The implosion of cylindrical shell structures in a high-pressure water environment

C. M. Ikeda, J. Wilkerling and J. H. Duncan

The implosion of cylindrical shell structures in a high-pressure water environment is studied experimentally. The shell structures are made from thin-walled aluminium and brass tubes with circular cross sections and internal clearance-fit aluminium end caps. The structures are filled with air at atmospheric pressure. The implosions are created in a high-pressure tank with a nominal internal diameter of 1.77 m by raising the ambient water pressure slowly to a value, $P_c$, just above the elastic stability limit of each shell structure. The implosion events are photographed with a high-speed digital movie camera, and the pressure waves are measured simultaneously with an array of underwater blast sensors. For the models with larger values of length-to-diameter ratio, $L/D_0$, the tubes flatten during implosion with a two-lobe (mode 2) cross-sectional shape. In these cases, it is found that the pressure wave records scale primarily with $P_c$ and the time scale $R_i\sqrt{\rho/P_c}$ (where $R_i$ is the internal radius of the tube and $\rho$ is the density of water), whereas the details of the structural design produce only secondary effects. In cases with smaller values of $L/D_0$, the models implode with higher-mode cross-sectional shapes. Pressure signals are compared for various mode-number implosions of models with the same available energy, $P_c V$, where $V$ is the internal air-filled volume of the model. It is found that the pressure records scale well temporally with the time scale $R_i\sqrt{\rho/P_c}$, but that the shape and amplitudes of the pressure records are strongly affected by the mode number.

1. Introduction

The study of the implosion of shell structures that are filled with low-pressure gas and located in high-pressure
water environments is of practical interest both for the design of structures that are resistant to implosion and for the assessment of possible damage to nearby structures owing to implosion-generated waterborne pressure waves. Some applications include submarine vehicles and underwater piping, sensors, floats, lamps and acoustic devices.

Implosions typically occur when the ambient water pressure becomes high enough for the structure to reach its elastic stability limit. Early theoretical studies of this limiting pressure (also referred to as the collapse or implosion pressure, \( P_c \)) for cylindrical shell structures were conducted by Southwell [1], von Mises [2] and Sturm [3]. In these theories, for a given wall-thickness-to-diameter ratio (\( w/D_0 \), where \( D_0 \) is the outer diameter of the cylinder) and Poisson’s ratio for the material, a family of curves are predicted on a plot of \( P_c/E \) versus \( L/D_0 \), where \( E \) is the elastic modulus of the material and \( L \) is the internal length of the model. Each curve corresponds to a collapse mode designated by the number of lobes in the cross-sectional shape of the model as the unstable deformation begins. Figure 1 shows a family of these curves from Sturm’s theory for models created from a brass tube with \( w/D_0 = 0.022 \). (Sturm’s theory, which considers lateral and axial pressure loading on all sides of the cylinder, was used throughout this work. The end conditions, which apply to ends of the air-filled section of the cylinder, are clamped, that is, there is no deformation of the cylinder in the radial direction, and the slope of the deformed cylinder must be zero at all points of the circumference.) The implosion pressure and mode number for a given model are determined from the mode curve with the lowest pressure at the design value of \( L/D_0 \). There are numerous additional theoretical and experimental studies of the stability limit of cylindrical shell structures, including the work by Carman [4,5], Carman & Carr [6], Cook [7], Southwell [8], Winden [9] and Saunders & Windenburg [10].

The previously mentioned theories assume that the cross section of the cylindrical model is perfectly round. Park & Kyriakides [11] conducted a study on the presence of a dent on an infinitely long cylinder. They studied three different diameter-to-wall-thickness ratios, and dents were created with two different-sized indenters. These cylinders were measured for their ovality, \( \Delta = (D_{\text{max}} - D_{\text{min}})/(D_{\text{max}} + D_{\text{min}}) \), prior to implosion. It was found that the collapse pressure decreased with increasing \( \Delta \), and that \( \Delta = 0.1 \) causes the collapse pressure to be reduced to 50% of the value for a circular cylinder without defects. It was also found that for \( \Delta \) values less than 0.005, there was no notable change in the collapse pressure from the corresponding case of a round cylinder.

Relatively few studies have considered the more complex problem of the full implosion of the structure with the generation of pressure waves. This phenomenon involves large material deformations, material fracture and tearing (in some cases), and compressible flow. Orr &
Schoenberg [12] conducted laboratory and field tests of the implosion of glass spheres that were weakened by grinding a flat on one side of the sphere. The amount of weakening was quantified by a flat thickness ratio defined as the ratio of the wall thickness at the flat location to the full wall thickness. It was found in the laboratory experiments that a linear relationship exists between the implosion pressure and the flat thickness ratio, whereas, in field tests, this relationship was not duplicated, perhaps owing to insufficient data. From measurements of the far-field pressure waves, it was found that the percentage of the total available energy (equal to $P_c V_w$, where $V$ is the air-filled internal volume of the sphere) that was converted to acoustic energy decreased with decreasing diameter of the spheres (21% for 43.2 cm diameter, 17% for 33 cm diameter and 15% for 23.5 cm diameter).

Turner [13] examined the underwater implosion of glass spheres with both experiments and computer simulations. Four identical spheres were imploded in a laboratory pressure tank, and the pressure versus time at three near-field locations was recorded. The pressure records for the four implosions were nearly identical. Computer simulations with the implosion modelled as a spherical bubble created peak pressures at the measurement sites that over predicted the measured peak pressures by 44%. On the other hand, simulations with the sphere modelled as a layer of sand initially supported by a rigid shell produced accurate pressure signatures when an effective failure propagation velocity along the surface of the rigid shell was taken as 275 m s$^{-1}$.

