Contributions of John Henry Poynting to the understanding of radiation pressure

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The name of Poynting is universally recognized for his development of the well-known expression for the flow of electromagnetic energy. Not so well known is Poynting’s series of papers on radiation pressure, with 2011 marking the centenary of the last of his 15 publications on this topic. This paper reviews and assesses his radiation-pressure work, with a level of coverage aimed at the reader familiar with the Maxwell electromagnetic theory and interested in the current understanding of radiation pressure. We begin with brief details of Poynting’s life, followed by accounts of the relevant publications by others before and during his period of activity in the field from 1903 to 1911. His contributions to the understanding of radiation-pressure effects in the solar system, and the linear and angular momenta of light are discussed, with evaluations from a modern perspective.

Keywords: solar system; linear and angular optical momentum; Abraham–Minkowski

1. Introduction

Poynting’s name is familiar to every student of electromagnetism as the man who, in 1884, published a theorem for the conservation of energy in an electromagnetic field, as well as an expression for the flow of energy, now known, respectively, as Poynting’s theorem and the Poynting vector. At a later stage in his career, in the early years of the twentieth century, he published a series of groundbreaking papers on radiation pressure, covering both measurements and theories of the effect. Our main concerns in this article, 100 years after his final publication in the field, are with the basis of his work in the Maxwell theory, a review of his radiation-pressure papers, and an assessment of his current standing in the field. The historical setting and evolution of Poynting’s experimental and theoretical activities are well covered in our cited references. It is, however, useful to begin with a brief account of his life.

John Henry Poynting (1852–1914) was born in Monton, a suburb of Manchester, UK, about 8 km west of the city centre, where his father was Unitarian Minister. He was educated initially in his father’s school, and in 1867,
he entered Owens College (to become Manchester University in 1880), obtaining an external London University BSc degree in 1872. He was awarded an entrance scholarship to Trinity College, Cambridge in the same year, to study for a bachelor’s degree in mathematics. He achieved the position of third highest-scoring student with First Class Honours in the Mathematical Tripos of 1876. He then returned to Manchester to take up a position as demonstrator in the physical laboratory at Owens College, where J. J. Thomson, one of the discoverers of the electron (Pais 1986, pp. 78–86) and winner of the 1906 Nobel Prize in Physics, was one of his students. Back to Cambridge in 1878, on his election as a Fellow of Trinity College, he worked under Maxwell in the Cavendish Laboratory on experiments to determine the mean density of the Earth. In 1880, Poynting married Maria Adney Cropper, the daughter of another Unitarian Minister at Stand, part of Whitefield, another suburb about 8 km north of Manchester.

In the same year of 1880, Poynting was appointed to the Chair of Physics at the new Mason Science College (to become Birmingham University in 1900), and he stayed there until his early death from diabetes. His career in Birmingham was marked by a series of distinctions, including the award of a Cambridge ScD in 1887; election as a Fellow of the Royal Society in 1888; awards of the Adams and Hopkins prizes at Cambridge in 1893 and 1903; election as President of the Physical Society in 1905; and awards of Royal Medal and Bakerian Lectureship by the Royal Society in 1905 and 1910, respectively. He was also one of the founders of the UK National Physical Laboratory in 1907. In addition to Poynting’s pioneering work in the new physics of electrodynamics and radiation pressure, he was active in areas outside the physical sciences, making, for example, a study of the drunkenness statistics of large towns in Great Britain and a statistical analysis of changes in commodity prices on the London Stock Exchange. We are not aware of any full-scale biography of Poynting, but there are extensive accounts of his life and achievements in obituaries published shortly after his death by Larmor (1914), Lodge (1914) and Thomson (1916). They all testify not only to Poynting’s professional qualities, but also to his modesty and to the catholic nature of his interests.

