Revisiting Barry Cox and James Hill’s theory of superluminal motion: a possible solution to the problem of spinless tachyon localization

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1. Introduction

A superluminal particle, or tachyon, is defined as a particle which travels faster than light in vacuum (FTL). The dawn of the scientific study of tachyons may be ascribed...
to papers by Oliver Heaviside, who in 1888, just two decades after the discovery of Maxwell’s
equations of classical electromagnetism, studied the motion of FTL electric charges [1].
Heaviside’s pioneering work on FTL particles is worth mentioning here because it has been
largely forgotten to the extent that, out of two recent detailed biographies, only one devotes a
scanty section to this topic [2] while the other completely ignores it [3].

In 1904, Arnold Sommerfeld published a paper concerning the electromagnetic radiation
of tachyons. Both Heaviside and Sommerfeld foreran the discovery of Cerenkov radiation.
Sommerfeld’s paper is re-examined in [4]; a modern synthesis of Sommerfeld’s paper is found in
the book by Fayngold [5]. But, in 1905, Albert Einstein published his celebrated paper on special
relativity (SR). He showed clearly that a massive particle needs an infinite amount of energy to be
accelerated to the light barrier, let alone to surpass it. It is the dynamic objection to the existence
of tachyons [6].

Another objection (of purely kinematic type) was devised by Einstein and soon popularized
by Richard Chace Tolman in 1917 [7]. Resorting only to the basics of space–time Minkowski
diagrams, he proved that tachyons might be used to send signals into the past, thereby violating
a basic tenet of physics: retarded causality. It is the so-called anti-telephone paradox, widely quoted
in almost all introductory books to SR [8,9].

The quite rapid acceptance of SR by mainstream physicists (especially the youngest and most
brilliant ones), with the exception of a few old opposers, froze the study of tachyons for decades.
Only a few rather obscure hints to these particles appeared every now and then in the scientific
literature [10]. The study was revived in the 1960s when it was realized that there is no need to
accelerate through the light barrier to get a tachyon, thus overcoming the dynamic objection.

Gerald Feinberg described the following example. If we did not know photons, one could
deny the existence of luxons (particles travelling at exactly the speed of light) because SR predicts
that an infinite amount of energy is needed to accelerate a massive particle up to the speed of
light. But we know that photons do travel exactly at the speed of light without acceleration
or deceleration. They are massless particles which propagate at the same speed from emission
to absorption. It is well known that luxons (photons) are obtained from subluminal particles
(renamed bradyons). For instance, when an electron–positron couple is annihilated, two photons
are created. Other (still unknown) reactions between bradyons may create tachyons which travel
faster than light from the start. They would feel the light speed as a downward barrier, as noted by
Sudarshan and co-workers [11,12].

A theory of classical tachyons, called extended relativity, was developed by Sudarshan, Recami
and others within a suitable extension of SR. Its main feature is that an imaginary rest mass is
attributed to tachyons; however, energy and momentum remain real numbers [13]. Kinematic
objections were faced by introducing a ‘switching principle’ modelled on Richard P. Feynman’s
idea that anti-particles can be conceived as particles travelling backwards in time.

In essence, the ‘switching principle’ reinterprets a free tachyon travelling backwards in time
(which is endowed with unphysical negative energy, as it can be easily shown) as a well-behaved
anti-tachyon, endowed with positive energy and travelling forward in time [14].

Unfortunately, as was soon pointed out by several opponents of the tachyon concept, the
‘switching principle’ also exchanges cause and effect. There is no harm while tachyon emission
and absorption remain spontaneous, uncontrollable phenomena (hence not suited to signalling),
but paradoxes arise if emission is intentional [15].

