending rupture of the most stretched plane. The presence of ramps linking weak horizons or décollements before any folding were discovered by Rich (1934) in a work in the Cumberland Mountain (USA), proving that the order of cause and effect, folding before faulting, could be reversed. Concomitant folding and faulting were recognized by Dahlstrom (1970) in a study of the Canadian Rocky Mountains. Weak horizons and the concomitant developments of the ramp and the fold formed the basis of geometrical constructions of fault-bend folds (Suppe 1983) and FPFs (Suppe & Medwedeff 1984, 1990). The latter construction (FPF) is central to this contribution.

The initial work of Suppe & Medwedeff (1984) has attracted a lot of attention from the structural geology community and the following review is certainly not complete. Jamison (1987) amended the geometrical rules to produce forelimb thickness changes motivated by a comparison of the final fold geometries and field examples. Chester & Chester (1990) extended the FPF construction to start from a ramp of finite length. Mosar & Suppe (1992) proposed to include simple shear within the internal layers of the FPF, resulting in a rotation of the kink fold ahead of the propagating fault. The generalization to three dimensions of the FPF construction with lateral variations in the fault geometry and slip rates was proposed by Wilkerson et al. (1991). Salvini & Storti (2001) studied the migration of the axial surfaces in terms of local history of the activated deformation mechanisms, preparing the grounds for a comparison between field observations and theoretical predictions.

Mechanical analyses should provide some insight on the governing properties and processes controlling faulting and folding. Hill & Hutchinson (1975) have extended the seminal work of Biot (1965) and established the bifurcation conditions for strain-rate independent elasto-plastic materials, relevant to the folding of brittle rocks in the upper crust. The local conditions for the onset of shear banding or faulting are essentially due to Rudnicki & Rice (1975). The bifurcation conditions applied to layered structures provide the stress conditions for the onset of folding, which can only prevail if the shear banding conditions are not met (Triantafyllidis & Leroy 1997). Computational methods are required to follow the fold and fault developments, as carried out by Barnichon & Charlier (1996) to reproduce the thrusting in sand boxes and Cardozo et al. (2003) to inspect the velocity field ahead of a propagating fault. The difficulty of the numerical approaches is that the solution search occurs close to or beyond the local conditions of shear banding for which the strain is localized on a length scale not resolved by the discretization. New algorithms are required to adapt this discretization (Braun & Sambridge 1994) or a characteristic length should be included, as with the discrete-element technique (Hardy & Finch 2007).

The approach followed here is very different from these numerical methods and relies on the overall shape of the structures that is proposed by the structural geologists based on simple geometrical rules. The structure shape is indeed not sought as the solution of boundary-valued problems. Faults are introduced explicitly and most of the calculations remain analytical without the need for numerical interpolation. The new idea of this contribution, compared with Maillot & Leroy (2006) and Kampfer & Leroy (2009), is to show that any geometrical construction of folding could become mechanically balanced in the sense that mechanical equilibrium and material strength are accounted for. The now classical example of the FPF is considered for that purpose. Two sets of
rules are proposed for the construction of this asymmetric fold (Jamison 1987; Suppe & Medwedeff 1990), and we choose here the set based on conservation of bed thickness and bed length. The deformation has to be accommodated by bedding plane slip as material crosses axial surfaces of the fold. The fold is characterized by a kink fold, bounded by several propagating axial surfaces, ahead of the region where the ramp propagates.

Concepts of mechanics required to complement the geometrical constructions are included by application of the maximum strength theorem. It corresponds to the kinematic approach of the limit analysis (Salençon 1974, 2002) developed initially for predicting bearing capacities of civil engineering structures. This theorem relies on a weak form of the equilibrium equations and on the existence of the strength domain, convex in the appropriate stress space. This domain is the set of admissible stresses that can be sustained by a given material. The case of frictional and cohesive materials is considered here, and the strength domain is bounded by the Coulomb criterion. These two concepts, mechanical equilibrium and maximum material strength, provide an upper bound of the internal dissipation and subsequently of the applied force.

The paper contents are as follows. Section 2 is devoted to the presentation of the geometrical construction of the FPF, which is further analysed in the electronic supplementary material, §1 and §3. It is shown that the velocity field associated with the FPF could be estimated in a rather systematic manner with the help of Hadamard’s jump condition (electronic supplementary material, §2). This condition relates the differences in velocities and in transformation gradients between the two sides of the discontinuity and its propagation (non-material) velocity. Section 2 also contains a description of the geometry of the BTF. The maximum strength theorem, which is presented with the help of a simple example in the electronic supplementary material, §4, is applied to the development of the FPF and its potential arrest by BTFs in §3. Analytical expressions are provided for the upper bounds of the forces necessary for the FPF and the BTF and the maximum strength theorem ensures that the smaller of two loads determines which deformation mode dominates. Also, restrictions on the geometry of the FPF based on the frictional properties of the axial surfaces are established. The competition between FPFs and BTFs is explored in §4 by comparing the least upper bound to the force for BTFs and FPFs. An estimation of the amount of shortening the FPFs can accumulate prior to the onset of the BTFs is proposed. In particular, it is shown that the friction angle of the fold axial surfaces is the key parameter to determine this critical shortening.

2. The geometry

The objective of this section is to describe the geometry of the FPF during its development and also the geometry of the BTF, which could occur at any step of this development.

(a) The geometry of the fault-propagation fold

The two-dimensional structure includes an initially horizontal, competent layer \(A''OG'B''\) of length \(L\), thickness \(H_s\) composed of a material of density \(\rho_s\).
(figure 1a). The segment $GG'$ acts as a décollement. Shortening is initiated by displacing the back wall towards the left. An inviscid fluid-like material of density $\rho_f$ is on top of the competent layer to account for the effect of burial. This fluid-like material may represent poorly consolidated sediments such as shales: its weight is important for the underlying rock, but it cannot resist the lateral tectonic forces. The fluid-like layer has a constant thickness $H_f$ with time because it is connected to an external reservoir of infinite extent, illustrated on the left-hand side of figure 1a.