Ambrico & Turner [14] investigated the pressure waves resulting from the underwater implosion of models created from aluminium cylinders. The models imploded in mode 2. Turner & Ambrico conducted four experimental runs and numerical computations using a fluid–structure interaction finite-element code. In the experiments, pressure sensors were placed at three positions along the model length, all at a radial distance of four model diameters from the model centreline. It was shown that the computed and measured pressure signals agreed well. The computational model gave the shape versus time of the collapsing cylinder, which allowed the authors to relate various features of the pressure signals to the behaviour of the shell walls, even though there were no direct visual observations of the models during the experiments.

In this study, laboratory experiments on the implosion of cylindrical shell structures are conducted in a water-filled high-pressure tank (nominal internal diameter of 1.77 m) with the aim of exploring the physics of the implosion process and the generation of pressure waves. The measurements include high-speed movies of the imploding models and pressures at eight locations around the midplane of the model. The study can be broken into two parts. In the first part, four model designs, all of which implode with a mode-2 cross-sectional shape, are studied. The models include a range of materials and geometry that result in implosion pressures ranging from about 7 to 28 bar above atmospheric pressure. In the second part, the effect of mode number on the implosion process is explored. These experiments are performed with models that have the same energy available to drive the implosion process ($P_c V_w$, where $V$ is the air-filled internal volume of the model) while varying the diameters and length-to-diameter ratios of the models. In all of these studies, the pressure waves are compared, and the high-speed movies are used to shed light on the shell motion events that create various features of the pressure records.

The remainder of the paper is divided into three sections. In §2, the details of the experimental set-up, model construction and test procedures are presented. The presentation and discussion of the results are given in §3. Finally, the conclusions of the study are given in §4.

2. Experimental methods

(a) Facilities

The implosion experiments were conducted in a steel high-pressure tank that consists of a vertically oriented cylindrical middle section (height 0.85 m and internal diameter 1.77 m) capped by ‘elliptically’ shaped top and bottom sections (figure 2a,b). The maximum internal height of the tank is 1.77 m. The tank is rated for static internal pressures up to 40 bar (gauge). There are 10 window ports with diameters of 10.2 cm in the tank walls; eight of the ports are located on the circular horizontal midplane of the tank, whereas two ports are located on the top (figure 2).
During the experiments, three of the ports along the midplane of the tank were fitted with glass windows, whereas the other seven were fitted with steel plates. There are also two 50.8 mm diameter openings, one in the top of the tank and one in the bottom. The opening at the bottom is fitted with a ball valve and is used for draining the tank. The opening at the top is fitted with a piping tree, including openings for two pressure gauges, openings to pressurize and depressurize the tank with nitrogen gas and a 50.8 mm opening with a ball valve for filling the tank with water.

A quick-opening manhole on the side of the tank allows for access to the interior for the placement of models and pressure probes. The models and equipment were suspended inside the tank on fishing lines (monofilament nylon, rated for a tension load of 178 N), which were attached to 12 eye bolts welded to the inside surface of the tank. Cables for the pressure sensors entered the tank via high-pressure feed-through fittings that were placed in ten of the 25.4 mm diameter couplings located on the bottom of the tank.

It is important to consider acoustic reflections inside the tank during the implosion experiments. If an imaginary sphere of radius 0.885 m were centred inside the tank, then it would touch the tank walls along a circle at the midplane of the cylindrical middle section of the tank and at single points on the top and bottom. The remaining parts of the internal surface of the tank, which comprises nearly all of the internal surface, are farther than 0.885 m from the tank centre. (The longest distance from the centre to a point on the wall is \( \approx 0.92 \) m.) Given a sound wave speed in water of 1482 m s\(^{-1}\) at 20°C, the acoustic reflection time for a wave travelling from the centre of the tank to the closest points on the tank wall and back to the centre is approximately 1.19 ms. However, because the remainder of the internal surface is farther from the tank centre, and because the tank wall internal surface normals for nearly all of the internal surface are not directed towards the centre, most of the energy from spherical pressure waves generated at the centre of the tank returns after multiple reflections with reduced amplitude and is spread out over times greater than 1.19 ms.

**Figure 2.** Schematic drawings of the high-pressure tank. (a) Side view with piping tree shown on top. (b) Plan view showing the eight viewing windows on the horizontal midplane of the tank and the set-up of the high-speed digital movie camera and lights.

(b) Cylindrical model construction and measurements

The cylindrical models were assembled from thin-walled tubes, end caps, washers and screws. A schematic drawing of the arrangement of the tube and end caps is shown in figure 3. To construct a model, the tube was first cut to length, \( L_t \), using a lathe. The end caps were fabricated...
Each cylindrical model was mounted vertically in the centre of the tank using fishing lines threaded through the holes in the two screw heads on the model and the eye bolts on the inside surface of the tank, see §2c for details. The cylindrical models were primarily constrained in the vertical direction, so the model could move horizontally and deform freely during the implosion event. In addition, the vertical constraint, while strong enough to hold the model against the forces of gravity and buoyancy, did not appear to impede shortening of the model in the vertical (axial) direction during implosion. This mounting system was chosen to simulate, as closely as possible, the implosion of untethered models, as would typically be found in the field for devices such as autonomous underwater vehicles.

The outer diameters of the tubes were measured both before and after the tubes were cut to length. In the early experiments, the diameters were measured photographically, and in the later experiments, they were measured using a micrometer. A reference line was first drawn onto the tube along its axis. The diameter was measured at axial distances spaced 150 mm apart before the tubes were cut and 25 mm apart after they were cut. These measurements were made at 12 equally spaced angular positions around the tube. Comparison of the measurements before and after cutting revealed that there was no measurable change in tube diameter or cross-sectional shape caused by the cutting process.

The tube wall thickness was measured using a micrometer with a ball-tip attachment. These measurements could only be made at the tube ends because of the limitations of the micrometer. As the end caps penetrate 0.5D₀ into the tubes, the micrometer measurements were not made over the central air-filled section of the tube. The ball tip was placed on the inside surface of the tube
Figure 4. A schematic plan view showing the cylindrical model surrounded by the eight midplane pressure sensors. In this schematic, the dots represent the position of the Tourmaline crystals located inside the pressure sensors. The sensors are placed, so that the crystals are all located at a radius of \(2.5R_0\) away from the centre of the cylindrical model, where \(R_0\) is the outer radius of the cylindrical model.

to allow for one point of contact on the interior of the tube. The wall thicknesses were measured near each end of the tube at three locations (increments of 6 mm along the axis) and at increments of 20° around the tube.