Some authors believe that the joint effect of the above-mentioned paradoxes is to write off
tachyons from theoretical physics, at least as long as we want to fully comply with SR [16]. The
only way out is to run against the spirit of SR, which sets all inertial frames on an equal footing.
Einstein’s great leap forward in 1905 was to deny the existence of a privileged reference frame
(PRF) in which the aether is stationary, which existed in a theory by Hendrik Antoon Lorentz [17].
However, a clever analysis of Einstein’s theory soon revealed that the theory rested upon a
subtle assumption concerning clock synchronization, as explained in a book by Jammer [18]. In
his doctoral thesis at Stanford University, CA, USA (1959), Frank Robert Tangherlini created a
theory which adopts a synchronization procedure different from SR, later re-examined by other
In spite of its deep significance, Tangherlini’s theory is not widely known; even Jammer in his thorough essay ignored it.

The main consequence of Tangherlini’s theory is the existence of a PRF which sets unambiguously the past and future of any inertial observer, forbidding any exchange of cause and effect. That is why kinematic objections to tachyons vanish in Tangherlini’s theory [20], which I will subsequently call privileged relativity (PR).

In 1965, just a few years after Tangherlini’s doctoral thesis, Arno A. Penzias and Robert W. Wilson discovered a radio noise of celestial origin which Robert H. Dicke interpreted as the relic radiation of the Big Bang: a breakthrough in cosmology [21]. The reference frame in which the cosmic background radiation is isotropic seems to materialize as a PRF in the Universe.

Extensions of general relativity (GR) which contain a PRF have been constructed. While in GR, a free-falling observer sees a locally flat and Lorentz invariant space–time (as in SR) and the maximum speed limit of light is valid, in the above-mentioned extensions of GR Lorentz invariance is broken for free-falling observers and no universal maximum speed limit exists [22]: hence, superluminal signalling becomes feasible.

In essence, the cosmic censorship conjecture by Roger Penrose assumes that no naked singularity exists in the Universe, that is to say, any singularity is clothed by an event horizon and it is invisible from the outside (as in black holes). However, numerical solutions of GR are known which contain naked singularities [24] so that the validity of the cosmic censorship remains an open question.

From a technological perspective, even if naked singularities really exist somewhere in the Universe, the engineering challenge posed by their exploitation for superluminal travel looks daunting even for an extremely advanced civilization. Hence, GR is far from supplying us with any viable solution to superluminal communication and/or travel in the foreseeable future. For instance, in spite of the popularity gained by ‘warp drives’ in mass media during recent years, they require absurdly huge amounts of unphysical ‘negative energy’ [25].

In the early 1980s, the detailed observation of a jet expelled from quasar 3C273 thanks to the high-resolving power of very long baseline interferometry caused a stir. The velocity of the jet perpendicular to the line of sight was estimated to be about 10 times the speed of light, if the red shift of the quasar radiation had a cosmological origin and the distance of the quasar was computed using Hubble’s law. However, it was soon realized that superluminal motion was not real; it was rather a sort of optical aberration [26].

The effect had been predicted by Sir Martin J. Rees in 1966 for a blob moving close to the line of sight towards the observer at a very high (but subluminal) speed [27].

An exhaustive study of tachyons cannot be based only upon SR or GR because tachyons, if they exist as free objects (not only as virtual particles, as in quantum field theory, QFT), are likely to share the atomistic structure of all known matter. A macroscopic tachyon should be composed of microscopic entities subject to quantum constraints.

The Soviet physicist D. I. Blokhintsev is believed to be the first who tried to develop a QFT including tachyons in 1946 [28]. Controllable tachyons, suitable for FTL signalling, cannot fit within standard QFT, which is based upon SR and incorporates a micro-causality condition. This condition asserts that operators at space-like separation always commute, thus forbidding signalling over space-like intervals [29].

To incorporate controllable tachyons, QFT must be rebuilt from the ground up. During the 1960s, some authors, for instance Gerald Feinberg (who coined the word tachyon), repeated the attempt to construct a SR-based QFT of tachyons but ran into difficulties [30].