The ramp dips at the pre-determined angle $\gamma_B$ and emanates from point $G$; its length is zero at the onset. The material to the right of this point $G$, in the back-stop, is displaced towards the left by the quantity $\delta$ increasing with time and denoting the shortening at the back wall. The left boundary of the back-stop is the axial surface $B$. This surface is dipping at the angle $\theta_B$, which is half the complementary angle of $\gamma_B$ ($2\theta_B = \pi - \gamma_B$). This assumption on the angle $\theta_B$ is classical in the geometrical constructions of folds and implies the conservation of the thickness of the beds crossing the axial surface. Material points coming from the back-stop and crossing the axial surface $B$ are entering the region $GBB'RS$, which is the sub-domain HW1 of the hanging wall. Note that all the geometrical attributes are defined in table 1.

The development of the fold is marked by the propagation of the ramp with the same constant dip $\gamma_B$. Point $S$ in figure 1b marks the limit of the material initially on the décollement now found on the ramp after shortening by $\delta$. The dotted line $SS''$ dipping at $\theta_B$ was initially on the axial surface $B$ and is bounding to the left by all the material coming from the back-stop and now in the hanging wall. The selection of the angle $\theta_B$ to conserve bed thicknesses means also that the distance $SG$ is equal to the shortening $\delta$. The ramp has propagated further than this distance and its tip is now at point $P$. The bed passing through point $P$ is marked by the dashed line in figure 1b. Material points above and below this line have different kinematics. Region $P'PRS'S''B'AA'A''$ deforms in a kink mode by the propagation of axial surfaces. The axial surface $A'$ is dipping at $\theta_A$, an angle chosen to be half the dip of the layering within the kink band $PRA'$, a choice made again to preserve bed thickness ($2\theta_A = \pi - \gamma_A$). Axial surfaces $A$ and $B'$ are parallel to the surfaces $A'$ and $B$, respectively. The relation between the angle $\gamma_A$ and the propagation ratio $k = PG/SG$ in terms of $\gamma_B$ is provided in the electronic supplementary material, §1.

The velocity field of the FPF is also presented in the electronic supplementary material by applying in a systematic manner Hadamard’s jump condition for non-material interfaces. This condition is found particularly useful to justify that the sub-domains HW1 and HW4 of the hanging wall have the same velocity and thus that the segment $RS$ is not a dissipative interface. This information is important for the determination of the upper bound of the force necessary for the FPF development.

(b) The geometry of the break-through fault

The BTF is defined by the instantaneous propagation of the ramp $GP$, which is extended by the new fault, segment $PP'$, upon further shortening of the whole structure (figure 1d). The dip of this last segment could differ from the dip of the fold ramp $\gamma_B$ and is denoted as $\gamma_{BTF}$. The faulted structure is composed
Figure 1. (a) The geometry of the structure at the onset and (b) during the development of the FPF. (c) Definition of the adopted terminology and of the four sub-domains composing the hanging wall of the FPF. (d) The geometry of the BTF, which could arrest the FPF development.
Table 1. Material properties and geometrical characteristics of the FPF and the BTF. The interfaces and axial surface cohesions are not indicated; they are used in the definitions of the upper bounds and denoted by the letter $C$, with the same subscript as that of the friction angle. The data that are varied are indicated as ‘var.’.

<table>
<thead>
<tr>
<th>symbol</th>
<th>FPF</th>
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<tbody>
<tr>
<td>$\delta$</td>
<td>shortening</td>
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<tr>
<td>$h$</td>
<td>current height of ramp tip $P$</td>
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<tr>
<td>$\gamma_B$</td>
<td>ramp dip</td>
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<tr>
<td>$\theta_B$</td>
<td>axial surfaces $B$ and $B'$ dip</td>
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<td>$\gamma_A$</td>
<td>interface $PR$ dip</td>
</tr>
<tr>
<td>$\theta_A$</td>
<td>axial surfaces $A$ and $A'$ dip</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>friction angle of the axial surfaces</td>
</tr>
<tr>
<td>$\phi_B$</td>
<td>friction angle of the bedding</td>
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<table>
<thead>
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<th>value</th>
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<tr>
<td>$\gamma_{BTF}$</td>
<td>break-through fault $PP'$ dip</td>
<td>var.</td>
</tr>
<tr>
<td>$\theta_{BTFu}$</td>
<td>upper axial surface dip</td>
<td>var.</td>
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<tr>
<td>$\theta_{BTFl}$</td>
<td>lower axial surface dip</td>
<td>var.</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>friction angle of competent layer</td>
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<td>$\phi_D$</td>
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of four regions, the back-stop (region $GG'B''G''$ denoted $BS$), the lower and upper hanging walls (region $PP''PG''PP''$ and region $P'PP''$ denoted HWl and HWu, respectively) and the foot wall (region $OGPP'A''$ denoted FW). The boundary between the upper and the lower parts of the hanging wall is the axial surface $PP''$ dipping at the unknown angle $\theta_{BTFu}$. The transition from the back-stop to the hanging wall is the axial surface $GG''$ dipping at the unknown angle $\theta_{BTFl}$, which could differ from $\theta_B$ (the FPF axial surface) because the points $G''$ and $B$ do not necessarily coincide.

### 3. Bounds of the applied forces

The objective of this section is to obtain the upper bounds of the force necessary for the development of the FPF and the BTF. These bounds are obtained by application of the maximum strength theorem that is presented in the electronic supplementary material, §4, with the help of a simple example, the sliding of a block up an inclined ramp. This example, corresponding to the geometry of the central region of our structure, should help the reader understand why the
exact velocity field is not a pertinent velocity field for the application of the maximum strength theorem. This point is discussed in §3a. Section 3b,c presents the expressions for the bounds for the FPF and the BTF. Section 3d reveals that FPFs may be impossible if the axial surfaces are assigned large friction angles.