The main influence on all of the activity in electromagnetic theory during the later years of the nineteenth century came from Maxwell’s famous treatise (Maxwell 1873). Poynting was a member of the group of young physicists led by Heaviside, Fitzgerald, Lodge and Hertz who developed Maxwell’s electromagnetic theory in the years following his death in 1879. They transformed his 1873 presentation into the formalism recognizable today as Maxwell’s equations. The detailed historical accounts by Hunt (1991) and Warwick (2003) describe Poynting’s contributions to electromagnetism, mainly during the 1880s. His name is more familiar to students of electromagnetic theory than those of other important members of the group on account of the widespread use of his eponymous energy-conservation theorem and energy-flow vector. These energy relations are not directly relevant for the electromagnetic momentum of interest here, but the expressions for energy density \( U \) and Poynting vector \( S \) do occur in the discussions of momentum in §2.

The plan of this review is as follows. In §2, we outline the relevant theories of Maxwell, which form the main basis for Poynting’s work on radiation pressure. Despite the apparent lack of any awareness of each other’s publications, we comment on the very relevant calculations by Minkowski and Abraham published

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Poynting was not the first to think about such effects, but, in regarding light as simply a wave capable of carrying both energy and momentum, he initiated an approach in the early 1900s that remains familiar today. The then-current orthodoxy required the existence of a material medium for transmission of light, but Poynting assumed only that the ether was a passive bystander in the exchange of momentum between a light ray and a material body (Poynting 1904a, p. 538), a role assigned today to the electromagnetic field alone.

2. Background to Poynting’s radiation-pressure work

Poynting’s collection of scientific papers has been published in a single substantial memorial volume (Shakespear & Barlow 1920). His 15 radiation-pressure publications during the years 1903–1911 occur together in the reference list. In the present section, we review the work prior to 1903 on which Poynting’s ideas and calculations were based, also the relevant papers by others that were published during his period of activity up to 1911.

The first recognition of the existence of what we would understand today as radiation pressure appears to date from speculation by Kepler in the early seventeenth century to the effect that light streaming away from the Sun is responsible for the orientation of a comet’s tail in a similar direction. The subsequent history of work on the measurement and theory of radiation pressure makes a fascinating story, whose twists and turns are associated with the rival corpuscular and wave theories of light. The story is covered in detail by Jones (1953), Schagrin (1974) and Worrall (1982), and there are shorter accounts in the introductions to papers on laboratory measurements of radiation pressure in the early twentieth century (Nichols & Hull 1903; Lebedew 1910).

Poynting’s early work was mainly based on Maxwell’s treatise, which had posthumous second and third editions in 1881 and 1891. It is convenient here to refer to the more accessible third edition (Maxwell 1891), where §§792 and 793 in volume 2 present the theory of radiation pressure exerted by light in free space, with estimates of its value for sunlight falling on the surface of the Earth. The theory in §792 shows that, contrary to the claims of proponents of the corpuscular nature of light, an electromagnetic wave indeed exerts a pressure...
in its direction of propagation. The pressure is determined by the components of the electromagnetic stress $T_{ij}$, with $i, j$ any two Cartesian coordinates, and where its magnitude equals the energy per unit volume $U$ in the wave. For the propagation of a transverse electromagnetic wave parallel to $z$, with $\mathbf{E}$ parallel to $x$ and $\mathbf{H}$ parallel to $y$, the pressure parallel to the $z$-axis and the energy density, re-expressed here in the SI units now current, are

$$T_{zz} = U = \frac{1}{2}(\epsilon_0 E^2 + \mu_0 H^2) = \epsilon_0 E^2 = \mu_0 H^2, \quad \text{as} \quad H = \sqrt{\frac{\epsilon_0}{\mu_0}} E. \quad (2.1)$$

In §793, Maxwell took the value of 83.4 foot pounds per second per square foot for the energy of sunlight falling on the Earth’s surface. The corresponding pressure on the Earth, obtained on division by the velocity of light, is about $10^{-7}$ pounds per square foot, equivalent to about 4.5 g per hectare in metric units. In his first paper on radiation pressure, Poynting (1903) emphasized the small value of the radiation pressure of sunlight on the Earth by its size of ‘less than one two-hundred-thousandth of a grain on the square-inch’, which turns out to be the same as that quoted above! The practical measurement of solar radiation pressure on the Earth is clearly a considerable challenge.