It is very easy to write a Klein–Gordon PDE for spinless tachyons, substituting an imaginary mass into the standard Klein–Gordon PDE [31].
Unfortunately, particles described by such a PDE are not localizable, pointlike particles: it is the localization objection not quite to the existence of tachyons but to their creation in a laboratory, which is necessarily a local process [32]. The problem of spin-1/2 tachyon localization has been addressed in a recent paper [33]. The authors’ opinion is that ‘rather painful choices have to be made in order to incorporate tachyonic spin-1/2 particles into QFT’.

It is well known that Einstein never accepted quantum mechanics (QM) as a fundamental theory of Nature not only because of its indeterminism, but also because of what he called ‘spooky action at a distance’ between entangled particles. His celebrated EPR paper, written in 1935 with his collaborators Boris Podolsky and Nathan Rosen, is reprinted in [34]. The EPR paper originated a line of thought which led to another famous paper by John S. Bell in 1964, reprinted in [35]. The paper contains an inequality which must be satisfied by any local theory (i.e. a theory without superluminal influences between events at space-like separation).

Successive experiments, mainly those by Aspect et al. [36], have proved beyond any reasonable doubt that no local theory (with or without hidden variables) can reproduce all the predictions of QM. Hence, non-locality is an intrinsic feature of Nature, not only of QM. However, this kind of non-locality cannot be used for superluminal signalling [37]. For instance, Nick Herbert devised a scheme for quantum superluminal communication in 1982. An account of this scheme (known by the acronym FLASH) is found in [38].

Soon after, it was proved that FLASH could not work because of the ‘no-cloning theorem’ by Wootters & Zurek [39], which forbids the creation of identical copies of an arbitrary unknown quantum state. Quantum communication is feasible and useful for security purposes (quantum cryptography), but the quantum channel is always coupled to a classical (electromagnetic) subluminal channel.

Standard QM is a linear theory. As other physical theories lack linearity (the most outstanding example is GR), nonlinear modifications of QM have been studied, for example by Weinberg [40]. Gisin in 1990 [41] and Polchinski in 1991 [42] proved that Weinberg’s nonlinear QM needs restraints to avoid superluminal communication.

Although it had already been shown in 1914 by Arnold Sommerfeld and Leon Brillouin that in no circumstance are electromagnetic waves suitable to send FTL signals (even if phase and group velocity may become superluminal) [43], a few authors (for instance, Recami) have continued the search for FTL loopholes in the realm of classical electromagnetism (X-shaped waves) [44,45].

Gunther Nimtz claimed the achievement of FTL signals in the microwave region of the electromagnetic spectrum at 4.7 times the speed of light using evanescent waves, which arise in waveguides when they are forced to operate below their cut-off frequency. However, his claim, supported by several papers and a book [46], was soon disputed as untenable [47]. Other authors claimed a similar goal in the optical region of the electromagnetic spectrum [48]. Raymond Chiao studied a similar phenomenon which is found in quantum tunnelling. Tunnelling speed can be superluminal: it was found out by E. P. Wigner. But the phenomenon is considered of no avail to send FTL signals by the overwhelming majority of physicists [49].

A small minority persist in the opposite interpretation of quantum tunnelling. Just a few months ago, Gunther Nimtz published a rebuttal in this never-ending controversy [50], which sounds like the debate on perpetual motion in past centuries on a smaller scale.

Much of the debate concerns the still controversial definition of signal velocity in the superluminal domain [51]. A concise and simple introduction at the undergraduate level can be found in [52], but the subject is complex and prone to misunderstandings to the extent that a whole book has been devoted to it very recently in view of the latest experimental data [53].

More promising (although still debated) results were found in the field of particle physics. After failed experimental searches of tachyons during the 1960s [54], an unconfirmed detection in cosmic rays [55,56] and the hypothesis that the anomalous decay rate of orthopositronium may be connected to tachyon emission [57], close attention was paid to neutrino mass, which was assumed to be exactly zero in the standard model of particle physics. However, landmark experiments at Super-Kamiokande, Hida, Japan, and the Sudbury Neutrino Observatory, Lively, Ontario, Canada, have proved conclusively that neutrinos do have mass.
Several attempts to measure the rest mass of electron neutrinos have been performed using tritium decay [58,59]. Surprisingly, almost all early experiments, during the 1980s and 1990s, gave negative results for the neutrino mass squared, which were officially deemed unphysical and attributed to unknown or partially known systematic errors [60,61].