(a) Strength domain and support function

Dissipation occurs over ramps, axial surfaces and décollement, which are velocity discontinuities within the structures or on their boundaries. The set of these surfaces is denoted $\Sigma_U$. Each member of this set labelled $i$ is a Coulomb interface and has a limited strength defined by the domain

$$G_i = \{ T | |\tau| + \sigma_n \tan \phi_i \leq C_i \}, \quad (3.1)$$

in which $\phi_i$ and $C_i$ are the friction angle and the cohesion, respectively. The stress vector $T$ in (3.1) is the sum $\tau t + \sigma n$, in which $\tau$ and $\sigma$ are the shear and the normal components in the orthonormal right-handed basis $\{n, t\}$, where $n$ is the normal vector (figure 2a). Note that the adopted sign convention is that stresses in tension are positive.

We shall work with kinematically admissible (KA) velocity fields that, by definition, satisfy the essential boundary conditions. These conditions in our problem stipulate that the foot wall remains at rest and that the material points of the back-stop in contact with the back wall are displaced at the same velocity. The set of KA velocity fields includes the exact field. A KA field is denoted by a superposed hat: $\hat{U}$. The velocity jump over any dissipative interface $[\hat{U}] = \hat{U}^+ - \hat{U}^-$ is defined as the difference between the velocity on the + and − side of the discontinuity (the normal points towards the + side), figure 2b.
The exact stress field required to define the internal power \( \langle \hat{U} \rangle \cdot T \) is unknown, and no attempt is made to determine it in this contribution. It is proposed to find an upper bound of the internal power and, for that purpose, we search for the maximum power that can be dissipated at every point along the surfaces in \( \Sigma_U \).

It can be shown (Salençon 1974, 2002 and more recently Maillot & Leroy 2006) that there is indeed a maximum power because the strength domain \( G_i \) in (3.1) is convex in the two-dimensional stress space. This maximum is called the support function and is defined for the Coulomb strength domain by

\[
\begin{aligned}
\text{case 1 : } & 0 \leq |\eta| < \frac{\pi}{2} - \phi_i, \quad \pi_i(\langle \hat{U} \rangle) = \hat{J} C_i \cotan(\phi_i) \cos \eta, \\
\text{case 2 : } & |\eta| = \frac{\pi}{2} - \phi_i, \quad \pi_i(\langle \hat{U} \rangle) = \hat{J} C_i \cos \phi_i,
\end{aligned}
\]

and

\[
\begin{aligned}
\text{case 3 : } & \frac{\pi}{2} - \phi_i < |\eta| \leq \pi, \quad \pi_i(\langle \hat{U} \rangle) = +\infty,
\end{aligned}
\]

in which \( \hat{J} \) is the norm of the velocity jump on the interface of interest. Three cases are defined in (3.2) and illustrated in figure 2c depending on the angle \( \eta \) between the velocity jump and the normal (\( \langle \hat{U} \rangle = \hat{J}(\cos \eta n + \sin \eta t) \)). Cases 1 and 2 are of interest because the support function is finite. The velocity jump vector for those two cases is oriented within (case 1) or at the boundary (case 2) of the cone of direction \( n \) and internal angle \( \pi/2 - \phi_i \). Choosing a velocity jump outside this cone leads to an upper bound that is infinite and thus of no interest, case 3 in (3.2). The velocity jumps that are oriented within or at the boundary of the cone are the only ones of interest and are said to be pertinent. Note that the exact velocity field is not pertinent for our analysis since in that instance \( \eta = \pi/2 \).

The support function is integrated over all the interfaces composing \( \Sigma_U \) to obtain the maximum, resisting power that is bounding the unknown, internal power. The theorem of virtual power (Malvern 1969) stipulates that the internal power and the external power are equal for any KA velocity field. It is the combination of this latter theorem and the bounding proposed above that constitutes the maximum strength theorem and provides the bounds of the applied force. There is an optimum velocity field (uniqueness is not ensured) that yields the least upper bound of the tectonic force. This optimum velocity field defines the dominant collapse mechanism, a term often used in structural engineering. In this contribution, two collapse mechanisms are studied, the FPF and the BTF. The interested reader is referred to the electronic supplementary material, §4) to find further information on this theorem and also the expressions for the maximum resisting power and the external power for the two collapse mechanisms. Only the relevant KA velocity fields and the expression for the upper bounds of the tectonic forces are provided in what follows.

(b) Faulting breaking through the fault-propagation fold

The boundaries between the different regions of the faulted structure defined in §2b together with the segmented ramp and the décollement are dissipative interfaces and they constitute the set of surfaces \( \Sigma_U \).

The KA velocity field of the faulted structure is constructed as follows, with respect to the foot wall that is at rest ($\hat{U}_{FW} = 0$, figure 3a). The velocity $\hat{U}_{BS}$ of the back-stop is oriented to be pertinent with respect to the décollement $GG^{′}$. Its angle $\eta$ with respect to the normal to the décollement needs to be less than or equal to $\pi/2 - \phi_D$. It is proposed to set $\eta = \pi/2 - \phi_D$, case 2 for the support function in (3.2), to minimize the power against gravity, as shown in the electronic supplementary material of Cubas et al. (2008). The same reasoning is applied to the lower and the upper parts of the hanging wall. The velocity $\hat{U}_{HWl}$ is oriented in the direction dipping at $\gamma_B + \phi_R$, and the upper part is assigned a velocity $\hat{U}_{HWu}$ dipping at $\gamma_{BTF} + \phi_s$. The subscripts ‘D’, ‘s’ and ‘R’ to the friction angles are referring to the décollement, the solid layer and the ramp, respectively. The ramp could have a lower friction than the bulk material of the solid layer because of damage accumulation upon shortening. These three velocities ($\hat{U}_{BS}$, $\hat{U}_{HWu}$ and $\hat{U}_{HWl}$) define the KA field for the BTF and are presented in figure 3a, including their orientations.