Experimental techniques for the measurement of radiation pressure employed an evacuated chamber, in which a thin vane was suspended from a torsion fibre. The asymmetric incidence of light from the Sun or a bright laboratory source on the vane should then cause it to rotate. The earlier observations of such rotations have mainly been interpreted as caused instead by other forces on the vane, which is inevitably heated by convection currents in the residual gas (Jones 1953; Schagrin 1974; Worrall 1982). The first reliable laboratory measurements of radiation pressure (Lebedew 1901; Nichols & Hull 1901, 1903) were made with carbon-arc light sources and with more sophisticated techniques to overcome the convection-current problem. The 1901 results showed that the radiation pressure was observable, but there was only approximate agreement with the Maxwell theory. Much better agreement, at the 1 per cent level, was claimed in 1903 with the use of further improvements in experimental techniques, although doubts have been cast on their true accuracy (Bell & Green 1933; Worrall 1982). Later laboratory measurements by Lebedew (1910) of the pressure exerted by light on gases were in rough agreement with the Maxwell theory.

The existence of the radiation-pressure phenomenon shows that electromagnetic waves carry momentum in addition to their more familiar energy content (Thomson 1893). Poynting (1905a) considered a light beam as a ‘stream of momentum’, whose flux (momentum per unit cross-sectional area per unit time) equals its energy density $U$. Formal theories of the electromagnetic energy and momentum in dielectric materials were developed by Minkowski (1908) and by Abraham (1909, 1910). Their papers provide relativistic treatments of electromagnetic theory in moving dielectric bodies, and the results of interest here are obtained by setting the material velocity equal to zero. The Minkowski and Abraham formalisms give identical expressions for the electromagnetic stress and energy density. Equation (2.1) now becomes

$$T_{zz} = U = \frac{1}{2}(\epsilon_0 \varepsilon E^2 + \mu_0 \mu H^2) = \varepsilon_0 \varepsilon E^2 = \mu_0 \mu H^2, \quad \text{as} \quad H = \sqrt{\frac{\varepsilon_0 \varepsilon}{\mu_0 \mu}} E. \quad (2.2)$$

where $\epsilon$ is the relative electric permittivity and $\mu$ is the relative magnetic permeability of the material. The energy flow, or Poynting vector, is also the same in both formalisms, given by

$$S = E \times H = \sqrt{\frac{\epsilon_0 \epsilon}{\mu_0 \mu}} E^2 \hat{z} = \left( \frac{cU}{\eta} \right) \hat{z}, \text{ where } \eta = \sqrt{\frac{\epsilon}{\mu}}$$  \hspace{1cm} (2.3)$$

is the usual refractive index of a dielectric and $\hat{z}$ is a unit vector. However, the momentum density vectors (momentum per unit volume) are different, given by

$$G_M = D \times B = \epsilon_0 \epsilon_0 \mu_0 \mu S = \left( \frac{\eta}{c} \right)^2 S \quad \text{and} \quad G_A = \frac{E \times H}{c^2} = \frac{S}{c^2}$$  \hspace{1cm} (2.4)$$

for the Minkowski and Abraham theories, respectively. Their magnitudes are expressed as

$$G_M = \frac{\eta U}{c} \quad \text{and} \quad G_A = \frac{U}{\eta c},$$  \hspace{1cm} (2.5)$$

with the use of equations (2.2) and (2.3). These forms are used in our analysis below and they are discussed in detail by Baxter & Loudon (2010). Both expressions reduce to $U/c$ in free space.