An imaginary rest mass would provide exactly the negative mass squared observed but, apart from errors in these difficult experiments, some authors found out tentative solutions to the tritium endpoint anomaly which do not resort to tachyons [62] so that the so-called tritium endpoint anomaly is not a smoking gun for neutrino superluminality.

Suggestions that neutrinos might be tachyons were put forward by Chodos [63], and J. Rembieliński developed a theory of the tritium endpoint anomaly based upon the tachyonic neutrino hypothesis within the framework of PR [64]. The hypothesis of tachyonic (or space-like) neutrinos seemed to find an earth-shaking confirmation in September 2011 when the spokesman for the Oscillation Project with Emulsion-TRacking Apparatus (OPERA) experiment at the European Organization for Nuclear Research (CERN) announced the detection of FTL neutrinos [65]. A preprint followed soon after [66]. The news spread at lightning speed on popular media all over the world because it was perceived by laymen as a refutation of Einstein’s relativity, although most experts were very cautious [67]. Unfortunately, the astonishing OPERA result was soon attributed to a trivial cause (a malfunction in electronics, which was equivalent to a systematic error) [68]. Moreover, the OPERA data were found at variance with astrophysical data gathered from the SN1987a supernova, although in 1998 Simone Giani at CERN had claimed that SN1987a data were compatible with slightly FTL neutrinos [69]. The last nail in the coffin of CERN’s untimely announcement was struck by other neutrino experiments.

The amount of controversy that has surrounded tachyons since their inception is such that a few authors even hinted at the possibility that tachyons were recorded during past decades, for instance in old photographic emulsions, but were overlooked by experimenters not versed in their discovery [70] or perhaps deliberately ignored in order to avoid disputes concerning the first experimental detection of anti-protons (a Nobel Prize-winning experiment) [71]: a sort of unlikely ‘conspiracy theory’ for tachyons, which sounds like the alleged ‘conspiracy of silence’ for UFOs and other pseudoscience topics.

Another controversy surrounds string theory (ST), which is a candidate that goes beyond the well-established QFT.

Tachyons are found in ST but as undesired guests [72]. As Joseph Polchinski recollects, ST was taken seriously only when it got rid of tachyons, that is, when tachyon-free versions of ST appeared [73]. However, ST cannot say much about tachyons because it is itself plagued by a raging controversy. Critics accuse ST of a massive disproportion between its incredible amount of mathematical development and a total lack of experimental evidence [74], all the more because experiments at the Large Hadron Collider (LHC) have failed to reveal any hint of supersymmetry (which should be the first step towards ST) up to now [75].

In 2012, Barry Cox and James Hill re-examined the extension of SR to tachyons and proved known formulae in a different way, but the most interesting suggestion found in Cox and Hill’s paper is an alternative dynamics for tachyons [76]. Their suggestion will be developed later in this paper, but in a different context.

2. A concise review of Frank Robert Tangherlini’s theory

Light pulses are used in SR to synchronize distant clocks. However, a difficulty arises because a wide consensus exists that only the two-way velocity of electromagnetic waves is measurable [77]. The difficulty was noted by Hans Reichenbach in 1928.

Einstein supposed that not only the two-way velocity of electromagnetic waves in the vacuum is constant and isotropic for all inertial observers (as a consequence of the Michelson and Morley experiment), but the one-way velocity as well. Tangherlini instead adopted a different synchronization scheme in his doctoral thesis (1959). The essential difference is that the two-way velocity of electromagnetic waves in the vacuum remains isotropic and constant for all
inertial observers in Tangherlini’s theory, but the one-way velocity is isotropic only in a privileged inertial reference frame (IRF) [78]. Hence, I will call Tangherlini’s theory PR to distinguish it from orthodox SR. Thus, although PR shares many features of SR, it differs radically from it because it violates the absolute equivalence of all inertial observers, which is a basic tenet of SR. PR singles out an IRF to provide a notion of absolute past and absolute future valid for all other inertial observers, so that even space-like events are uniquely ordered in time. The same concept has been translated to GR [79].