Attention is now turned to the intensity of these three velocity vectors. The intensity of the back-stop velocity is arbitrary and will be used later on as a normalizing factor. The intensity of the velocity in the lower part of the hanging wall is determined by inspection of the axial surface $GG^{′′}$, which is assigned the friction angle $\phi_s$ as for the new part of the ramp. This interface accommodates the jump in virtual velocity ($\hat{U}_{HWl} - \hat{U}_{BS}) = \hat{J}_{GG^{′′}}$, a vector that is oriented to be pertinent according to case 2 of the support function (3.2), as illustrated in the hodogram of figure 3b. The law of sines applied to this triangular construction provides

$$\frac{\hat{U}_{HWl}}{\sin(\phi_D + \theta_{BTF} + \phi_s)} = \frac{\hat{U}_{BS}}{\sin(\gamma_B + \phi_R + \theta_{BTF} + \phi_s)} = \frac{\hat{J}_{GG^{′′}}}{\sin(\phi_R + \gamma_B - \phi_D)}.$$  

(3.3)

The same reasoning is considered for the upper region of the hanging wall separated from the lower part by the axial surface $PP^{′′}$. The velocity of the upper
region $\hat{U}_{HWu}$ makes the angle $\phi_s$ with the ramp and its norm $\hat{U}_{HWu}$ is chosen such that the jump in velocity across the axial surface $PP''$ is also oriented according to case 2 of the support function for the friction angle $\phi_s$. The corresponding hodogram is presented in figure 3b and reveals
\[
\frac{\hat{U}_{HWu}}{\sin(\gamma_B + \phi_R + \theta_{BTFu} + \phi_s)} = \frac{\hat{U}_{HWl}}{\sin(\gamma_{BTF} + 2\phi_s + \theta_{BTFu})} = \frac{\hat{J}_{PP''}}{\sin(\gamma_{BTF} + \phi_s - \gamma_B - \phi_R)}.
\]

We now apply the maximum strength theorem, and the expressions for the maximum resisting power and the external work are found in the electronic supplementary material, §4. The upper bound of the force necessary for the BTF reads
\[
Q \leq Q^u_{BTF},
\]
with
\[
Q^u_{BTF} \cos \phi_D = \left( p_A L_A' \sin(\phi_s + \gamma_{BTF}) + \frac{p_A + p_A'}{2} L_{AA'} \sin(\phi_s + \gamma_{BTF} - \gamma_A) + p_A L_{AB} \sin(\phi_s + \gamma_{BTF}) \right) \hat{U}_{HWu}
+ \left( \frac{p_B + p_B'}{2} L_{BB'} \sin(\phi_R + \gamma_{BTF} - \gamma_B) \right) \hat{U}_{HWl}
+ p_B \sin(\phi_D + S_{HWu} \hat{U}_{HWu} \rho_s g \sin(\phi_s + \gamma_{BTF})
+ S_{HWl} \hat{U}_{HWl} \rho_s g \sin(\phi_R + \gamma_B) + S_{BS} \hat{U}_{BS} \rho_s g \sin(\phi_D)
\right)
+ \hat{U}_{HWu} C_s \cos(\phi_s) L_{PP} + \hat{J}_{PP''} C_s \cos(\phi_s) L_{PP''}
+ \hat{U}_{HWl} C_R \cos(\phi_R) L_{PG} + \hat{J}_{GG''} C_s \cos(\phi_s) L_{GG''}
+ \hat{U}_{BS} C_D \cos(\phi_D) L_{GG'},
\]
in which the jumps and the velocities with a superposed tilde have been obtained by the normalization with the arbitrary velocity of the back-stop $\hat{U}_a = \hat{U}_a / \hat{U}_{BS}$. The first four lines and the first term of the fifth line in the expression for the upper bound are due to the external work and correspond to the work of the velocity field on the fluid pressure at the boundary between the competent and the fluid-like layer. The pressure at any given point $M$ is denoted as $p_M$, and the length between points $M$ and $M'$ as $L_{MM'}$. The pressure at any point $M$ of the top of the competent layer is $\rho_M g D_M$, where $D_M$ is the depth of this point, measured from the stress-free top surface. The depth of each point is found directly from the geometrical description of the FPF given in §2. The second term in the fifth line and the next line in the expression for the upper bound in (3.5) correspond also to the external work: the work of the velocity field on the gravity field. Surfaces are denoted by $S$, and the subscript identifies the sub-domain considered.
The last three lines correspond to the maximum resisting power over the five discontinuities composing the set $\Sigma_U$. All these interfaces are in case 2 of the support function. Note that equation (3.5) was set up for the faulting event defined in figure 3a. The new fault extending the ramp reaches the competent layer at point $P'$ between $A''$ and $A'$. Similarly, points $P''$ and $G''$ are presented to the right of points $B'$ and $B$, respectively. The results presented in the next section could rely on different geometries, and proper modifications of this expression are accounted for in a straightforward manner, which is not discussed further here. Note also that the contribution of the fluid-like pressure on the back wall over the depth $H_f$ has been disregarded from the bound in (3.5) for the BTF. The reason is that the same term contributes to the least upper bound determined next for the FPF, and what is of interest is not the exact value of these bounds, but their difference.

Note finally that (3.5) is an upper bound that can be minimized by selecting the values of the the dips $\theta_{BTFU}, \theta_{BTFr}$ and $\gamma_{BTF}$. The minimum upper bound is referred to as the least upper bound.

