How could the Viking Sun compass be used with sunstones before and after sunset? Twilight board as a new interpretation of the Uunartoq artefact fragment

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Vikings routinely crossed the North Atlantic without a magnetic compass and left their mark on lands as far away as Greenland, Newfoundland and Baffin Island. Based on an eleventh-century dial fragment artefact, found at Uunartoq in Greenland, it is widely accepted that they sailed along chosen latitudes using primitive Sun compasses. Such instruments were tested on sea and proved to be efficient hand-held navigation tools, but the dimensions and incisions of the Uunartoq find are far from optimal in this role. On the basis of the sagas mentioning sunstones, incompatible hypotheses were formed for Viking solar navigation procedures and primitive skylight polarimetry with dichroic or birefringent crystals. We describe here a previously un conceived method of navigation based on the Uunartoq artefact functioning as a ‘twilight board’, which is a combination of a horizon board and a Sun compass optimized for use when the Sun is close to the horizon. We deduced an appropriate solar navigation procedure using a twilight board, a shadow-stick and birefringent crystals, which bring together earlier suggested methods in harmony...
and provide a true skylight compass function. This could have allowed Vikings to navigate around the clock, to use the artefact dial as a Sun compass during long parts of the day and to use skylight polarization patterns in the twilight period. In field tests, we found that true north could be appointed with such a medieval skylight compass with an error of about ±4° when the artificially occluded Sun had elevation angles between +10° and −8° relative to the horizon. Our interpretation allows us to assign exact dates to the gnomonic lines on the artefact and outlines the schedule of the merchant ships that sustained the Viking colony in Greenland a millennium ago.

1. Introduction

The Sagas and the Old Icelandic law book the Grágás are main sources of information on the navigation methods and routes of Viking sailors. Coastal navigation and relying on cues taken from characteristic landmarks were preferred, but Vikings also were able to cross open seas. A favoured method was latitude sailing, which means sailing directly to west or to east along latitudes marked by prominent settlements or coastal landmarks [1,2]. Even the southern tip of Greenland was connected to Norway by merchant ships sailing more than 2500 km (1350 nautical miles) along the 61° latitude. Vikings did not have a magnetic compass. However, they divided the horizon into eight named sections forming the ‘attir’ system, a primitive analogue of the modern compass points. This attir served as an independent reference to record bearings, but it was also used for coding azimuths of celestial bodies to provide directional and temporal references. Bright and relevant stars were used as navigation cues at night. The first of these was the northern Pole Star, Polaris, even though it was located 6° 14′ off the celestial North Pole during the Viking era [2].

The Viking navigation season covered the summer months with almost perpetual daylight in northern latitudes, thus solar navigation was also important. It is accepted by the scientific community that Vikings used primitive solar navigation instruments such as the horizon board (figure 1a; [2]) and the Sun compass (figure 1b; [1]), which look similar but are fundamentally different. In this work, we suggest a third alternative solar navigation method based on a so-called twilight board (figure 1c) that could be used with the existing artefact sundial fragment and brings together the functions of the horizon board and the Sun compass.

The horizon board is a primitive but effective tool suggested by Karlsen [2] for interpreting directional references of the Grágás (figure 1a). It consists of a flat board with a central hole and a series of holes on the perimeter coding seasonal azimuth angles $\varphi_S$ of the rising and setting Sun at the latitude chosen for crossing the sea. Based on an artefact sundial fragment found under the ruins of an eleventh-century Benedictine convent in Uunartoq in Greenland, Vikings are alternatively thought to have used an inverted sundial during the day [1–3]: the navigator held the disc of the sundial horizontally and rotated it around the vertical axis of the gnomon until the tip of the gnomon’s shadow reached a pre-drawn gnomonic line (hyperbola or straight line), then the symmetry axis of the gnomonic line pointed to the geographic north (figure 1b).

A further mysterious tool called a ‘sunstone’ is mentioned in the Saga of King Olaf the Holy, that allowed the king to see the Sun when it was occluded by clouds. No authentic description of its nature and use is known, but the high value of a sunstone is indicated by its mentions in medieval treasure inventories. It is a widely accepted hypothesis that it was a birefringent crystal with which Vikings performed crude skylight polarimetry and located the occluded Sun to add sky polarimetric navigation to solar navigation (figure 1) [2,4,5]. The basic idea of this hypothesis was first introduced by Ramskou [4]. Although he made some mistakes, which unfortunately were later continued by Thirslund [1] and Karlsen [2], Ramskou was a pioneer in this respect. A recently found archaeological artefact raised the possibility that this method was still practised even in the sixteenth century [6].
Figure 1. Instruments for solar navigation possibly used by Viking navigators to maintain course in open sea. (a) A horizon board bears holes on its perimeter coding the azimuth angles $\psi_S$ of the rising and setting Sun at a given latitude. Shift of the rising and setting Sun during the whole navigation season can be followed by choosing appropriate holes along the perimeter. Pegs inserted into the holes help to read the bearings. The navigator can derive true directions at sunrise and sunset by aiming at the Sun with the central and the peripheral pegs, and also at noon by recording the direction of the shortest shadow. Seasonal change of $\psi_S$ can be followed by using different holes. The accuracy of orientation is limited by the exact timing of readings, which is difficult at northern latitudes because of the small angles $\gamma$ between the celestial path of the Sun and the horizon. (b) A Sun compass bears a thin and high central gnomon and one or more matching gnomonic lines. During the day, the navigator can derive true directions by fitting the shadow tip to the gnomonic line. At low solar elevation angles $\theta_S$, the shadow tip falls off the board, thus orientation is not possible. (c) A twilight board is a combination of a horizon board and a Sun compass bearing a short conical gnomon and a matching gnomonic line (thick solid line). The gnomon base is tangential to the asymptotes (dashed arrows) of the gnomonic line. At low solar elevations $\theta_S$, the edge of the shadow lies on an asymptote of the gnomonic line. The navigator can derive true directions by fitting the shadow edge to the outer section of the gnomonic line. The ‘cast’ shadow (dash-dotted line) of an imaginary down-pointing gnomon (dashed circle) when ‘illuminated’ by the set Sun from below the horizon would act likewise.
The Uunartoq artefact is one half of a round dial. It bears deliberately incised lines which were interpreted as gnomonic lines (figure 2a) valid on days of Equinox and summer solstice at the 61st latitude, which was the transatlantic Viking route between Norway and Greenland. Although this find could, indeed, be a fragment of a Sun compass, its dimensions are far from optimal in this role. The artefact compass disc has a diameter of 70 mm. It has a large central hole with a diameter of 17 mm, possibly a socket for a grip, beside which its incised gnomonic lines terminate. It has also been hypothesized that this hole might have housed a lost central part with a pin-like gnomon and a complement of the gnomonic lines [1]. The artefact can also be associated with different navigation procedures [7,8].