(c) Implosion measurements

The ambient water pressure in the tank was obtained from measurements of the pressure of the nitrogen gas using a slow-response pressure transducer (Honeywell, model TJE, range 0–68 bar, resolution 0.068 bar). This sensor was calibrated by the manufacturer before and after the experiments were performed. The calibration curve changed by less than 0.3% at typical operating pressures (≈7–24 bar). Owing to the hydrostatic pressure gradient in the water, the pressure at the model is about 0.08 bar higher than the pressure in the nitrogen gas.

The dynamic pressure field in the water was measured with eight underwater Tourmaline-crystal-based sensors (PCB Piezoelectronics, model nos 138A02 and 138A01). These sensors were attached to a thin stainless steel support frame, so that the sensing elements formed a ring of radius \(r_s = 2.5R_0 = 1.25D_0\) (figures 2 and 4). The model was positioned at the centre of this ring using the following arrangement. As shown in figure 2, the support lines coming from the tank walls were connected to two rings positioned above and below the model, along its axis. Both the model and the support frame were attached to these rings via fishing lines, thus helping to maintain the relative alignment between the sensors and the model. The pressure sensors have a rise time of 1.5 μs and a useful range of ±138 bar. Each sensor was connected to a signal conditioner that gives a ±10 V analogue output signal. The signals were then sent to simultaneous sample-and-hold analogue-to-digital (A/D) converters with a sample rate of 2 MHz per channel. A program in LABVIEW was used to record the final output data into the computer and convert the voltage signal into units of pressure. The sensors respond only to rapid changes in pressure; any non-zero static voltages in sensor output immediately before an implosion event were subtracted from the record during the implosion for each sensor.

The ambient pressure at which a given model will implode is not known precisely. Thus, in order to capture the pressure signals, the A/D system operates continuously filling the system memory in a first-in-first-out mode, holding more than 2 s of data at all times. Then, when the
operator hears a noise from within the tank, indicating that the model has imploded (the pressure waves hitting the tank walls from within), she manually triggers the A/D system, which is set to save 2 s of data ending at the moment the trigger was initiated.

The motion and deformation of the model during an implosion was recorded with high-speed photography, using a Phantom V7.2 camera (Vision Research, Inc.) with an 800 × 600 pixel sensor. The camera was set to record 27 000 pictures per second at an image size of 256 × 256 pixels and was post-triggered in the same way as the A/D system. A timing system triggers the A/D and camera simultaneously, so that the pressure signals over the 37 µs period that each image is captured can be examined. The camera is positioned to view the implosion through one of the glass windows in the midplane of the tank, and the two adjacent windows are used to project light from two 650 W flood lamps onto the model (shown in figure 2b). The inside surface of the tank is painted black to provide contrast. From the movies of the implosions, both qualitative information and quantitative measurements can be extracted. The qualitative information includes the orientation of the collapsed model with respect to the tank and the state of the model at any time. Quantitative measurements include the implosion time of the model.

(d) Test procedure and plan of experiments

To perform an experiment, the model and pressure sensors are first placed in the tank. The front manhole and drain are closed, and the tank is filled with water through the 50.8 mm diameter ball valve in the piping tree at the top of the tank until ≈5.7 l of air remains. The ball valve is then closed, and the tank is slowly pressurized with nitrogen gas through a solenoid valve in the piping tree. Immediately after the model implodes, anywhere from 2 to 5 min after the pressurization process begins, the nitrogen inlet valve is closed, and a separate valve is opened to depressurize the tank. After each experiment, the water inlet valve is opened, and the tank is drained through the valve at the bottom.

Table 1 shows the run designation; implosion mode number observed (n); the material, outer diameter (D₀) and wall thickness (w) of the tube; the ambient implosion pressure (P_c) and the available energy (P_cV), where \( V = 0.25\pi(D_0 - 2w)^2L \) is the internal, air-filled, volume of the model. These experiments were done in two series. In the first series, models were created from the same tube stock but with different lengths. As the model length is reduced, the internal volume decreases and, as can be seen from figure 1, the mode number of the instability increases as does \( P_c \). It was found that, for a given tube, it is possible to choose several model lengths so that the models imploded in modes 2, 3 and 4, but with the same available energy. Details of the model selection process and the results for this series of experiments are given in §3b(i).

In the second series of experiments, two model designs were made from brass tubes with different diameters. The model lengths were chosen so that one imploded in mode 2, whereas the
other imploded in mode 4, and so that both $P_c$ and $V$ (and therefore $P_cV$) were the same for both models. Details of the model selection process and results for this series of experiments are given in §3b(ii).

Both the above-described series 1 and series 2 experiments on the effect of mode number required accurate curves of $P_c/E$ versus $L/D_0$; these were obtained experimentally, as described in detail in §3b(ii), by performing implosions of a number of additional models. In these experimental runs, only $P_c$ was recorded.

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Table 1. Summary of models. Characteristics of the primary models imploded in this study. In the names given for each model, the first digit is either an 'A' for aluminium or 'B' for brass. The mode number follows. For the brass models, the third digit is a 'D' for diameter followed by numbers representing the diameter in millimetres, while for the aluminium models, the third digit is an 'A' for alloy followed by a '2' for the 2024 alloy or a '3' for the 3003 alloy. Finally, there is an 'r' followed by a number representing the run number for the given geometry and material. The volume $V$ used in computing $P_cV$ is the volume of air in the model, i.e. $V = 0.25\pi(D_0 - 2w)^2L$. 