(c) The least upper bound for the fault-propagation fold

The frictional properties of the interfaces composing the FPF are first discussed. The ramp has the friction angle $\phi_R$ assumed again to be lower than the friction angle of the pristine material because of damage accumulation. It is also assumed that the competent rock is layered. The rock composing each layer has the friction angle $\phi_s$ considered in the previous section. The layers are separated by weak interfaces corresponding to the bedding planes, typically composed of shales and having the smaller friction angle $\phi_B \leq \phi_s$. These weak planes facilitate the activation of the axial surfaces, and a detailed discussion of the axial surface mechanism is given by Maillot & Leroy (2003). It is assumed here that the effective properties of the axial surface are cohesive and frictional, the friction angle being denoted as $\phi_*$ and assumed smaller than $\phi_s$.

The velocities of the back-stop and of the region HW1 in the hanging wall are constructed similar to the velocities of the back-stop and of the lower hanging wall of the BTF, respectively. The velocities $\hat{U}_{BS}$ and $\hat{U}_{HW1}$ are oriented by the angles $\phi_D$ and $\phi_R$ from the directions of the décollement $GG'$ and of the ramp, respectively. The norm of $\hat{U}_{HW1}$ is found from the application of the law of sines to the hodogram for the velocity jump across the surface $B$, of norm $J_B$, presented in figure 4b,

$$\frac{\hat{U}_{HW1}}{\sin(\phi_D + \theta_B + \phi_*)} = \frac{\hat{U}_{BS}}{\sin(\gamma_B + \phi_R + \phi_* + \theta_B)} = \frac{J_B}{\sin(\phi_R + \gamma_B - \phi_D)}.$$  

(3.6)

The velocity of the region HW3 of the hanging wall is oriented by the angle $\phi_*$ from the axial surface $A'$. Its norm is found by requiring that the velocity jump over the axial surface $PR$ dips at $\phi_B + \gamma_A$. It is indeed the bedding friction angle
that has to be used here because the surface $PR$ is parallel to the bedding in HW3. The application of the law of sines to the hodogram corresponding to the jump across $PR$, of norm $\hat{J}_{PR}$, presented in figure 4b, yields

$$
\frac{\hat{U}_{HW3}}{\sin(\gamma_B + \phi_R + \gamma_A + \phi_B)} = \frac{\hat{U}_{HW1}}{\sin(\theta_A + \phi_s + \phi_B + \gamma_A)}
= \frac{\hat{J}_{PR}}{\sin(\theta_A + \phi_s - \gamma_B - \phi_R)}.
$$

(3.7)

The velocity of the triangular region HW2 in the hanging wall is found by studying the axial surfaces $A$ and $B'$. The jump in velocity across the axial surface $A$ is oriented with the dip $\theta_A + \phi_s$. This vector is thus parallel to the velocity vector $\hat{U}_{HW3}$ and, consequently, the unknown velocity vector $\hat{U}_{HW2}$ has to have the same direction, as illustrated in figure 4b. The norm of this last vector remains unknown and is determined by inspection of the axial surface $B'$. The velocity jump over the axial surface $B'$ has the norm $\hat{J}_{B'}$ and is dipping at $\theta_B + \phi_s$. The
The corresponding hodogram is presented in figure 4b and the application of the law of sines reveals

\[
\frac{\hat{U}_{\text{HW}2}}{\sin(\gamma_B + \phi_R + \theta_B + \phi_*)} = \frac{\hat{U}_{\text{HW}1}}{\sin(\theta_B + 2\phi_*) + \theta_A)}
\]

\[
= \frac{\hat{J}_{B'}}{\sin(\theta_A + \phi_* - \gamma_B - \phi_R)},
\]

(3.8)

providing the unknown \( \hat{U}_{\text{HW}2} \). The velocity field just constructed only exists under certain constraints that are deduced from the four hodograms of figure 4b. These constraints will be presented in §3d.

The maximum strength theorem is now applied, and the expressions for the maximum resisting power and the external work are presented in the electronic supplementary material, §4. The least upper bound is

\[
Q \leq Q_{\text{FPF}}^\text{lu},
\]

(3.9)

with

\[
Q_{\text{FPF}}^\text{lu} \cos \phi_D = \frac{p_A' + p_A}{2} L_{A'A'} \sin(\phi_* + \gamma_A + \theta_A) \tilde{U}_{\text{HW}3}
\]

\[
+ p_A L_{A'B'} \sin(\phi_* + \theta_A) \tilde{U}_{\text{HW}2} + \frac{p_B' + p_B}{2} L_{B'B} \sin(\phi_*) \tilde{U}_{\text{HW}1}
\]

\[
+ p_B L_{B'B'} \sin(\phi_D) \tilde{U}_{\text{BS}} + S_{\text{HW}3} \tilde{U}_{\text{HW}3} \rho_s g \sin(\phi_* + \theta_A)
\]

\[
+ S_{\text{HW}2} \tilde{U}_{\text{HW}2} \rho_s g \sin(\phi_* + \theta_A) + S_{\text{HW}1} \tilde{U}_{\text{HW}1} \rho_s g \sin(\phi_R + \gamma_B)
\]

\[
+ S_{\text{BS}} \rho_s g \sin(\phi_D) + \tilde{U}_{\text{HW}3} C_s \cos(\phi_*) L_{A'P} + \tilde{J}_{PR} C_B \cos(\phi_B) L_{PR}
\]

\[
+ \tilde{J}_{A} C_s \cos(\phi_*) L_{AR} + \tilde{J}_{B'} C_s \cos(\phi_*) L_{RB'} + \tilde{U}_{\text{HW}1} C_R \cos(\phi_R) L_{PG}
\]

\[
+ \tilde{J}_{B} C_s \cos(\phi_*) L_{GB} + \tilde{U}_{\text{BS}} C_D \cos(\phi_D) L_{GG'},
\]

in which again all velocities have been normalized by the arbitrary velocity of the back-stop. The bound in (3.9) is the least because there is no degree of freedom, either in the geometry or in the velocity field that could be optimized.

\[(d) \text{ Locking of the fault-propagation fold}\]

This section is devoted to the constraints due to the construction of the KA velocity field of the FPF. The hodograms presented in figure 4b were plotted for realistic orientations of the sense of shear on all the interfaces. The reversal of a sense of shear would not be consistent with the development of the structure. It is said that in this instance, the FPF would lock. The condition for this locking to occur is, typically, that two of the three vectors used for one of the hodograms become linearly dependent. These conditions are now presented and the consequences for the development of the FPF discussed.