In this work, we suggest that the large central hole of the Uunartoq sundial fragment might have a key role in navigation, and its diameter might have been purposefully chosen (figure 2b): it could house a broad and short conical gnomon optimized for use at low solar elevation angles (figure 1c). Before sunset, a Viking navigator could orient himself by aligning the straight edge of the metre-long shadow of such gnomons to the outermost visible point of the gnomonic line. In the moments of sunset and sunrise, the instrument could function as a horizon board. Assuming the use of two sunstones and an appropriate shadow-stick, the instrument could have functioned even after sunset until the end of civil twilight. Because it is a combination of a horizon board and a Sun compass designed to be used in the matutinal and crepuscular periods, we call the hypothesized instrument a ‘twilight board’ or ‘twilight compass’. To test this hypothesis, we calculated the systematic navigation error of the twilight board for various latitudes, and determined the time period when the twilight compass could be useable. Furthermore, in the field, we tested a counterpart twilight board in situations when the artificially occluded Sun had elevation angles between $+10^\circ$ and $-8^\circ$ relative to the horizon. From this, we determined the error with which the true north can be appointed with this instrument, which we found to be a functional skylight compass.

2. Material and methods

(a) Twilight board

Our proposed twilight board is characterized by the following features (figure 1c). (i) Its broad conical gnomon casts a shadow, the edge of which lies on the asymptote of the hyperbolic path of the shadow tip at low solar elevations. (ii) Unlike a horizon board, a twilight board bears one or more gnomonic lines. At intermediate solar elevations, the shadow tip falls on the compass dial, and the twilight board functions as a Sun compass. (iii) Unlike the gnomon of a Sun compass, the broad gnomon of a twilight board unavoidably covers the central sections of gnomonic lines. At low solar elevations, the shadow edge is used as a pointer to be fitted to the outermost section of a given gnomonic line. (iv) The base of the gnomon of a twilight board should be nearly or perfectly tangential to the asymptotes of the gnomonic lines. (v) Unlike the dial of a Sun compass, the dial of a twilight board may be small and bears only the inner section of the gnomonic line that significantly deviates from its asymptote (figure 1b,c). Note that the asymptotes point towards the setting and rising Sun. They are marked out by the outermost point of the gnomonic line and the base of the gnomon, thus the twilight board can also function as a horizon board (figure 1a,c).

(b) New interpretation of the artefact dial fragment

To estimate the accuracy of the Uunartoq artefact (with a diameter of 70 mm) in the role of a twilight board, a digital model (figure 2) was created on the basis of the information from [1]. The height and position of the gnomon were calculated using this digital model. Parameters of gnomonic lines and asymptotes valid at a given latitude on a given day of the year were calculated by means of a computer program that we have written. Atmospheric refraction was not considered.
Figure 2. Engraved markings on the artefact dial fragment found in Uunartoq in Greenland. The existing fraction of the dial is shown in grey; the centre of the supposed outlines was taken as the position of a central gnomon. Thick solid lines represent incisions which were proven to be engraved deliberately. The dark grey oval on the right-hand side represents a depression that is hypothesized to be the marking of east [1]. (a) Thorslund [1] interpreted the straight incised line as the path of the tip of the gnomon shadow on the day of the Equinox. The series of notches were supposed to be an approximate marking of north. Dashed-dotted lines show the appropriate gnomonic lines at the 61st latitude on the days of the Equinox, summer solstice and on 8 April, the date on which the shadow tip would follow the inner section of the curved incised line. Thin solid lines on the left-hand side represent the asymptotes of the gnomonic lines. The perimeter of the inner hole is nearly tangential to the asymptotes of the marked gnomonic lines. The dial can be used as a twilight board. Orientation using the outermost section of the gnomonic line instead of the asymptote leads to a navigation error. (b) An alternative interpretation of the markings on the artefact. The short notches in the upper part are considered to be the marking of the north–south line, if the dial is rotated by 8.7°. Incised lines fit to the path of the shadow on 10 and 31 March. These gnomonic lines are the northern and southern branches of a hyperbola with asymptotes nearly tangential to a central conical gnomon with a base diameter of 16 mm. The dial can be used either as a twilight board or as a twilight compass. The depression in the eastern part could be a correction mark for the erroneously incised outer section of the southern gnomonic line.
In earlier studies of the artefact, a series of notches were considered to be a rough mark of north, an easily visible depression was assumed to be a rough mark of the direction of east, whereas the straight incised line was believed to represent the equinoctial line which is the straight path of the shadow on the day of Equinox running from west to east (figure 2a). These assumptions demand a gnomon with a height of about 4.3 mm that would cast a shadow on the dial only at solar elevation angles between 7° and 27°. We analysed two alternative arrangements of the artefact dial: (i) the northern incised line was considered to be the equinoctial line as suggested in earlier studies [1] demanding a gnomon height of 4.3 mm; (ii) as an alternative, the series of notches were considered to mark true north, necessitating an 8.7° rotation of the model, but demanding the same gnomon height (4.3 mm). Using the northern incised line, the date was assigned by selecting the gnomonic line, the asymptote of which is parallel to the incised line itself. Using the southern incised line, the date was assigned by selecting the gnomonic line, the inner section of which fits best to the inner section of the incised line. All dates are presented here according to the Gregorian calendar system, and considering the recent positions of the equinoctial points.