3. Radiation pressure in the solar system

His first publication on radiation pressure (Poynting 1903) is remarkable partly for the choice of journal, The Inquirer, a Unitarian periodical still in existence today. The choice was possibly influenced by his religious background and by the wish to write an article accessible to the layman. Poynting outlined the then-recent laboratory experiments (Lebedew 1901) and described them as a triumph for the theoretical predictions of Maxwell (1873). He also considered the solar radiation pressure, which is much smaller than the solar gravitational force for the Earth. However, he pointed out that their relative sizes are reversed for smaller particles sufficiently close to the Sun, with astronomical consequences.

Poynting (1904a) later suggested that the solar radiation pressure would cause small orbiting particles gradually to lose momentum and plunge into the Sun. The effect arises because light from the Sun impinges on the particle not directly side-on, but slightly on its leading surface. The absorbed light energy is subsequently re-radiated isotropically, thus leading to a net force with component against the direction of motion. The magnitude of the retarding force is proportional to the rate of energy re-radiation in Poynting’s calculation. He also published shorter papers on the same topic in the following two years (Poynting 1904b, 1905b, 1906a). The gravitational force and the energy radiation rate are proportional to the cube and square, respectively, of the particle’s linear dimension. The retarding effect is thus relatively greater for smaller particles. Poynting’s original paper contained a number of important typographical mistakes, which were corrected in his collected papers (Shakespear & Barlow 1920). However, the calculations are not easy to follow and Poynting’s own in situ notes and corrections can add to the
confusion. The several criticisms and the contributions by later authors, including Larmor (1912, 1918), were carefully evaluated and summarized by Robertson (1937).

Robertson also provided a fuller treatment of the problem consistent with relativity theory, and he established the existence of a drag of the same nature as that predicted by Poynting on classical grounds. This radiation-pressure-induced retardation is now known as the Poynting–Robertson effect or Poynting–Robertson drag. The drag component of the radiation-pressure force has the form

\[ F_{\text{drag}} = -\frac{S\sigma v}{c^2} \sim \frac{r^2}{\rho^{2.5}}, \]

where \( S \) is the Poynting vector of the solar radiation, \( \sigma \) is the particle cross section, \( v \) is its velocity, \( r \) is the particle radius and \( \rho \) is its orbital radius. The improved expressions (Robertson 1937) for the retarding force on a spherical particle reflecting or absorbing/re-emitting the radiation received from the Sun are modified by numerical factors from those obtained both by Poynting and Larmor, even in the approximation where only terms of first order in the particle velocity are retained. Nevertheless, despite these corrections, it was Poynting, who first identified the force on a particle in the solar system as the sum of gravitational, radiation-pressure and momentum-drag components.

More recent work (Burns et al. 1979) on the Poynting–Robertson effect has clarified and simplified the Robertson derivation. The earlier expression was also generalized to allow for the physical properties of interplanetary dust available from subsequent measurements.

4. Theory and experiments on the linear momentum of light

This section reviews Poynting’s contributions to the measurement and theory of the radiation pressure or linear momentum of light in free space and in dielectric media. Much of his work provided the first detailed accounts of the radiation-pressure phenomena he studied.

With the electromagnetic momentum flux in free space equal to the energy density, Poynting (1905a) quoted theoretical values of

normal pressure: \((1 + R)U \cos^2 \theta \) and tangential stress: \( \frac{1}{2}(1 - R)U \sin 2\theta \)

(4.1)

for the components of the force exerted by light incident at angle \( \theta \) to the normal on a surface that reflects a fraction \( R \) of the incident intensity. These results are straightforwardly calculated by resolution of the incident momentum flux \( U \cos \theta \) on unit area of surface into its normal and tangential components. Insertion of the reflected contributions with appropriate signs then leads to equation (4.1). The tangential stress, more easily measured than the normal pressure, vanishes for perfect reflection and it has a maximum value of \( U/2 \) for total absorption of light incident at 45°. These predictions were confirmed to reasonable accuracy in experiments with light incident on blackened and silvered discs hung on quartz fibres, all performed with the assistance of Guy Barlow. They used light from a Nernst lamp, which consists of a ceramic rod electrically heated to incandescence.
(Mendelsohn 1973). More details of the experiments are given in Poynting (1910c, pp. 53–58), where an improved 5 per cent accuracy is reported. This work confirmed the free-space value of radiation pressure.