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Figure 5. The various constraints obtained from the hodograms of the FPF KA velocity jumps (figure 4b) defining an admissible domain for the selection of the ramp angle and of the axial surface friction angle.

For the hodogram for the axial surface $\mathcal{B}$ in figure 4b, the three conditions are

$$
\begin{align*}
\phi_D - \phi_R &\leq \gamma_B, \\
\phi_* + \phi_R &\leq \frac{\pi + \gamma_B}{2}, \\
\phi_* + \phi_D &\leq \frac{\pi + \gamma_B}{2}.
\end{align*}
$$

(3.10)

and

The décollement friction angle is less than the ramp friction angle, and the first condition is always met. Small values are expected for $\phi_*$ and $\phi_R$ so that the second condition is also not restrictive. Consequently, only the third condition remains. This is presented in figure 5, where the axial surface friction angle is plotted as a function of the ramp dip, as a dotted line labelled with the letter $\mathcal{B}$. The series of barbs mark the region that is restricted.

The second hodogram in figure 4b corresponds to the interface $PR$, and the three conditions to ensure its validity are

$$
\begin{align*}
\phi_R - \phi_* &\leq \frac{\pi - \gamma_A - \gamma_B}{2}, \\
\phi_* + \phi_B &\leq \frac{\pi - \gamma_A}{2}, \\
\phi_R + \phi_B &\leq \pi - \gamma_A - \gamma_B.
\end{align*}
$$

(3.11)

and
The first and second conditions correspond to the dotted curve $PR_1$ and $PR_2$ in figure 5, with the second being plotted for three values of the bedding friction angle. The third condition is always respected.

The third hodogram in figure 4b for the axial surface $A$ is based on three collinear vectors and the appropriate sense of shear requires that $\hat{U}_{HW3} \geq \hat{U}_{HW2}$. Comparing the second and third hodograms in figure 4b, it is concluded that this condition is equivalent to

$$\theta_B + \phi_* \geq \gamma_A + \phi_B. \tag{3.12}$$

This constraint is presented in figure 5 where it is labelled $A$ and plotted for three values of $\phi_B$.

The fourth hodogram is for axial surface $B'$ and the first of the three conditions for its validity is identical to the first in (3.11). The two others read

$$\phi_* \leq \frac{\gamma_A + \gamma_B}{4} = \phi_*^c \sqrt{\frac{3}{2}}$$

and

$$\phi_R + \phi_* \leq \frac{\pi - \gamma_B}{2},$$

defining the critical friction angle $\phi_*^c$. The first of these two conditions is the only relevant one and is plotted in figure 5 with the label $B'$.

These various inequalities define the admissible domain for the couple $(\gamma_B, \phi_*)$ in figure 5. It is found that for a ramp dip between 17 and 29°, there is a minimum and a maximum value for $\phi_*$ to avoid the fold locking. The minimum value is zero for $\gamma_B \geq 29°$, and the maximum decreases significantly for larger $\gamma_B$.

The physical interpretation of these constraints is better explained in the space of forces than in the KA velocity space. The example of the first constraint in (3.13), which is characteristic of the region HW2, is chosen to illustrate this point. For $\phi_* > \phi_*^c$, the forces on the two axial surfaces $A$ and $B'$ cannot balance the vertical weight of the region HW2. There is thus a locking of this region if the friction angle is larger than the critical, maximum value $\phi_*^c$. The proof of this statement in terms of forces is presented in the electronic supplementary material, §5, with a force balance analysis of the free body diagram of this region HW2. The fact that the restrictions based on forces and on KA velocities are the same provides a non-trivial illustration of the equivalence between the force approach and the maximum strength theorem.

4. Competition between folding and faulting

The main outcome of §3 is the estimation of upper bounds $Q_{lu}^{BTF}$ and of the least upper bound $Q_{lu}^{FPF}$ of the forces necessary for the BTF and for the development of the FPF, respectively. Only bounds were obtained for the force required for the BTF because several angles necessary to describe this deformation mode and the associated velocity field are not yet optimized. This section has two objectives. The first consists of conducting this optimization to obtain the least upper bound for faulting $Q_{lu}^{BTF}$. The second objective is to compare $Q_{lu}^{BTF}$ and $Q_{lu}^{FPF}$ to decide on the dominant mode, faulting or folding.
(a) Optimizing the break-through fault

The need to optimize the BTF is first assessed by considering the upper bound $Q_{BTF}^{\text{u}}$ obtained for a fault $PP'$ extending the initial ramp $GP$ with the same dip ($\gamma_{\text{BTF}} = \gamma_B$) and the upper and lower axial surfaces coinciding with the segments $PB'$ and $GB$, respectively ($\theta_{\text{BTFu}} = \gamma_A$, $\theta_{\text{BTFl}} = \gamma_B$). The optimized BTF is obtained by numerical means searching among all possible orientations $\gamma_{\text{BTF}}, \theta_{\text{BTFu}}$, and $\theta_{\text{BTFl}}$ the set of three angles leading to the smallest upper bound according to (3.5). Results, presented in Kampfer (2010) show that there is a reduction by 45 per cent of the bound by optimizing the BTF at every step of the development of the fold and in the absence of the fluid layer. The magnitudes of these forces are rather independent of the amount of shortening. This reduction between the upper and the least upper bound for thrusting is of the order of 15 per cent only for a 500 m thick overburden. This is simply the consequence of the dominant role played by the fluid-like pressure on the top of the competent layer. These numbers lead to the conclusion that optimization of the BTF is necessary to warrant a precise comparison with the FPF.