$P_cV = \frac{\rho}{E} \frac{D_0^3}{36} \frac{1}{w^2} \left( \frac{1}{L} - \frac{2}{D_0} \right)^2$.
3. Results and discussion

Here, the pressure fields generated during the implosions are examined in the light of the model behaviour as seen in the high-speed movies. As discussed already, the following is divided into two subsections with results from the experiments on the effect of model characteristics on mode-2 implosions presented in §3a and the effect of implosion mode number in §3b. This latter subsection is further divided into results from experiments using models with the same outer diameter presented in §3b(i) and results from experiments using models with different diameters presented in §3b(ii).

(a) The effect of model design on mode-2 implosions

As can be seen in table 1, there are four model designs that imploded in mode 2. Two of the designs are made from brass tubes and have different geometries (B2D25 and B2D16), and two have the same geometry but are made from tubes of different aluminium alloys (A2A2 and A2A3). Sample records of pressure \( P(r_s = 2.5R_0) - P_c \) versus time \( t \) from the eight sensors around the midplane of the model are shown for a single implosion (case B2D25r2, \( D_0 = 2.54 \text{ cm}, P_c = 7.1 \text{ bar} \)) in figure 5. The time \( t = 0 \) is taken as the average of the times of the maximum pressure \( (P_{\text{max}}) \) for the eight sensors and the record begins at \( t \approx -1.2 \text{ ms} \) when the pressure, which is dropping slowly at first, reaches \(-2\% \) of the average peak pressure at \( t = 0, \bar{P}_{\text{max}} \). In addition to the pressure records, images of the model from the high-speed movie are shown at three times during the implosion. The pressure records and model behaviours for all the other mode-2 implosions are qualitatively similar to those shown in the figure.

In the initial period \( (t < -0.8 \text{ ms}) \), the tube wall begins to move inwards, with the maximum tube wall displacement occurring midway between the two ends of the model, in the plane of the ring of pressure sensors. During this time, the pressures measured at the sensor locations drop relatively slowly. This negative pressure region has a minimum of about \(-24 \text{ bar} \) at \( t \approx -0.4 \text{ ms} \).

At about \( t = -0.2 \text{ ms} \), a narrow pressure spike occurs with a maximum value of about 2 bar above ambient and a width of about 0.006 ms at 50\% of the local peak pressure. Examination of the high-speed movie of the implosion indicates that this peak occurs at about the time of the first impact of the tube walls at the centre plane of the model. After this narrow spike in pressure, the pressure rises again, reaching its absolute maximum of about 4 bar at \( t = 0 \). The width of this peak is about 0.085 ms at 50\% of the peak pressure. At this point in time, the high-speed movie indicates that the tube has reached its minimum, i.e. fully imploded, internal volume. (It should be kept in mind that there are about 400 pressure samples but only about five movie frames in the 0.2 ms time interval from the initial pressure spike to the main pressure peak. Thus, statements about the timing of events from the movies relative to the pressure records have limited accuracy.) Up to this point in time, the readings from the eight pressure sensors are qualitatively similar to those shown in the figure.

Contour plots of pressure on a polar coordinate plane with time in the radial direction and probe position around the model in the azimuthal direction for the three models of design B2D2 are displayed in figure 6a–c. The data in figure 6a are the same as those in figure 5. The grey scale of the surface gives the pressure. The straight line plotted on top of the contours shows the orientation of the post-implosion flattened surface of the model, and this line gives a reference with which to interpret the pressure contours. The orientation of the imploded model was measured from the high-speed movies and the accuracy is estimated to be ±15\°. The pressure impulse for each probe, the integral of the pressure over a given time interval, is also plotted in figure 6. Two integration time intervals were chosen: one from the first zero up-crossing after the small pressure spike to the first zero down-crossing after the main pressure peak (plotted as black squares) and one from the beginning of the initial low-pressure region to the first down-crossing after the main pressure peak (plotted as black diamonds; figure 7). The points are plotted at the azimuthal locations of the respective pressure probes, and the radial distance is proportional...
Figure 5. Pressure \( P(r_s, t) - P_c \) versus time \( t \) for the implosion of model B2D25r1 (brass 260, \( D_0 = 25.4 \text{ mm}, L = 231 \text{ mm} \)). The model imploded in mode 2 at an ambient pressure of \( P_c = 7.1 \text{ bar} \). Each line is the pressure signal from one of the eight probes surrounding the model at its midplane. The time \( t = 0 \) is taken as the average of the times of the maximum pressure for the eight sensors. The three still images shown in the figure are from the high-speed movie taken during the implosion. These images were taken at the times corresponding to the minimum pressure, the first small positive pressure spike and the maximum positive pressure peak. The images show the centre section of the model (the end caps are not visible). The bright objects at the mid section of each image are the probes and probe-mounting ring.

Figure 6. Pressure contours (shown as grey level) on a polar plot with time in the radial direction and angular position of the sensors around the model in the azimuthal direction. For the contours, the origin corresponds to the time when pressure first falls to \(-2\%\) of the peak pressure. Data are shown for the three mode-2 implosions of the brass model design with \( D_0 = 25.4 \text{ mm} \) (B2D25r1, B2D25r2, B2D25r3), and the values at the circumference of the polar plot are (a) a time of 4 ms and 20 Bar-ms (impulse units), (b) a time of 4 ms and 19 Bar-ms, and (c) a time of 5 ms and 25 Bar-ms. The raw data in subplot (a) are the same as those in figure 5. The straight line plotted on top of the contours shows the orientation of the flat surface of the imploded model, accuracy \( \approx \pm 15^\circ \). Symbols (located at radial positions proportional to positive values of the quantity): filled squares, pressure impulse computed for the time interval from the up-crossing before the maximum pressure to the first down-crossing after the maximum pressure; filled diamonds, pressure impulse from the time when the pressure drops by 2\% of the maximum pressure to the first down-crossing after the maximum pressure. See figure 7 for time interval definitions.
Figure 7. Pressure versus time for each of the eight pressure sensors for a single implosion. The definitions of the time intervals for the calculations of the two pressure impulses plotted in figure 6 and similar plots are shown. The plotting symbols in the annotation on the plot are the same as those used in figure 6 and later contour plots.