A second paper (Poynting 1905b) is important for its treatment of the radiation pressure within a dielectric material. It presented the first ever such discussion, predating the more comprehensive calculations (Minkowski 1908; Abraham 1909, 1910). Poynting also discussed the pressures associated with different kinds of wave motion, particularly elastic waves, and the effects of radiation pressure in the solar system. He argued that because ‘the pressure is the momentum given out or received per second, and the pressure is equal to the energy density in the train, the momentum density is equal to the energy-density \( \div \) wave-velocity’. The momentum density for electromagnetic waves is then the same as the Minkowski value from equation (2.5), and this assumption is inherent in Poynting’s subsequent calculations.

Poynting’s main concern, however, was with the reflection and transmission of light for general incidence angles at the interface between two transparent media, both assumed to be non-magnetic with \( \mu = 1 \). We consider here only normal incidence, where equations (2.2) and (2.3) give

\[
T_{zz} = U = \varepsilon_0 \eta^2 E^2 \quad \text{and} \quad S = \varepsilon_0 c \eta E^2 = \frac{c U}{\eta}, \quad \text{where} \quad \eta = \sqrt{\varepsilon}. \tag{4.2}
\]

Thus, with the radiation pressure equal to the electromagnetic stress or energy density as shown above, the pressure on the interface, determined by the stresses (4.2) in the two media with signs appropriate to the propagation directions of the incident, reflected and transmitted light beams, is

\[
T_{\text{inter}} = (1 + R) U - U' = \frac{2 U (\eta - \eta')}{(\eta + \eta')}. \tag{4.3}
\]

The variables in transmission and incidence media are here denoted by primed and unprimed quantities, respectively, and the reflection coefficient is given by the usual expression (Jackson 1999). The force on the interface is thus predicted to act outwards from the material with the higher refractive index, as the push from the reflected light is overwhelmed by the pull from the transmitted light. The magnitude of \( T_{\text{inter}} / U \) for \( \eta' / \eta = 1.5 \) agrees with the tabulated value (Poynting 1905b) of 0.4 for \( \theta = 0 \), but equation (4.3) did not appear in his paper. The values tabulated for oblique incidence agree with a generalized version of the theory outlined above (L. Allen 1994, personal communication). The expression in equation (4.3) is again based on a choice equivalent to the Minkowski momentum, and this result is criticized in §6.

Poynting (1905b) also reported two further measurements of the free-space momentum associated with a light beam, performed in collaboration with Guy Barlow. Thus, one end of a suspended rectangular glass block was illuminated by a beam that is twice internally reflected before its emergence from the other end of the block. A more accurate experiment (Poynting 1905b, 1910c) used a pair of glass prisms attached to a brass arm suspended from a fibre. Further measurements by Barlow (1912) had the glass block or prism pair replaced by a glass cube. In all three versions, the double internal reflection within the glass block or prism pair ensured that the momentum stream of the transmitted beam

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had a parallel shift from that of the incident beam, equivalent to a counterclockwise couple on the light beam. The observed clockwise reaction couple on the glass in the series of experiments was in increasing agreement with calculations.

The other Poynting papers that include material on radiation pressure in the laboratory environment consist of a review article (Poynting 1906a), an abstract of a review lecture (Poynting 1906b), a conference review talk (Poynting 1909a) substantially repeated in French (Poynting 1910b) and a preliminary account (Poynting 1909b) of material later expanded (Poynting 1910a). The latter paper reproduces the 1910 Royal Society Bakerian lecture by Poynting and Barlow, where they considered the recoil momentum imparted to a source in the emission of light. They presented the theory of a set of experiments in which a beam of light fell on one of an array of four thin discs mounted in a mica plate suspended from a quartz fibre. The discs had front and back surfaces that were either totally absorbing (blackened: B) or totally reflecting (silvered: S). Schematically,

$$\begin{align*}
P \rightarrow & \ B|B \quad B|S \quad S|S \quad S|B \\
P \rightarrow & \ \frac{5P}{3} \quad 2P \quad 2P
\end{align*}$$