The geometry of the optimized BTF is presented in figure 6 with dotted, solid and dashed curves, for $\gamma_{\text{BTF}}, \theta_{\text{BTFu}}$, and $\theta_{\text{BTFl}}$, respectively. The dimensionless shortening is $\tilde{\delta} = \delta/L$. Results were obtained with $\gamma_B$, the fold ramp dip, being set to $30^\circ$. The three angles are found to have the same value, corresponding to $\pi/4 - \phi_s/2$, for zero shortening. This result is not influenced by the friction angle of the décollement nor by the length $L - D$ of the back-stop. An analytical proof of this surprising result is presented in the electronic supplementary material, §6. It is during the fold development that these various angles change drastically. The case with no overburden ($H_f = 0$) in figure 6a is first discussed. The evolution of the angles is partitioned in two phases. In phase 1, the upper axial surface and the thrust ramp are conjugate faults with the same dip decreasing to reach the critical value of $20^\circ$, marking the beginning of phase 2. At the transition and in phase 2, the upper hanging wall velocity $\hat{U}_{\text{HWu}}$ and the velocity of the lower part of the hanging wall $\hat{U}_{\text{HWl}}$ (see figure 3 for these definitions) are identical. It implies that $\gamma_{\text{BTF}} + \phi_s = \gamma_B + \phi_R$ and means that the hanging wall is acting as a single region. Thus, the upper axial surface does not exist any more in this phase 2, as it can be seen by the two insets in figure 6a. In phase 2, the thrust ramp keeps the same dip. The lower axial surface dip does not appear to be influenced by the transition between these two phases. Its dip decreases steadily during the fold development until it intersects the back wall, resulting in the kink in the curve of figure 6a for the dimensionless shortening of 0.15, approximately.

The evolution during the FPF development of the three dips of the BTF geometry is more complex in the presence of an overburden, figure 6b, and is partitioned into four phases. Phase 1 is identical to the one described above, except that the lower axial surface is now increasing its dip. In phase 2, the upper axial surface and the new segment of the ramp are not conjugate any more in terms of dip, the upper axial surface dip decreases more than the new ramp dip. It is found that the upper axial surface intersects the top of the competent layer exactly at the boundary between the back-stop to the hanging wall of the fold (point $B$ in figure 3). In phase 3, the selection of this special point $B$ ceases, and
the upper axial surface remains within the hanging wall. Its dip increases sharply. During the same phase, the dip of the new segment of the ramp continues to decrease to reach the conditions $\gamma_{\text{BTF}} + \phi_s = \gamma_B + \phi_R$, for which the upper axial surface does not exist. Phase 4 is thus identical to phase 2 in figure 6a where the hanging wall of the BTF behaves as a single region.

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Figure 7. The least upper bounds for the FPF (solid curves) and for the BTF (dashed curves) for different values of the fluid-like layer thickness. The circles indicate the intersection of the two curves and signal the end of the fold development.

(b) The dominant mode of deformation

The objective is now to compare the two least upper bounds to determine the lifespan of the FPF. This lifespan is defined by the amount of accumulated shortening at the transition from folding to the BTF.

This comparison in terms of least upper bounds is presented in figure 7 where the forces for the FPF and the BTF are presented as solid curves and dashed curves, respectively. Forces are normalized by $r_s g L^2$. Results are obtained for $\phi_s = 16^\circ$. The bounds for the FPF are initially always below the bounds for the BTF and the circles mark the transition with increasing shortening from the former to the latter. The FPF least upper bound without any overburden is an increasing function of the shortening because of the increasing relief during fold development. For $H_f = 100 \text{ m}$, the same least upper bound first decreases with the accumulated shortening, corresponding to a phase where the fluid pressure on the top of the competent layer decreases on average. The kink on that curve marks the shortening at which the relief pierces the fluid layer. The FPF least upper bounds for thicker overburdens are all similar: a slow decrease of the FPF bound with increasing shortening. The critical shortenings at the intersections of the bounds, marking the transition from FPFs to BTFs, are pinpointed by circles. This critical shortening is a slightly decreasing function of $H_f$ if larger than 200 m. This dependence is more pronounced and of opposite sense for $H_f$ less than 200 m.

These comparisons are now summarized in figure 8, which can be seen as a deformation mechanism map, in the space spanned by the dimensionless shortening and thickness of the overburden. The map is partitioned into two
domains, one to the left (where the FPF develops) and one to the right (where the BTF has stopped this development). The boundary between the two regions is the collection of the circles found in figure 7. The boundary between the BTF and the FPF domains is established for different values of the axial surface friction angle $\phi_*$: the larger the $\phi_*$, the smaller the region of dominance of the fold. Note the kink in the domain boundary for $\phi_* = 16^\circ$ corresponding, for the smaller values of $H_f$, to conditions for which the fold pierces the weak overburden. The influence of $H_f$ is weak if this piercing does not take place.

5. Conclusion

The first of the two objectives of this contribution was to show that geometrical constructions of folds could be complemented by the account of mechanical equilibrium and of the material limited strength. The second objective was to show that the resulting mechanically balanced constructions could be compared so as to explain, for example, the possible development of the FPF or its arrest by faulting through its fore limb (BTF).