Figure 7. Pressure versus time for each of the eight pressure sensors for a single implosion. The definitions of the time intervals for the calculations of the two pressure impulses plotted in figure 6 and similar plots are shown. The plotting symbols in the annotation on the plot are the same as those used in figure 6 and later contour plots.

to the impulse magnitude. As can be seen in the plots, there is little or no correlation of either impulse or the location of the peak pressure with the orientation of the collapsed model.

In order to compare the pressure fields generated with the four model designs with mode-2 implosions, records of pressure versus time for one implosion run for each design (B2D25r2, B2D16r1, A2A2r2 and A2A3r1) are shown in figure 8. For each of the plots, the pressure signals from all eight probes are shown, and the time axis is defined such that the maximum positive pressure peak ($P_{\text{max}}$) occurs at time $t = 0$. The records begin when the pressure falls to $-0.02P_{\text{max}}$, as described earlier for figure 5. As can be seen in the plots, the variations of the minimum and maximum pressures from one probe to another are approximately the same percentage of the average peak pressure for each of the four models. Photographs of four models after implosion, one from each model design, are shown on the right-hand side of the photograph in figure 9. The collapsed tube shapes are similar; however, more tearing of the metal is observable at the internal edge of the end caps in the two aluminium tubes. In addition, it can be seen from figure 8 that from one model design to another, the time scales and pressure scales vary dramatically. Model B2D25r2 has the smallest maximum pressure and the longest collapse time, defined as the time between when the pressure first drops to $-0.02P_{\text{max}}$ to the time when it reaches its maximum value. Model B2D16r1 has a higher amplitude and the shortest collapse time. Finally, models A2A2r2 and A2A3r1 have the highest maxima and the second longest collapse times.

In order to compare the pressure signals from all of the mode-2 implosions, pressures from the eight sensors at each time instant were averaged to create one record of average pressure versus time for each run, and the resulting data for all 13 implosions are shown on a single plot in figure 10. As can be seen from table 1, the B2D25, B2D16, A2A2 and A2A3 models include a fairly wide range of parameters: three materials (brass 260, aluminium 2024 and aluminium 3003), outer diameters ranging from 16.6 to 38.1 mm, $L/D_0$ ranging from 6.33 to 9.15, $w/D_0$ ranging from 0.013 to 0.0234, and implosion pressures ranging from $P_c = 7.1$ to 28.3 bar. As can be seen from the figure, the time scales of the pressure signals (for instance, the time from the first small pressure spike to the main pressure peak) are exceedingly consistent from one run to another for a given model design. The run-to-run variations of the magnitudes of the pressure signals for the two model designs constructed from brass tubes are relatively small: for the B2D25 and B2D16 models, the ranges of the maximum pressure signal over the maximum pressure averaged over
Figure 8. Pressure \((P_{rs}, t) - P_c)\) versus time \((t)\) for single runs for each of the four model designs that implode in a mode-2 cross-sectional shape (a) model B2D25r1 \(P_c = 7.3\) bar, (b) model B2D16r1 \(P_c = 26.6\) bar, (c) model A2A2r2 \(P_c = 26.6\) bar and (d) model A2A3r1 \(P_c = 28.3\) bar). The model characteristics can be found in table 1. For each implosion, records are shown for all eight pressure sensors. The time \(t = 0\) is taken as the average of the times of the maximum pressure for the eight sensors in each run.

all the runs for a given model design were 14.0% and 6.2% (each over three runs), respectively, whereas the range of collapse pressures, \(P_c\), were only 5.5% and 1.5%, respectively. For the models constructed from aluminium tubes, A2A2 and A2A3, which have the largest diameters of all the models, the variations in peak pressure were 55% (over five runs) and 23.6% (over two runs), respectively, whereas the variations in the ambient collapse pressure \(P_c\) were only 12.8% and 1.4%, respectively.

To explore the physical processes that control the variations in time and pressure scales from one model design to another, dimensional analysis was used with the data in figure 10. In creating the non-dimensional pressure and time for a new plot, it was theorized that as the tube cross section becomes out of round in the beginning of the implosion, the shell structure becomes exceedingly weak compared with the pressure from the surrounding water. If this idea is valid, the pressure versus time data might successfully be scaled with parameters used to scale collapsing cavitation bubbles. In the typical scaling for cavitation bubbles, the pressure is
non-dimensionalized by the pressure difference causing the implosion, $P_c$. Thus, dimensionless pressure is given by

$$P' = \frac{P - P_c}{P_c}. \quad (3.1)$$

In addition, in bubble dynamics, the time is non-dimensionalized by the time scale of the implosion of a spherical bubble, $T_b$ which is proportional to $R_i \sqrt{\rho / P_c}$, where $\rho$ is the density of water and $R_i$ is the radius of the bubble, or in the present case, the inside radius of the tube. Thus, dimensionless time is given by

$$t' = \frac{t}{R_i \sqrt{\rho / P_c}}. \quad (3.2)$$

The curves of $P - P_c$ versus $t$ in figure 10 are replotted in $P' - t'$ coordinates in figure 11. The non-dimensionalization of the time axis works quite well up to about $t' = 0.4$. The non-dimensionalization of the pressure shows more variation. The magnitude of the peaks ranges...
between 0.5 and 0.65, and the minimum of the initial dip in pressure at \( t' \approx -1 \) ranges between \(-0.3\) and \(-0.2\). To explore this effect further, the peak pressure for each run is plotted in figure 12 against \( M' \), the ratio of the mass per unit length of the metal tube divided by the mass of water displaced by the tube per unit length,

\[
M' = \frac{\rho_m \pi (R_0^2 - R_i^2)}{\rho \pi R_0^2},
\]

where \( \rho_m \) is the density of the tube material. The idea behind this scaling is that while the stiffness of the shell may not be important during the mode-2 implosions, perhaps the added inertia of the shell might decrease the tube wall acceleration and the wall velocity just before impact, thereby decreasing the maximum pressure generated in the water. As can be seen from the plot, the average dimensionless peak pressure is nearly the same for the two models created from brass tubes, but about 40% higher for the two models constructed from aluminium tubes. The plot also shows that the scatter in the amplitude of the curves in figure 11 is primarily the result of the variability in the implosions of the aluminium tubes, all of which have identical design parameters, with the exception of the properties of the two alloys and the collapse pressures, which vary by only 8% over all the aluminium models.