Advocates of Tangherlini’s theory remark that a PRF seems to exist in the Universe: the reference frame in which the cosmic background radiation is isotropic. Obviously, a debate on the pros and cons of PR versus SR is out of place here. Although experiments to distinguish between PR and SR have been proposed [80], supporters of PR believe that it is compatible with all known experimental evidence of SR but it is conceptually superior to SR in the explanation of certain phenomena (for instance, the Sagnac effect) [81]. The situation is similar to that of Bohmian mechanics (BM) versus orthodox QM [82]. Although BM is believed to reproduce all predictions of QM (see [83] for a discordant view), its supporters complain that BM is ignored by most university courses in favour of QM. (By the way, BM, although an explicitly non-local theory with hidden variables, does not allow superluminal signalling [82].)

Almost all university courses are centred on SR and PR is mentioned hardly ever. Barone [84], which is an authoritative Italian textbook for a modern university course on SR at the graduate level, devotes a quite slim section to PR and denies its validity without pointing to precise evidence against it.

For simplicity’s sake, I shall deal only with the unidimensional case along the $x$-axis. If $t_P$ and $x_P$ are time and abscissa measured in the PRF, $t_I$ and $x_I$ are the same quantities measured in another IRF which translates at constant speed $v_{PI}$ along the $x$-axis with respect to the PRF, then the kinematic transformations of PR in infinitesimal form are

$$\frac{dt_P}{dt_I} = \frac{1}{\sqrt{1 - \left(\frac{v_{PI}}{c}\right)^2}}$$

and

$$dx_P = \sqrt{1 - \left(\frac{v_{PI}}{c}\right)^2} \cdot dx_I + \frac{v_{PI}}{\sqrt{1 - \left(\frac{v_{PI}}{c}\right)^2}} \cdot dt_I,$$

which take the place of the kinematic transformations by Lorentz. $c$ is the speed of electromagnetic waves in the vacuum, as usual.

Unlike the corresponding Lorentz transformation, (2.1) does not mix time and space coordinates: that is why all observers agree on the time ordering of events, even when space-like (or superluminal, or tachyonic) particles are involved. So no anti-telephone paradox arises for tachyons in PR.

The law of composition of unidimensional velocities, deduced from (2.1) and (2.2), is

$$v_{PA} = \left[1 - \left(\frac{v_{PI}}{c}\right)^2\right] \cdot v_{IA} + v_{PI},$$

where $v_{IA}$ is the speed of a particle A in the IRF and $v_{PA}$ is the speed of particle A in the PRF.

It is easy to see from (2.3) that, although the one-way speed of light is anisotropic in the IRF, the two-way speed remains isotropic. In fact the one-way speeds of light in the vacuum measured by the IRF in opposite directions are $c^2/(c + v_{PI})$ and $c^2/(c - v_{PI})$, hence light takes time $2L/c$ even in the IRF to propagate along a distance $L$ in a two-way path. Similar transformations are found in the dynamics of PR. The energy of a free particle A having rest mass $m_R$ and speed $v_{IA}$ in an IRF is

$$E = \frac{m_R \cdot c^2}{\sqrt{(1 - v_{PI} v_{IA}/c^2)^2 - (v_{IA}/c)^2}}.$$
The corresponding formula of SR is recovered when \( v_{\text{Plx}} = 0 \), so that the IRF coincides with the PRF. It is a general rule that PR recovers the formulae of SR in this case. The momentum is

\[
p_{\text{Plx}} = \frac{m_{\text{R}} \cdot v_{\text{la}}}{\sqrt{1 - v_{\text{Plx}} v_{\text{la}}/c^2}}.
\]