(c) Calculation of the accuracy of the twilight board

The optimal base diameter of the gnomon could be calculated for any combination of dates and latitudes, but no existing circle is tangential to asymptotes of all gnomonic lines that are valid on different days. Thus, applying one single gnomon with a given base diameter results in a navigation error that changes during the year and may limit the period in which a twilight board can effectively be used. As in the case of using a wrong gnomonic line on a Sun compass, the navigation error owing to a wrong base diameter in the morning and evening would compensate for each other, but their absolute value may be significant. The optimal gnomon diameter was calculated for the days between the winter and summer solstices for the latitudes of 46°, 51°, 56°, 61° and 66° N covering the area of Viking marine activities. It is the smallest on the days of the Equinoxes (figure 3a). The gnomon height was set to gain a 7.9 mm long noon shadow on the day of the Equinox at a given latitude. The length of the period covered by a single gnomon was defined by a range of the optimal base diameter not wider than 1.1 mm and causing an orientation error smaller than ±1°.

A navigation error also originates from fitting the shadow edge to the outermost shown section of the gnomonic line instead of to its asymptote (figure 2a). This is a systematic error that also could be compensated by measurements performed at dawn and dusk. Before the date of the gnomonic line, this error will even reduce the error originating from using a wrong gnomonic line in the Northern Hemisphere, and the net navigation error of the twilight board may be relatively small. After the date of the gnomonic lines, the errors add up. However, in the season of the Equinox, even the inner sections of the gnomonic lines approach their asymptotes, thus this error is negligible and may be disregarded (figure 2b).

(d) Shadow-stick

A shadow-stick is an acute object that can be used in unfavourable weather for replacing the missing shadow of the gnomon at selected solar elevation angles (figure 3b). It bears a series of sockets located at distances from the tip of the stick equalling the distances between the gnomon tip and the shadow tip at selected solar elevations. The appropriate socket should be fitted to the gnomon tip, and the stick should aim at the anti-solar meridian. Distances between the tips of the gnomon and the shadow could easily be calculated or found empirically.

A twilight board requires a shadow-stick that also codes the width of the shadow at the edge of the compass dial (figures 1c and 3b). Distances between the stick tip and the sockets corresponding to low solar elevations may be arbitrarily chosen, because the shadow tip would fall off the dial anyway. However, the width of the stick at a distance from the socket equalling that between the gnomon tip and the dial edge must equal the shadow width at this point (figures 1c,
Figure 3. (a) Fitted polynomial functions representing the optimal base diameters of conical gnomons for a twilight board at the 46th, 51st, 56th, 61st and 66th latitudes providing 7.9 mm length of the cast shadow on the day of the Equinox. The difference between a single base diameter of the gnomon and the actual optimal base diameter was required to be less than 0.55 mm (straight dashed lines) defining the longest possible season of operation. At the 61st latitude (solid line), which is the traditional Viking sailing route to Greenland, the above requirement defines a vernal navigation season between 24 February and 15 April in the Gregorian calendar system. The season is significantly extended at southern latitudes.

(b) Characteristic sizes of the hypothesized shadow-stick to be used with the Uunartoq artefact dial shown in 1:1 scale.

(c) A Viking bone pendant of a circular design from Estonia, from the tenth–eleventh century AD, shown in 1:1 scale (source: http://www.icollector.com/Viking-bone-pendant-of-a-circular-design-and-wolf-tooth-pendant-from-Estonia-10th-11th-century-AD_i8646474). Its markings do not form a pattern coding evenly distributed elevation angles, but naive imitations of ‘navigator pendants’ and genuine shadow-sticks are supposed to take the form of pendants with similar shape, size and decorations.
3b and 4). Because, at a solar elevation of $0^\circ$, the shadow tip lies in infinity, the corresponding width of the shadow-stick equals the base diameter of the gnomon. These requirements and the practical arrangement of the sockets dictate a characteristic arrowhead shape for the shadow-stick (figures 3b and 4).