Four contour plots of pressure and the pressure impulses values with one plot for each of the four model designs are given in figure 13. (The plot in figure 13a is identical to that shown in figure 6a.) As can be seen in the figure, there is little apparent correlation between the orientation of the surface of the imploded model (given by the straight line) and the azimuthal location of the maximum pressure or the maximum impulse. To examine this finding quantitatively, a plot of the distribution of the relative orientation angle (\( \Delta \theta \)) between the model and the maximum positive impulse is given in figure 14a and between the model and the maximum pressure is given in figure 14b, with data shown for all 13 implosions. The angles are defined to have a range from 0 to 90°. As can be seen in the plot, there is no evidence of a preferred value for \( \Delta \theta \) in either plot. It should be kept in mind, of course, that these pressure measurements are taken only at a radial distance of \( 2.5R_0 \) from the centreline of the model.
Figure 12. Dimensionless maximum pressure, \((\bar{P}_{\text{max}} - P_c) / P_c\), versus mass ratio, \(M' = \rho_m \pi (R_0^2 - R_i^2) / (\rho \pi R_0^2)\) for all 13 models that imploded in a mode-2 cross-sectional shape.

Figure 13. Pressure contours (shown as grey level) on a polar plot (see the caption of figure 6 for definitions and details). Data from single implosion events from all four model designs that implode in mode 2 are shown: (a) model B2D25r1, and the values at the circumference of the polar plot are at a time of 4 ms and 20 Bar-ms; (b) B2D16r1, and the values at the circumference of the polar plot are at a time of 6 ms and 36 Bar-ms; (c) A2A2r1, and the values at the circumference of the polar plot are at a time of 4 ms and 66 Bar-ms; and (d) A2A3r2, and the values at the circumference of the polar plot are at a time of 4 ms and 77 Bar-ms.
Figure 14. The distribution of the relative angle ($\Delta \theta$) between the flattened surface of the imploded cylindrical shell model (mode-2) and (a) the azimuthal angle of the probe with the largest impulse of the pressure peak and (b) the azimuthal angle of the probe with the maximum peak pressure during the implosion.

(b) The effect of mode number

In exploring the effect of mode number on the pressure field during the implosion of a cylindrical model, the choice of model designs is somewhat complicated. In the present experiments, it was decided to compare implosions with the same available energy, $P_c V$. During an implosion, this available energy is used to deform the model, create kinetic energy both in the model and the surrounding water, generate sound waves and compress the internal air volume. Two methods of choosing models with different implosion mode numbers but the same available energy were used. In the first, the models were created from different lengths of the same tube stock, brass tubes with $D_0 = 25.4$ mm. These models have different values of $P_c$ and $V$, but nearly identical values of $P_c V$. In the second method, the models were created from two tube stocks (brass with $D_0 = 25.4$ mm and $D_0 = 16$ mm) and designed to have the same $P_c$ and the same $V$. These two series of experiments are described in the following subsections.

(i) Series 1: models created from the same tube stock

In order to select the lengths ($L$) of models that implode with the same available energy, $P_c V$, and are created from the same tube stock, it is first necessary to have reliable curves of $P_c$ versus $L/D_0$. In preliminary testing, it was found that the theoretical curves shown in figure 1 were not accurate enough in the region of higher-mode implosions, particularly near the mode transitions, to achieve this available energy equality. Thus, a set of experiments was performed to determine the mode number and implosion pressure for models of various lengths. The resulting data are shown on a plot of $P_c/E$ versus $L/D_0$ in figure 15 (the models used in these tests are not included in table 1). The mismatch between the theory and the experiments is particularly evident at the boundary between the mode-3 and -4 implosions for brass tubes with $w/D_0 = 0.013$, where the transition boundary is found to be about $0.2 \times 10^{-5} P_c/E$ higher than predicted. The theoretical and experimental data for the model designs constructed with the 25.4 mm diameter brass tubes in this plot are then replotted in $P_c V$ versus $L/D_0$ coordinates in figure 16. As can be seen from the plot, a horizontal line can intersect the theoretical curve in several places, and tube designs corresponding to each intersection point would have the same values of $P_c V$. Because the theoretical curve was inaccurate, the horizontal line is drawn through
Figure 15. The theoretical implosion boundaries are shown for the models constructed from the 25.4 mm diameter brass tubes (solid curve), the 16.6 mm diameter brass tubes (dotted curve) and the 38.1 mm diameter aluminium tubes (dashed curve) on a plot of $P_c/E$ versus $L/D_0$. Results from the experiments are plotted as individual data points. The largest discrepancies between the theoretical curves and the experimental data are on the left side of the plot where $P_c/E$ changes rapidly with $L/D_0$.

Figure 16. Available energy, $P_cV$, versus model aspect ratio, $L/D_0$, for the 25.4 mm diameter brass tubes. The curved solid line is from the linear stability theory (figure 1), and the data points are from experiments. The horizontal solid line is drawn at the available energy value chosen for further study, $P_cV = 83.0$ N m.

The experimental data points at $P_cV = 83$ N m. The model designs selected for further experiments were B2D25 ($L/D_0 = 9.1$, three models), B3D25 ($L/D_0 = 3.3$, five models) and B4D25 ($L/D_0 = 2.3$, seven models). From the final experiments, the average and standard deviation of the values of $P_cV$ for each model design were $82.1 \pm 2.25$ N m, $83.1 \pm 4.7$ and $83.0 \pm 6.1$ N m, respectively (table 1).