(4.4)

showing the direction of a normally incident beam of light that exerts a steady pressure $P$ on the front surface of the disc. Each disc had an asphaltum layer heated by the incident light until a steady state was reached. With the front and back surfaces at the same temperature, the layer then acted as a source of re-radiated energy and produced a recoil pressure on the disc. The calculated values in the last line of equation (4.4) are obtained as follows: disc (1) has equal amounts of re-radiation from its two sides and only the incident pressure contributes, disc (2) re-radiates only from its front side with a cosine-law spatial distribution that adds $2P/3$ to the incident pressure, discs (3) and (4) do not absorb and the pressure is that of specular reflection from their front surfaces. Although there was reasonable agreement only between calculated and measured values, this is an important experiment as it clearly demonstrated the presence of recoil momentum associated with re-emission of radiation. A particularly clear and succinct account of the work occurs in Poynting’s (1910b) review lecture. This is yet another experiment that confirmed the free-space value of the radiation pressure.

His last publication (Poynting 1911) on radiation pressure calculated the strength of the longitudinal elastic waves generated by the pressure from an electromagnetic plane wave in a dielectric. The pressure is proportional to the square of the field, as in equation (2.2), and the elastic wave had twice the frequency of the light wave. He showed that the energy content of the material waves was an infinitesimal fraction of the energy in the electromagnetic wave itself.

Remarkably, none of the later Poynting papers reviewed here makes any reference to the work of Minkowski (1908) and Abraham (1909, 1910) on the electromagnetic energy and momentum densities in dielectric materials. It may be that papers published in Göttingen or Palermo in German or Italian were not easily available or understandable in Birmingham. However, in compensation, although Minkowski and Abraham both mention the Poynting vector, neither of them refers to the relevant papers of Poynting on radiation pressure.
5. Optical angular momentum

The notion that a beam of circularly polarized light carries angular momentum was an original contribution by Poynting, apparently not previously mentioned in the literature. Much of his single publication on this topic (Poynting 1909c) presented a mechanical analogue of the optical effect, in which he treated the propagation of torsional elastic waves in a cylindrical shaft. The calculation followed his work (Poynting 1905b) on the linear pressure associated with transverse waves in elastic solids, mentioned above. It led to the expression

\[ \Gamma = \frac{\lambda U}{2\pi} = \frac{cU}{\omega} \]  

(5.1)

for the torque exerted on unit area of an absorbing surface by a circularly polarized free-space light beam of wavelength \( \lambda \). The paper ends with an ingenious proposal for the practical measurement of optical angular momentum by the passage of a plane-polarized beam upwards through a stack of quarter-wave plates. Each of the odd-numbered set of alternate plates, suspended from a torsion fibre, generates circular polarization and therefore experiences a torque. Each of the interleaving even-numbered fixed set of plates restores the linear polarization. The addition of all the torques on the suspended set of plates thus amplifies the angular momentum transfer. However, Poynting concluded that ‘... even with such multiplications, my present experience of light forces does not give me much hope that the effect could be detected ...’.

Despite Poynting’s pessimism concerning the practical detection of optical angular momentum, it was successfully measured by Beth (1936); this paper and that of Poynting (1909c) are the first two in a modern compilation (Allen et al. 2003). The experiment followed Poynting’s original idea, except that Beth used a single 25 mm half-wave plate with direct and reflected light from a filament lamp. The measured angular momentum of circularly polarized light agreed with the prediction in equation (5.1).