For symmetry reasons, if both branches of the hyperbola functioning as the valid gnomonic line are shown in the dial of the twilight board, then both can be used with a shadow-stick (even if the Sun is below the horizon) providing a true twilight compass capacity. The tip of the shadow of an imaginary downward-pointing conical gnomon placed in the northern focus of the hyperbola of the real gnomonic line and ‘illuminated’ by the set Sun from below the horizon would follow the real gnomonic line (figures 1c and 5). When the Sun is just a few degrees below the horizon, the edge of this imaginary shadow also lies on the asymptote of the real gnomonic line. The shadow-stick may be used to visualize this imaginary shadow after sunset on the upper surface of the dial (figures 1c and 4a). To find the appropriate socket on the shadow-stick, one may take the absolute value of the now negative solar elevation angle. The socket corresponding to this value should be fitted to the centre of the imaginary gnomon positioned along the north–south line, whereas the edge point of the stick representing the matching width of the shadow should be aligned with the asymptote (figure 4a). This is possible only if the place of the second gnomon is marked on the compass, otherwise the appropriate position of the shadow-stick is ambiguous. Because the place of the imaginary gnomon changes day by day, the task is hardly achievable. Luckily, however, one can take further advantage of symmetry and rotate the compass by $180^\circ$ (figure 4b). Now, the real and imaginary gnomons switch places, and the shadow-stick can be used with the other branch of the hyperbola, as before sunset. A pair of sunstones could be used for estimating the solar azimuth and elevation by performing skylight polarimetry (figures 1 and 6b,c). Below, we refer to the set of the twilight board, shadow-stick and sunstones as the twilight compass.

As the Sun moves deep below the horizon, the tip of the imaginary shadow also follows a hyperbolic path, being nearly identical to that of the real shadow tip (figures 1c and 5). When the compass is rotated to use the real central gnomon in the place of the imaginary one, the northern branch of the hyperbola will move to the right position and may be used with the shadow-stick as long as the Sun’s position can be estimated. This northern branch may be drawn even without a sophisticated geometrical construction on a day when the Sun is seen to set $180^\circ$ away from the rising Sun of the date of the southern gnomonic line (figure 5). The positions of the shadow tip may easily be marked on the corresponding days to gain both branches of the hyperbola. Such pairs of days can be appointed by counting matching numbers of days ahead and back from the day of the Equinox, and can be verified with the aid of the attir system. Because the inner sections of the gnomonic lines significantly deviate from their asymptotes in mid-summer, the method functions well only in the seasons of vernal and autumnal Equinox (figure 2).

**e) Estimation of the solar azimuth and elevation angles with sunstones**

Several atmospheric optical phenomena can be used to estimate the position of the occluded Sun, but the Vikings have been hypothesized to rely on the polarization pattern of the sky [1,2,4,5,8–11]. Skylight is formed predominantly by first-order Rayleigh scattering of sunlight on atmospheric molecules and aerosols, and is partially linearly polarized (figure 6). As a rule of thumb, the direction of polarization of skylight is perpendicular to the plane of scattering determined by the Sun, the observed celestial point and the observer. At sunset and sunrise, skylight is strongly polarized in a zone lying across the zenith perpendicular to the solar meridian [9,12] (figure 6b). Hence, the direction of skylight polarization is generally perpendicular to the celestial great circle connecting the Sun and the observed celestial point [13] (figure 6b,c). When the Sun is close to the horizon, the orientation of the solar meridian can be estimated by measuring the direction of polarization of skylight at the zenith. Estimating both the solar elevation angle and the orientation of the solar meridian is possible by measuring the direction of polarization of skylight at two or more points around the zenith to appoint celestial great circles connecting the Sun and the
Figure 4. Hypothesized use of the artefact dial with a shadow-stick. (a) At low positive solar elevation angles, when the shadow is not visible but the Sun's position can be estimated, the appropriate socket of the stick is fitted to the tip of the central gnomon (dark grey). After sunset the shadow-stick could represent the shadow of an imaginary gnomon ‘illuminated’ from below the horizon (light grey). The side point of the stick representing the shadow edge at the given solar elevation should be fitted to the asymptote, the matching socket should be fitted to the centre of the imaginary gnomon, and the dial should be rotated until the stick points towards the solar meridian. (b) When the Sun is below the horizon, it is easier to navigate with a compass turned upside down (by 180°). Now, the real gnomon plays the role of the imaginary one. The shadow-stick can be used to represent the imaginary shadow below the compass dial. The northern incised line represents the imaginary gnomonic line. The kit functions as an efficient twilight compass as long as the azimuth and elevation angle of the set Sun can be reliably estimated.
Figure 5. Geometrical arrangements of gnomons, gnomonic lines and their asymptotes. In summer in the Northern Hemisphere the shadow of a conical gnomon (solid circle) follows the southern branch of a hyperbola (thick solid line) with the gnomon in its focus $G$. The shadow of the same gnomon would approximate the northern branch (thick dashed line) with $IG$ in its other focus on the day in winter when the azimuth of the setting Sun is $180^\circ$ away from that of the rising Sun on the given summer day. Corresponding dates can be found by the span between the Equinox and them. For symmetry reasons, the ‘cast shadow’ (dashed-dotted line) of an imaginary down-pointing gnomon (dashed circle) placed in $IG$ and ‘illuminated’ by the set Sun would approximate that of the real shadow of the real gnomon. If the base of the conical gnomon is tangential to the asymptote, then one edge of the real and imaginary shadows will lie on the same asymptote in the periods around sunset and sunrise.

observed points (figures 1 and 6c). The intersection of two such great circles or an error triangle defined by three of them appoints the position of the Sun (figure 6c).