Figure 17 contains three plots of $P - P_c$ versus $t$, with one plot for a representative implosion of a model from each of the three model designs in this series. In each plot, data from all eight pressure sensors are shown. As noted in the figure and table 1, the average collapse pressures are 7.3 bar for the mode-2 implosion, 20.7 bar for the mode-3 implosion and 26.1 bar for the mode-4
Figure 17. (a–c) Pressure ($P(r_s, t) - P_c$) versus time ($t$) for single runs for each of the model designs B2D25 (mode-2 implosion, $P_c = 7.3$ bar), B3D25 (mode-3 implosion, $P_c = 20.7$ bar) and B4D25 (mode-4 implosion, $P_c = 26.1$ bar). Records from all eight sensors are shown in each plot. These models were created from the same 25.4 mm brass tube stock, and their lengths were chosen so that the same energy, $P_cV$, was available for each implosion. The arrows and corresponding call-outs in (b) and (c) indicate the times when the images in figure 18 were taken. The model characteristics can be found in table 1. The time $t = 0$ is taken as the average of the times of the maximum pressure for the eight sensors in each run.

implosion. As can be seen in the figure, there is considerably higher variation in the pressure versus time records from one probe to another in the mode-4 implosion, figure 17c, than in the mode-2 and -3 implosion records (figure 17a,b, respectively). Three images from each of the high-speed movies taken along with the pressure records in figure 17b,c are shown in figure 18. In these images from a single viewpoint, the complex three-dimensional nature of the model deformation makes it difficult to recognize events that would explain the corresponding features in the pressure records. The variations of the collapse times and pressure amplitudes with mode number will be discussed below.

Photographs of selected models imploded in this series of tests (B2D25 (one model), B3D25 (two models) and B4D25 (two models)) can be found in figure 9. As mentioned above, in mode-2 implosions, the final collapsed shape of the model is relatively simple and consistent from one run to another with the same model design. In addition, as can be seen in the photographs, the long axis of the model is still relatively straight after implosion. The mode-3 and -4 post-implosion model shapes are more complex. First of all, the cross-sectional shape of the model at the midway point between the end caps never had equally spaced lobes. Rather, in the mode-3 implosions, two of the lobes are typically found quite close together. In some mode-4 implosions, three of the lobes were very close together, while in others, the fourth lobe appeared to be only partially formed. The second main complexity of the imploded model shapes was that the tubes were bent along their long axes. This was particularly pronounced in the mode-4 implosions.
Figure 18. Selected images from the high-speed movies taken simultaneously with the pressure records in figure 17b (mode-3 implosion) and figure 17c (mode-4 implosion).

Figure 19. Pressure contours (shown as grey level) on a polar plot (see the caption of figure 6 for definitions and details): (a) a mode-3 implosion, model B3D25r3, and the values at the circumference of the polar plot are at a time of 4 ms and 2.25 Bar-ms and (b) a mode-4 implosion, B4D25r1, and the values at the circumference of the polar plot are at a time of 4 ms and 2 Bar-ms.

Contour plots of pressure and values of the pressure impulses are shown in figure 19a,b for a mode-3 implosion and a mode-4 implosion, respectively. Owing to the complexity and run-to-run variability of the cross-sectional shape of the model, it was not possible to record a reliable model orientation on the polar plot.

In order to compare the pressure versus time records for the three model designs in this series of tests, the pressures from the eight sensors at each time instant were averaged to create one record of average pressure versus time for each implosion. Such records for all 15 implosions are plotted in dimensional form \((\bar{P} - P_c) \text{ versus } t\) in figure 20 and in non-dimensional form \(((\bar{P} - P_c)/P_c \text{ versus } t/(R_i\sqrt{\rho/P_c})\)) in figure 21. As in figures 10 and 11, the pressure records from the various implosions are aligned in time with the peak pressure at \(t = 0\). The pressure records plotted in both figures show that the run-to-run variability in pressure at any time increases dramatically with mode number. The plot in dimensional variables in figure 20 shows a wide variation in collapse times and peak pressures. In the dimensionless plot in figure 21, the change in time scale has created a fairly universal implosion time of about 2.5, whereas the dimensionless peak pressure varies substantially from run to run and model design to model design. Thus,
Figure 20. Pressure versus time for each of the mode-2 (solid line), mode-3 (dotted line) and mode-4 (dashed line) implosions with models constructed from the same 25.4 mm diameter tube stock with lengths selected to have the same energy ($P_cV$) available for the implosions. Each curve is the average of the eight pressure sensors in the midplane around the model for one implosion.

Figure 21. Dimensionless pressure, $(\bar{P} - P_c)/P_c$, versus dimensionless time, $t/(R_i\sqrt{\rho/P_c})$, for each of the mode-2 (solid line), mode-3 (dotted line) and mode-4 (dashed line) implosions with models constructed from the same 25.4 mm diameter tube stock with lengths selected to have the same energy ($P_cV$) available for the implosions. Each curve is the average of the eight pressure sensors in the midplane around the model for one implosion.

The data show that the concept of the implosion behaving primarily as an imploding bubble is less appropriate when considering implosions with higher-mode shapes. This result is likely due to the increased strength of the model structure as it folds and implodes upon itself in the higher-mode cases.

(ii) Series 2: models with the same $V$ and $P_c$

Models that imploded with the same collapse pressure, $P_c$, and the same initial internal air-filled volume, $V$, were constructed from brass tubes with diameters of 16.6 and 25.4 mm. In order to
select the lengths of the models, the results of the preliminary testing discussed in §3b(i) were used. The length of the model design constructed from the 16.6 mm diameter tube stock was chosen so that it imploded in mode 2. Because the implosion pressure for mode 2 becomes nearly independent of the model length for $L/D_0 > 7$ ($P_c \approx 26$ bar for this tube stock), it was possible to keep $P_c$ constant while varying $V$. The model design constructed from the 25.4 mm diameter tube stock imploded with $P_c \approx 26$ bar when $L/D_0 = 2.3$. These implosions had a mode-4 cross-sectional shape. The length of the 16.6 mm diameter model design was then chosen so that the internal volume was equal to that of the 25.4 mm diameter models. Based on this analysis, the model designs selected for further experiments were B2D16 ($L/D_0 = 9.2$, three models) and B4D25 ($L/D_0 = 2.3$, seven models). From the final experiments, the average and standard deviation of $P_c V$ for each model design were $81.3 \pm 0.6$ N m for B2D16 and $82.9 \pm 6.1$ N m for B4D25 (table 1).