The FPF is composed of a kink fold bounded by several axial surfaces that are migrating during the fold development ahead of the region where the ramp has propagated. It is shown how these migrations can be characterized by application of Hadamard’s jump condition, providing a rather systematic construction of the exact velocity field (electronic supplementary material). The application of the maximum strength theorem to the FPF shows that there are some restrictions
Figure 9. (a–d) The BTF interrupts the FPF development at different amounts of shortening depending on the axial surface friction angle, except for case (d) for which the fold development is completed before faulting could occur. The faults and the associated axial surfaces are the dashed and the dotted lines, respectively. Note in (c) that the fault dip is less than the fold ramp dip and that there is no upper axial surface. The dotted line is a marker and the top solid line is the stress-free surface of the fluid-like overburden. The dotted-dashed segment in (c) illustrates the position of the faulting breaking through the FPF fore limb of the Puri anticline (Medd 1996).

on the range of admissible fault dips according to the possible values for the axial surface friction angle. There are no ramp dips less than approximately 20°. For ramp dips varying between 20° and 60°, the maximum axial surface friction angle decreases from 30° to 20°, approximately.

The comparison between the least upper bounds of the force to drive the BTF and to develop the FPF is directly interpreted in terms of the lifespan of the latter defined by the accumulated shortening prior to the onset of the former. These verdicts on the lifespan are summarized in figure 9 for various values of the fold axial surfaces friction angle $\phi_s$. The top layer is composed of a fluid-like layer (poorly consolidated sediments) and its top surface remains at a constant level during compression. The dotted line is a marker. For the largest value of the axial surface friction angle, the BTF dominates as soon as shortening is initiated and the dashed and the dotted segments in figure 9a mark the fault and the
associated axial surface, respectively, both dipping at 30°. This particular angle corresponds to the classical orientation for faulting $\pi/4 - \phi_s/2$ in compression, where $\phi_s$ is the friction angle of the bulk material. This orientation of 30° is independent of the friction on the décollement, as shown analytically in the electronic supplementary material, §6. The interpretation of figure 9a is that a fault-bent fold will take place instead of the FPF. Figure 9b,c shows that the fold develops until it is stopped by the onset of the BTF at a critical shortening, which increases with decreasing values of $\phi_s$. Note the difference between the BTF mechanisms in these two examples. In figure 9b, the top segment of the ramp (dip $\gamma_{BTF}$) is sub-parallel to the lower ramp (dip $\gamma_B$) used by the fold and is associated with two axial surfaces, the first at the base of the fold ramp and the second at its tip. In figure 9c, a single axial surface exists at the base of the fold ramp and the hanging whole is sliding as a single unit. In that instance, the dips are related by $\gamma_{BTF} + \phi_s = \gamma_B + \phi_{R}$, in which $\phi_s$ and $\phi_{R}$ are the friction angles of the upper and lower segment of the ramps, respectively. In the last example (figure 9d), the fold develops completely without being interrupted by the onset of faulting because of the small value of $\phi_s$. In the last three examples of figure 9, the fold pierces the fluid-like overburden. The thickness of this overburden has little influence on the shortening, marking the end of the FPF development once piercing of the overburden has occurred.

One should note that our predicted break-through ramp is dipping less than the ramp of the FPF because its friction angle is larger than the friction angle of the FPF ramp. Such a dip is certainly satisfactory in the early stage of the FPF development for predicting the ramp linking, for example, two weak décollements as seen by Rich (1934), but it is not consistent with the later stage of the FPF life. Consider for example the Puri anticline (Papua, New Guinea; Medd 1996), several faults are breaking through the front limb at a dip larger than the FPF ramp dip (see the illustration with a dotted-dashed segment in figure 9c). Three reasons could justify this discrepancy. First, one could argue that the faults are not emanating from the tip of the FPF ramp, as is done in this contribution, but rather initiate somewhere along the ramp, as is illustrated in figure 9c. A second reason could be related to the overburden considered as a fluid-like material, whereas the Puri anticline is within a triangular zone and the overburden must exert a certain amount of shear forces on the upper boundary of the FPF. Thirdly, and more theoretically, is this difference in dip an outcome of our choice to favour the dissipation along the ramp instead of the work done by the fault propagation? A comparison with the work-budget approach proposed by Cooke & Murphy (2004) could clarify this issue.

One of the main outcomes of the comparisons between the FPF and the BTF is the importance of the friction angle of the axial surfaces that compose the former. It is argued that this friction angle could be significantly smaller than the classical Coulomb friction angle of a pristine rock if slip parallel to the bedding is activated during the propagation of the axial surfaces. This mechanism has been studied in detail by Kampfer & Leroy (2009) for the development of a kink fold as well as by Maillot & Leroy (2003) for the axial surface of a fault-bend fold. The key to the weak axial surfaces of the kink fold in these two references was the introduction of a compacting deformation mechanism in the intrados of every bed crossed by the axial surface. Here, we assume simply that the axial surface friction angle is smaller than the pristine rock friction...
angle without explicitly introducing the details of the axial surface mechanisms. Extending the present analysis to include the detailed axial surface geometry and properties could certainly be carried out in the future, especially if field data were available to constrain the spacing of the bedding and laboratory experiments to quantify the bedding friction and the exact nature of this compaction mechanism.

In the meantime, it could be argued that the necessity to reduce our axial surface friction angle is needed to palliate the fact that the kinematics of the FPF is not yet optimized. Such optimization is certainly important because we have seen here that an optimized BTF could lead to the reduction in the applied force by up to 45 per cent. An optimized FPF could thus be more competitive with respect to the BTF with a large axial surface friction angle. Optimizing the FPF is beyond the scope of the present contribution, and it will require us to relax some of the assumptions adopted for the geometrical constructions. Choosing which assumption to relax, or which set of geometrical rules to select, should be guided by comparisons with field examples (Jamison 1987; Suppe & Medwedeff 1990). The first assumption that is known not to be optimum is the choice of orienting some of the axial surfaces at half the complementary angle of the ramp. This point was discussed by Maillot & Leroy (2003) theoretically and by Maillot & Koyi (2006) and Koyi & Maillot (2007) from the results of analogue laboratory experiments and field observations. The second assumption is the rate of propagation of the ramp. The rate in this contribution is either controlled by the FPF construction or is instantaneous if the BTF is dominant. A continuous transition from fold control to fault control would be of interest to pursue this comparison: folding or faulting?

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