6. Epilogue

Studies of the radiation pressure and torque, or equivalently the optical linear and angular momenta associated with light beams have burgeoned in the 100 years following Poynting’s pioneering contributions, particularly since about 1970. Much of the theoretical and experimental work on radiation pressure has been motivated by an interest in the fundamental nature of the effect. There are also important practical applications, as in the development of the photon-drag detector (Gibson et al. 1970), which relies on the transfer of momentum from light to the electrons and holes in semiconductors and provides a useful high-speed room-temperature device for optical intensity measurements. Radiation pressure and torque facilitate the manipulation of particles in suspension, both linear, as in the optical tweezers of Ashkin et al. (1986), and rotational, as in the optical spanner of He et al. (1995), Simpson et al. (1997) and Friese et al. (1998). Radiation-pressure effects are also important in the practical use of global positioning systems (Pratap & Enge 2001; Leick 2004), where solar and albedo perturbing forces provide the largest error in determining their accuracy. The
optical stretcher (Guck et al. 2002) uses two counter-propagating laser beams to trap and stretch individual biological cells. Finally, details of light-driven micromachines are provided by Rubinzstein-Dunlop & Friese (2002), and there has been newspaper coverage (e.g. Guardian and Financial Times) of proposals for spacecraft driven by the radiation pressure of sunlight.

Nevertheless, one may reasonably wonder at the sheer volume of work on a quite narrow research topic, as surveyed in a series of detailed review articles (Robinson 1975; Brevik 1979; Pfeifer et al. 2007; Baxter & Loudon 2010; Mansuripur 2010; Milonni & Boyd 2010; Kemp 2011). A main reason for the popularity of radiation-pressure studies lies in a conflict that developed between the physical interpretations of the two forms of momentum density in equation (2.5), known as the Abraham–Minkowski controversy. The work of Poynting represents the first blow in a debate that has been continued to the present time by many of the papers reviewed in the above articles. The currently available experiments predominantly measure the Minkowski momentum, the result assumed by Poynting. However, several later calculations show that an Abraham momentum transfer should occur in some interactions between light and dielectrics, often for transfers that are difficult to measure in practice.

The interpretation of radiation pressure at the time of Poynting was unaffected by the first developments of the quantum and relativity theories. However, modern discussions of radiation pressure and electromagnetic field momentum involve quantum and relativistic ideas. The quantum theory initiated by Planck in 1900 showed that a quantization of the energy of harmonic motion in units of $\hbar \omega$ accounted for the observed spectral distribution of radiation by a thermal source. Many recent publications on radiation pressure describe light in terms of the photon, its quantum of energy as named by Lewis in 1928. The equivalence of mass and energy identified by Einstein in 1905 has clear relevance to the propagation of energy and momentum by light, important in radiation-pressure theory. Einstein (1917) also introduced the momentum of the light quantum, with magnitude

$$p = \frac{\hbar \omega}{c}$$

in free space, equivalent to the de Broglie relation between particle wavelength and momentum. This is analogous to the classical electromagnetic momentum density $U/c$ familiar to Poynting, but the photon momentum $p$ has the nature of a particle momentum rather than a field momentum density. Equation (5.1) converts similarly to the angular momentum carried by a single photon, analogous to the single-photon linear momentum in equation (6.1), as

$$s = \hbar,$$

where the conventional $s$ notation reflects the spin-like nature of circular polarization. Beth (1936) himself expressed the results of his measurements as assigning an angular momentum $\pm \hbar$ to each photon of left or right circularly polarized light, respectively.

The dielectric momentum densities in equation (2.5) convert to the single-photon momenta

$$p_M = \frac{\eta \hbar \omega}{c} \quad \text{and} \quad p_A = \frac{\hbar \omega}{\eta c}.$$
The controversy between the two formulations is conveniently discussed in terms of these single-photon values, with one of them sometimes claimed to be correct and the other as incorrect. The arguments in favour of one or the other are summarized by Barnett & Loudon (2010). At the simplest level, the Minkowski, but not the Abraham, momentum satisfies the de Broglie relation between particle momentum and wavelength. However, the Abraham momentum is supported by a general condition for conservation of the centre of mass energy when a single-photon pulse is transmitted through a block of transparent medium with antireflection coatings (Balazs 1953). The condition for an unchanged centre of mass energy relative to an identical pulse that travels the same distance outside the block requires a displacement of the block, equivalent to its acquisition of a linear momentum