Figure 22a,b contains plots of $P - P_c$ versus time, for single representative mode-2 and mode-4 implosions, respectively. In each plot, data from all eight pressure sensors are shown. The average implosion pressures are 26.7 bar for the mode-2 implosions and 26.1 bar for the mode-4 implosions (table 1). As in the above-discussed higher-mode implosions, there is considerable variation in the pressure versus time records from one probe to another for the mode-4 implosions. A pressure contour plot on a polar coordinate plane is shown for one representative run for the mode-4 implosion in figure 19b. As was mentioned in §3b(ii), owing to the complexity and run-to-run variability of the cross-sectional shape of the models in the higher-mode implosions, it was not possible to record a reliable model orientation on this contour plot.

The average pressure over the eight probes for each run in this test series is plotted versus time in non-dimensional variables ($(\tilde{P} - P_c)/P_c$ versus $t/(R_i \sqrt{\rho/P_c})$ in figure 23. As in previous plots, the pressure records from the various implosions are aligned in time with the peak pressure at $t = 0$. The scaling has resulted in a fairly universal non-dimensional implosion time of about 2.5. As in previous plots for the mode-2 implosions, the dimensionless pressure–time records are quite similar from run to run, and show a very sharp and well-defined peak. The maximum dimensionless pressure is about 0.55. For the mode-4 implosions, the peaks are broader and show a large variation from run to run. For these cases, the peak dimensionless pressure varies from about 0.48 to 0.18. As previously stated, this large run-to-run variation is likely due to the increased strength of the model structure as it folds and implodes upon itself in the higher-mode cases.
Figure 23. Dimensionless pressure, $(\tilde{P}(r, t) - P_c)/P_c$, versus dimensionless time, $t/(R_i\sqrt{\rho/P_c})$, for models constructed from brass tubes with diameters of 16.6 mm (solid line, mode 2, $L/D_0 = 9.2$) and 25.4 mm (dashed line, mode 4, $L/D_0 = 2.3$). For these models, the implosion pressures and internal air volumes, and thus $P_c V$ are nearly the same (Table 1). Each curve is the average of the pressures from the eight pressure sensors in the midplane around the model for one implosion.

4. Conclusion

The implosion of cylindrical shell structures in a high-pressure water environment was studied experimentally. The shell structures were constructed from thin-walled aluminium and brass tubes and sealed with thick aluminium end caps. The models were filled with air at atmospheric pressure. The implosions were photographed with a high-speed digital movie camera, and the pressure field generated in the water during the implosions was measured with eight high-frequency pressure sensors located in a ring of radius $2.5 R_0$, where $R_0$ is the outer radius of the model, around the model centreline and located in the midplane between the two end caps.

Four of the model designs imploded with a two-lobe cross-sectional shape (mode 2). These designs were constructed from brass 260 (two diameters $D_0 = 16.6$ mm and 25.4 mm), aluminium 3003 ($D_0 = 38.1$ mm) and aluminium 2024 ($D_0 = 38.1$ mm) tubes. The length-to-diameter ratios of these four model designs ranged from 6.3 to 9.2, and the ambient water pressure that triggered the implosions ($P_c$) ranged from 7.1 to 28.7 bar above atmospheric pressure. The pressure field recorded by the sensors ($P - P_c$) first drops and then reaches a positive short-duration peak followed by a higher longer-duration peak. Comparison with the high-speed movies of the implosions indicates that the first peak pressure occurs when the walls of the tubes first meet at the mid-section of the model and the larger peak occurs at about the time when the model reaches its minimum volume. In dimensional units, the time scale of the implosions and the values of the peak pressures varied substantially from one model design to another. However, when the pressure is scaled by $P_c$ and the time is scaled by $R_i\sqrt{\rho/P_c}$, where $R_i$ is the internal radius of the model and $\rho$ is the density of the water, a substantial collapse of the data to a single curve is achieved. The success of this scaling, which does not include the geometrical (other than $R_i$) or structural properties of the tubes, seems to indicate that once the implosion begins, the structure becomes so weak that the influence of the details of its design on the pressure signals is secondary to the effect of the high-pressure water, which behaves in some ways as if the structure were replaced by a bubble of low-pressure gas. The least successful part of the scaling is the peak pressure whose average value over all implosions for each model design varied from about 0.55 to 0.75; however, it was found that this ratio correlated well with the ratio of the mass per unit length of the tube to the mass of water per unit length displaced by the tube.
In order to compare the pressure fields generated by models that imploded in different modes, designs were created that had the same available energy for the implosions, $P_c V$, where $V$ is the internal air-filled volume of the model. Two strategies to achieve this available energy equality between models were used. The first strategy used models created from the same tube stock. Here, the curve of $P_c$ versus $L/D_0$ was mapped out using theory and experiment. Noting that $P_c$ increases with decreasing $L/D_0$, values of $L/D_0$ were found that produced implosions in modes 2, 3 and 4 with the same available energy. The second method used models created from different tube diameters. Here, models that imploded in mode 2 and mode 4 were designed with the same $V$ and the same $P_c$. It was found that the higher-mode implosions created pressure records with wider peaks and with a much greater variation from run to run for the same model design. When the pressure records of the various model designs were non-dimensionalized using the above-described scheme, it was found that the mode number had little effect on the dimensionless time scale of the implosions, while the peak pressures did not scale as well. It is speculated that the strength of the structure as it folds upon itself in the higher-mode implosions, may be responsible for the higher variation in pressure magnitudes found in these cases.

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