A rhombohedron of Icelandic spar (a transparent birefringent calcite crystal occurring in raw form in eastern Iceland), for example, can be used as a sunstone [2,5,11]. An efficient method is covering all faces of this crystal in such a way that only slits perpendicular to the crystallographic $c$-axis of the calcite are left clear on the two greatest faces (see fig. 2 in Bernáth et al. [8]). Partially linearly polarized light passing through the entrance slit is divided into totally linearly polarized ordinary and extraordinary rays that diverge while crossing the crystal, and that form two parallel images of the entrance slit on the exit face. Slit images are equally bright when the axis of the entrance slit is $45^\circ$ to the direction of polarization of entering light. If the degree of skylight polarization is high enough, then the direction of polarization of skylight can be measured by a calcite sunstone with an accuracy of about $1^\circ$ [2,11].

(f) Field tests

In field trials, we assessed the accuracy of orientation with a counterpart twilight compass in clear weather during the autumn equinoctial period (24 August–19 October 2012) in Budapest, Hungary ($47^\circ 28'\ N; 19^\circ 4'\ E$; see the electronic supplementary material, figures S1–S3). Tests were carried out by six staff members of the Environmental Optics Laboratory, all being city-dwelling males aged between 25 and 40 but experienced in analysing skylight polarization patterns. The dimensions of the twilight compass were taken from the digital model of the artefact dial found at Uunartoq. Owing to the more southern latitude of Budapest, a higher gnomon was required for gaining a shadow that follows the approximate path marked by the supposed equinoctial line in the artefact dial, thus a broad conical gnomon with a height of 9.8 mm and a diameter of
Figure 6. (a) The setting or rising Sun may be obscured from surface observers by nearby fogbanks, but even thin distant clouds may obscure it because of the curvature of the Earth's surface. However, skylight produced by scattering in the high atmosphere will reach the observer on the surface, unless thick cloud layers are present. (b) Skylight is produced by the Rayleigh scattering of direct sunlight on air particles forming a characteristic linear polarization pattern being mirror symmetrical to the solar–anti-solar meridian that allows solar navigation. (c) The Sun’s position can be estimated by using two calcite sunstones. At low solar elevations, the degree of linear polarization of skylight is the highest in a zone (thick dashed line) passing perpendicular to the solar meridian (sm) at 90° from the Sun across the zenith. The direction of polarization is normal to the plane of scattering determined by the observed celestial point, the Sun and the observer (O). Two sunstones can be used to measure the direction of skylight polarization and marking out the great circles a and b lying in the plane of scattering. The intersection of a and b estimates the Sun’s position.
17 mm was used. Such a gnomon casts a shadow when the solar elevation angle is lower than 49°, and the tip of its shadow falls off the dial at solar elevations lower than 15.7°. Watertight dial cards bearing gnomonic lines calculated for the actual date and easily visible straight lines pointing towards true north and local magnetic north were printed and placed under the gnomon. Shadow-sticks representing the shadow of a gnomon with optimal base diameter (22 mm at the latitude of Budapest and a gnomon height of 9.8 mm) and solar elevations of 1.4°, 3.7°, 5.7°, 9°, 13.5° and 18° were created for every occasion. These elevation angles correspond to the average arcs subtended by the fingers and fists on the extended arm of the six participant observers and could be reliably estimated (see the electronic supplementary material, figure S2). A pair of non-optical quality but polished calcite rhombohedra (5 cm × 5 cm × 2.5 cm) were used as sunstones (see the electronic supplementary material, figure S2a, and see fig. 2 in Bernáth et al. [8]).

Measurements were carried out on an undisturbed terrace of a steel-frame building of the Eötvös University, Budapest, Hungary, in the morning and evening in clear weather at solar elevations lower than 10°. At the chosen point of the terrace, nearby buildings screened the eastern and western parts of the horizon, but the zenith could be freely seen (see the electronic supplementary material, figure S1a). A pendulum-levelled dial plate (125 mm × 245 mm) supported by a ball-and-socket joint of a portable tripod was used (see the electronic supplementary material, figure S1). The dial plate was divided into three panes: (i) a holder for a magnetic compass, (ii) a holder for a central gnomon and the dial card, and (iii) fixed black-and-white stripes which provided a contrast reference while photographing the dial plate. A magnetic compass (Freiberger Präzisionsmechanik Sport 4) with an opaque cover fixed on the dial plate served to measure the orientation error (see the electronic supplementary material, figures S1 and S3). The local magnetic deviation was calculated from the deviation of the magnetic compass, and the true north identified using the classic method of Indian circles [14]. Data on real solar elevations were obtained from the data service of the United States Naval Observatory (http://www.usno.navy.mil).