\[ p_{\text{block}} = \left(1 - \frac{1}{\eta}\right) \frac{\hbar \omega}{c}, \]

so that \( p_{\text{block}} + p_A = \frac{\hbar \omega}{c} \) (6.4) during transmission of the photon. The total momentum with the pulse inside thus equals the photon momentum in free space before and after its transmission.

A resolution of the conflict between the two momenta by Barnett (2010) identifies the Abraham form as the kinetic momentum, which is associated with the motion of a dielectric specimen as a whole. This association is clearly illustrated by the example in equation (6.4), where the block displacement is a consequence of a velocity imparted to it and hence a kinetic momentum. The Minkowski form is identified as the canonical momentum, which is associated with the motion of objects embedded in the dielectric (Barnett 2010). The accompanying de Broglie relation led Schrödinger to represent the quantum-mechanical momentum by the operator \(-i\hbar \nabla\), and the usual position–momentum uncertainty relation then follows. This canonical momentum operator is a generator of translations and, for a body immersed in a transparent medium, Minkowski provides the relevant momentum, as observed in a range of experiments. The original Barnett paper should be consulted for a full understanding of the two momenta, and there is also useful subsequent discussion of their subtleties (Barnett & Loudon 2010; Baxter & Loudon 2010; Milonni & Boyd 2010; Kemp 2011). In summary, the expressions attributed to Minkowski and Abraham in equation (6.3) represent different physical forms of momentum, and both are correct.

The very early contribution by Poynting (1905b) should now be evaluated against the background of these two varieties of momentum measurement. More recent calculations of radiation pressure are based on evaluations of the Lorentz force (Loudon 2002), a method initiated long ago by Lorentz himself (Lorentz 1909, §§23 and 24). These calculations find that the force on a dielectric interface acts into the material with the higher refractive index, consistent with the Abraham form of momentum in the dielectric. A simple example is provided by the transmission of a single-photon pulse through an antireflection-coated surface, where a calculation based on the Lorentz force leads to the same results as in equation (6.4). Thus, \( p_{\text{block}} \) is the momentum transferred to a dielectric as the pulse passes through its surface and \( p_A \) is the momentum of the photon within the block. This inward force on the material disagrees with the outward force predicted by Poynting in equation (4.3) and with his assumption of a momentum equivalent to the Minkowski form inside a material. However, the
measured forces on mirrors suspended in dielectric liquids (Jones & Leslie 1978) and on charge carriers in semiconductors (Gibson et al. 1970) are consistent with the Minkowski form of photon momentum, as taken effectively by Poynting, and these observations agree with the Lorentz-force calculations (Loudon 2002; Loudon et al. 2005).

Poynting’s results for the three areas of radiation-pressure theory and experiment covered in §§3–5, frequently the first contributions in their fields, have largely withstood the tests of time. For the solar system, he was the first to propose the effect now known as the Poynting–Robertson drag, a force important in the motions of small particles closer to the Sun. His estimate of its size was in error by a numerical factor, but its nature was correctly identified. For the radiation pressure or linear momentum of light, Poynting presented the theories for a range of laboratory experiments. These were mainly carried out in collaboration with Guy Barlow and they verified the size of the optical momentum in free space, whose flux equals the energy density $U$ and whose density equals $U/c$. Poynting was also the first to recognize the property that circularly polarized light carries an angular momentum. His calculated value of the associated torque, given in equation (5.1), agrees with that currently accepted. Poynting’s papers are regularly cited in relevant current publications and his work from at least 100 years ago remains a most important contribution to radiation-pressure studies.

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References


Larmor, J. 1918 Retardation by radiation pressure, a correction. See *Shakespear & Barlow 1920*, 754–757.