To orient our twilight compass, the position of the Sun obscured by a building was estimated with the two calcite sunstones. We performed all measurements under clear skies, when the concentration of air pollution was the lowest possible. Because the calcite sunstones that we used functioned best where the degree of skylight polarization was the highest, the test persons automatically measured the direction of skylight polarization in such celestial regions (approx. at 90° from the Sun) where it only slightly deviates from the Rayleigh model [13,15–17]. The solar elevation angle was estimated by counting fingers and fists on the extended arm needed to cover the angular distance between eye level and the estimated position of the Sun (see the electronic supplementary material, figure S2). The matching socket of the shadow-stick was fitted to the gnomon tip, and then the point of the stick was aimed towards the anti-solar meridian, that is, the direction of the natural shadow. The dial plate was rotated until the outermost marked point of the gnomonic line fitted to the edge of the shadow-stick (see the electronic supplementary material, figure S3a). At negative solar elevation angles, the dial plate was rotated by 180° and its marked south was taken as the true north. After the test person oriented the twilight compass, the cover of the magnetic compass was carefully removed, and the dial plate was photographed from above (see the electronic supplementary material, figure S3b). The error of true north was calculated by means of image-processing software (CORELDRAW9.0 script) from the deviation of the line considered to point to the magnetic north from the magnetic needle measured to 0.1°. The direction of the mean vector and the angular scatter was calculated to determine the navigation error, which is the deviation from true north [18].

3. Results

Assuming the use of a central gnomon with a height of 4.3 mm demanded by the northern incised line in the role of the equinoctial line on the artefact, no valid gnomonic line fitted to the southern line on any other date at the 61° latitude. This line was too close to the perimeter and its outer section ran far south from the direction expected in this position (figure 2a). Following
the interpretation of Thirslund [1], the gnomonic line valid on 8 April was found to coincide best with the inner section of the gnomonic line. Its asymptote is nearly tangential to the base of a gnomon with a diameter of 16 mm. Note that the equinoctial line is located 15.5 mm to north from the presumed centre of the gnomon (figure 2a).

Rotating the model by 8.7° (figure 2b) allows interpreting the instrument as a twilight board or a twilight compass. Now, the series of notches in the northern part marks true north, whereas the equinoctial line is unmarked. The northern incised line approximates well the gnomonic line valid on 10 March, whereas the gnomonic lines valid on the last days of March fit to the southern line. According to the twilight compass concept, this latter line should represent 31 March, and the gnomonic line calculated for this day fits well to the southern incised line on the artefact dial (figure 2b). The asymptotes of these gnomonic lines are nearly tangential to a gnomon with a base diameter of 15.7 mm.

While the Uunartoq artefact could have operated as a classical sundial for only 3 h in the forenoon and for 3 h in the afternoon, it could have operated as a twilight board for about 5 h after sunrise and for 5 h before sunset without auxiliaries. During the five midday hours, its gnomon has no cast shadow, thus the navigator would steer by wind, waves, cast shadows of ship parts and sighting of the Sun disc. Using the artefact as a twilight compass extends its operation for about 50 min before sunrise and for 50 min after sunset. During the remaining 9 h of the night, the sky is dark enough to take sightings of stars. Thus, in favourable weather, a Viking navigator could have been provided with sufficient cues to maintain course around the clock.

We found the equinoctial period to be the most appropriate for navigation with a twilight board defining vernal and autumnal navigation seasons, but mercantile considerations render Atlantic crossing during the latter one unlikely. The lengths of periods in which a single gnomon may be used are shorter at northern latitudes (figure 3a), but a single gnomon could be used even at the 66° latitude from 27 February until 10 April. On the 61° latitude, which was the traditional Viking sailing route between Norway and Greenland, a single gnomon could serve from 24 February until 15 April. This period contains all dates that are supposed to be marked by gnomonic lines on the artefact dial. At southern latitudes, the operation period is significantly longer: at the 46° latitude, the supposed position of Vinland, a single gnomon could be used for more than two months. However, in the sub-Mediterranean region, rapid sunsets and short civil twilight would constrain the usability of twilight boards to those of simple horizon boards.

In our field tests, the twilight board aided by a shadow-stick and a pair of calcite sunstones proved to be a functional medieval skylight compass, although its accuracy was incomparable to that of modern direction-finder instruments (figure 7). The direction considered to be true north had significant error with a wide interval of 95% confidence: $\alpha = 356.1^\circ \pm 2.2^\circ$. This suggests a significant uncertainty in estimating the Sun’s position, which is supported by the dispersion of the average and the wide interval of 95% confidence of the error of the estimated solar elevation angle ($\alpha = 2.5^\circ \pm 2.2^\circ$; figure 7). In our field trials, test persons had different systematic errors (table 1), which might suggest that training of the observer/navigator can improve the accuracy of navigation. The best performing test person reached an average bearing error of $\pm 1.5^\circ$. Although producing various average bearing errors, test people remained within a 95% confidence interval of about $\pm 11^\circ$. This demonstrates that individual variations in estimation methods raise different systematic errors and a competent mentor teaching an efficient method could greatly increase the accuracy of navigation. More sophisticated methods allowing more accurate estimation of the Sun’s position would decrease the orientation error [2]. However, such methods are not known from historical documents.

4. Discussion

The artefact dial found in Uunartoq is surely a navigational instrument, but its true designation is not confirmed by other archaeological finds. In the lack of counterpart artefacts and detailed authentic descriptions, it is hard to judge how it was used. The reconstruction of Thirslund
Figure 7. Histograms of bearings considered as true north, and of errors of the solar elevation angle estimated by six test persons (table 1) equipped with a twilight compass consisting of a twilight board, a shadow-stick and two calcite sunstones in Budapest, Hungary (47° 28′ N, 19° 4′ E), in clear weather during the autumnal equinoctial period. The 642 trials (a) were performed at solar elevation angles between +10° and −8°. Skylight polarimetry gets more difficult with the darker zenith region. Accuracy before sunset or after sunrise with the Sun above the horizon (b, c) should be compared with that during periods after sunset or before sunrise with the Sun below the horizon (d, e). Number N of individual trials, direction α of the mean vector, width of the 95% confidence interval, r-value of the Rayleigh statistics and its p-value of significance are given. Straight lines mark the expected direction, arrows mark the found direction and length of the average vectors. Solid circles mark the length of average vectors required for 1% significance of Rayleigh statistics. Likewise, dashed circles mark the length of average vectors required for 5% significance of Rayleigh statistics. Positive deviation marks overestimation of the solar elevation.
Table 1. Bearings considered as true north and error of solar elevation angles estimated by six test persons using a shadowstick, a twilight board and two calcite sunstones when the Sun (positioned between $+10^\circ$ and $-8^\circ$ elevation angles relative to the horizon) was occluded by a building.

<table>
<thead>
<tr>
<th>test person</th>
<th>number of trials</th>
<th>average bearing error (°)</th>
<th>interval of 95% confidence (°)</th>
<th>average solar elevation error (°)</th>
<th>interval of 95% confidence (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>103</td>
<td>1.5</td>
<td>± 5.6</td>
<td>2.1</td>
<td>± 5.5</td>
</tr>
<tr>
<td>2</td>
<td>114</td>
<td>1.5</td>
<td>± 5.3</td>
<td>2.2</td>
<td>± 5.3</td>
</tr>
<tr>
<td>3</td>
<td>152</td>
<td>3.2</td>
<td>± 4.6</td>
<td>1.4</td>
<td>± 4.5</td>
</tr>
<tr>
<td>4</td>
<td>64</td>
<td>4.8</td>
<td>± 7.1</td>
<td>3.9</td>
<td>± 7.1</td>
</tr>
<tr>
<td>5</td>
<td>114</td>
<td>5.5</td>
<td>± 5.3</td>
<td>1.1</td>
<td>± 5.2</td>
</tr>
<tr>
<td>6</td>
<td>95</td>
<td>6.6</td>
<td>± 6.0</td>
<td>4.8</td>
<td>± 5.8</td>
</tr>
</tbody>
</table>

[1] identifying it as a dedicated Sun compass aiding latitude sailing to Norway is a sound proposition, but it is not the only possible solution as shown by Bernáth *et al.* [7,8].

Solar navigation instruments are useless when the Sun is occluded, unless its position can be estimated on the basis of atmospheric optical phenomena. Theoretically, the polarization pattern of the sky may be analysed even under clouds [12,15–17,19], and dichroic or birefringent crystals (sunstones) can be used for analysing the polarization characteristics of skylight [2,9,11]. However, both the accuracy and the extra information provided by such measurements in the field were queried [5,10,13]. Such sunstones are more practical for estimating the position of the solar meridian at low solar elevations when the horizon is covered by fog or clouds, but the zenith is clear (figure 6) [2,8,16,19].

Our twilight board or twilight compass interpretation of the Uunartoq artefact dial explains both the small diameter of the dial and the missing outer sections of the gnomonic lines. Because these line sections do not deviate from the asymptote of gnomonic lines, they have no unique navigational function and may be omitted. The large inner hole in the disc also becomes reasonable: it might have been needed to house a broad and short conical gnomon. The weakest point of our interpretation is the non-fitting slope of the southern incised line on the artefact. Based on the above reasoning, we therefore have to assume that it was imperfectly formed. Erroneous carving surely happened at some points; the third beam from south on the compass rose was even corrected by the wood-carver [1]. On the other hand, taking the series of notches as a precise marking of north is plausible: this most cardinal direction must have been marked correctly. If the straight northern incised line is taken as an ordinary gnomonic line at the 61° latitude, the matching Gregorian date—10 March—is unambiguously determined by the angle enclosed by this gnomonic line and the marked direction of north. The southern line does not fit to any valid gnomonic line at this latitude, but the position of its inner section relative to the assumed central position of the gnomon would fit to those of gnomonic lines valid at the end of March. The deepening in the eastern side of the fragment is just about in a good position to serve as a correction mark of the gnomonic lines valid at the end of March (figure 2b). However, if the artefact is used as a twilight board, then the assumed erroneous outer section of the line would not constrain the usability of the dial: the edge of the shadow could have been fitted to its flawless inner section.

If either the twilight board or the twilight compass interpretation is accepted, the two gnomonic lines code very similar schedules which are quite rational for merchants based in Norway: on a twilight board, the two lines would be usable in two distinct periods, thus they would practically imply dates of outbound and inbound transatlantic crossings. The gnomonic line belonging to 10 March is unambiguously marked out by the asymptotes of the northern gnomonic line. The date of the southern line is ambiguous, but it is valid some time at the end of March. Assuming the existence of sunstones and a lost shadow-stick, and considering the artefact as the dial of a twilight compass defines 31 March as the date belonging to the southern gnomonic...
line. In this case, one line would represent the path of the tip of the imaginary shadow cast by the Sun from below the horizon, and would be used during dawn and twilight with the compass rotated by 180°. Because the lines can swap roles before and after the Equinox, this interpretation also links outbound sailing to 10 March and inbound sailing to 30 March. Considering the cruising speed of replica Viking ships, crossing the North Atlantic Ocean with a good wind could take about two to three weeks. A departure on the first day of March would predict landing in Greenland around the day of the Equinox, marked by the fest of Eostre. The twilight compass is quite accurate until the middle of April at the 61° latitude, allowing a safe return by the end of March. Should the northern incised line on the Uunartoq artefact dial be either the equinoctial line or a gnomonic line used solely during the twilight period, it could serve navigation at the end of March at the latest. Markings referring to dates months away from the start of the nautical season would have been pointless.

Because the twilight board is a sophisticated form of the horizon board suggested by Karlsen [2], astronomical records and historical documents underpinning the possible use of the latter would also fit to the former. Pegs placed on a twilight board at the intersections of the two asymptotes and of their northern run-off points on the perimeter would form an adequate horizon board. One of the advantages of the twilight board is the serial reading by using the gnomon shadow following the moving Sun as a pointer showing the azimuth of the setting/rising Sun for a long period in spite of the continuously changing solar azimuth angle. This allows navigators to read accurate bearings of the setting/rising Sun during a longer period, and not only during a few minutes. Atmospheric refraction increases the perceived elevation of celestial bodies close to the horizon by about 0.5° and should be taken into account in modern navigation processes. However, it only slightly affects the width of the gnomon shadow, thus in the case of a twilight board atmospheric refraction causes only a negligible error.

The extended operational period of the horizon board allows serial readings which reduce the bearing error. Under heavily overcast skies, its accuracy would significantly decrease, but the method may still function, if clear patches around the zenith are visible [16,17,19]. We found that calcite sunstones allow the solar elevation angle to be assessed with a bias of 9° and with a clear tendency of overestimating it during sunset and civil twilight in the equinoctial period (figure 7). This means that the azimuth angle of the obscured setting/rising Sun could be detected with a bias of 10° either in Budapest (47° 28’ N) or at the 61st latitude. This constrains the use of sunstones with a horizon board as suggested by Karlsen [2], even if the most precise location of the solar meridian is assumed. Because such a single reading would determine the course of the ship until the next sunset or sunrise, and its significant error cannot be reduced by averaging several measurements, it might have been a serious jeopardy.

Although a twilight board allows a quite precise orientation, it is not free from navigation errors. If the navigator finds true directions by fitting the edge of the shadow and the outset marked point of the gnomonic line instead of the asymptote, then an error will occur, because the two coincide only in the infinite (figures 2b and 5). However, as in the case of using inappropriate gnomonic lines (see the electronic supplementary material, Discussion), errors with equal absolute value but opposite signs will be made in the matutinal and crepuscular periods, and, thus, compensate for each other. In the periods of the Equinox when a single gnomon could be used on a twilight board for several weeks, even the inner sections of the gnomonic lines are nearly straight, thus the above error is negligible. A navigation error originating from using a gnomon with a wrong base diameter also causes systematic navigation bias. This error is not free from the actual solar elevation angle and could be compensated by readings performed in the morning and evening at identical solar elevations.

The accuracy of navigation is strongly influenced by the weather conditions and the error of estimating solar azimuth and elevation angles with the sunstones. In this work, we chose simple unaided estimation methods and could appoint true north with a bearable average error of about 4° (table 1 and figure 7). Some of our city-dwelling male test persons showed more talent than others and achieved an accuracy of 1.5° (table 1), which is comparable to that of modern magnetic pocket compasses. Obviously, a few years of orientation practice and a mariner background
could greatly improve their performance. This means that twilight boards and twilight compasses could provide a reference as reliable as aligning to the Pole Star, which later could cause a bearing bias of 12° 28′ in the Viking era. Magnetic compasses can cause even greater navigational errors in uncharted territories with unknown magnetic deviation, especially at high latitudes. Further tools, for example mirrors, mounts or improvised sextants, could be applied to improve the accuracy of the twilight board [2], but no such Viking equipment is known from historical documents or archaeological findings.

Developing a twilight board is surely not beyond the capabilities of sea-faring people. However, one must consider that the idea of a Viking-age skylight compass presumes a significant development of the navigation toolkit. Sunstones are mentioned in written sources and they could be used during civil twilight, although it is not known how one can accurately estimate the position of the Sun with them. The recently identified calcite crystal found among the navigational tools of a sixteenth-century European shipwreck hints that they could play some role in marine navigation [6]. But, there is no evidence on the use of shadow-sticks yet. Such small items would conserve badly, but some genuine examples or naive contemporary imitations may easily be part of artefactual collections without their true purpose being recognized (figure 3c).

Acknowledgements. G.H. thanks the German Alexander von Humboldt Foundation for donating equipment. B.B. is grateful for the research fellowship (3.3-UNG/1127933STP) received from the Alexander von Humboldt Foundation. We are grateful for the digital photographs of the artefact dial found in Uunartoq kindly provided by Dr Thorbjørn Thaarup of the Danish Maritime Museum. The critical comments of Professor Aurél Ponori Thewrewk and Lajos Bartha are highly appreciated. We are also grateful to three anonymous referees for their positive and constructive comments.

Funding statement. This work was supported by the Hungarian Science Foundation (grant no. OTKA K-105054: Full-Sky Imaging Polarimetry to Detect Clouds and to Study the Meteorological Conditions Favourable for Polarimetric Viking Navigation). S.Å. is supported by a project grant from the Swedish Research Council (621-2010-5584) and from a Linnaeus grant to the Centre for Animal Movement Research (CAnMove) from the Swedish Research Council (349-2007-8690) and Lund University.

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