The energy transformation in differential form is analogous to (2.1), with \( E \) in place of \( t \)

\[
dE_P = \frac{dE_I}{\sqrt{1 - (v_{\text{Plx}}/c)^2}}.
\]

The momentum transformation is analogous to (2.2), with \( p \) in place of \( x \) and \( E/c^2 \) in place of \( t \)

\[
dp_P = \sqrt{1 - (v_{\text{Plx}}/c)^2} \cdot dp_I + \frac{v_{\text{Plx}}}{\sqrt{1 - (v_{\text{Plx}}/c)^2}} \cdot \frac{dE_I}{c^2}.
\]

As in SR, superluminal \( v_{\text{Plx}} \) velocities in PR require a change in the square root, which becomes \( \sqrt{(v_{\text{Plx}}/c)^2 - 1} \) instead of \( \sqrt{1 - (v_{\text{Plx}}/c)^2} \).

### 3. Alternative tachyon dynamics in Cox and Hill’s theory

The paper by Cox and Hill [76] suggests a more radical departure from SR, based upon the notion of dynamical mass \( m_{\text{D}} \) (as opposed to the rest mass \( m_{\text{R}} \) mentioned before) and the identity

\[
\frac{dE}{dv_x} = v_x \cdot \frac{d(m_{\text{D}} \cdot v_x)}{dv_x}.
\]

Dynamical mass is often neglected in modern treatises of SR, which prefer to resort to rest mass only (invariant for all inertial observers).

Dynamical mass was popular in early presentations of SR, where the formulae

\[
E = \frac{m_{\text{R}} c^2}{\sqrt{1 - (v_x/c)^2}}
\]

and

\[
p_x = \frac{m_{\text{R}} v_x}{\sqrt{1 - (v_x/c)^2}},
\]

for energy and momentum, were rewritten as

\[
E = m_{\text{D}} c^2
\]

and

\[
p_x = m_{\text{D}} v_x,
\]

which is the same as in classical mechanics.

The departure of SR from classical mechanics was concentrated in \( m_{\text{D}} \), which varies according to

\[
m_{\text{D}} = \frac{m_{\text{R}}}{\sqrt{1 - (v_x/c)^2}}.
\]

So dynamical (or relativistic mass) increases with \( v_x \).

Apart from the fact that \( \sqrt{1 - (v_x/c)^2} \) is replaced by \( \sqrt{(v_x/c)^2 - 1} \) for superluminal particles, Cox and Hill do not take for granted that all formulae remain true for tachyons. They give up
(3.6) and explore the consequences of an arbitrary dependence of $m_D$ on $v_x$. Then they rely upon the energy balance

$$\frac{dE}{dt} = \frac{dp_x}{dt} \cdot v_x \tag{3.7}$$

(equivalent to (3.1)), transformed into

$$E = m_D \cdot c^2 - \int m_D \cdot v_x \cdot dv_x + a_C \tag{3.8}$$

(where $a_C$ is an additive constant), to find out new expressions of energy. The simplest application of (3.8) is the case

$$m_D = m_R, \tag{3.9}$$

that is to say, independence of $m_D$ from $v_x$ as in classical mechanics. The choice

$$a_C = 0, \tag{3.10}$$

so that $E = 0$ when $v_x = 0$, provides

$$E = \frac{m_R v_x^2}{2}, \tag{3.11}$$

which is the expression of kinetic energy for unidimensional motion along the $x$-axis in classical mechanics.

In order to recover SR, one must choose

$$m_D = \frac{m_R}{\sqrt{1 - (v_x/c)^2}} \tag{3.12}$$

(for subluminal particles, sometimes called bradyons) or

$$m_D = \frac{\mu_R}{\sqrt{(v_x/c)^2 - 1}} \tag{3.13}$$

(for superluminal particles or tachyons). $\mu_R$ is the fictitious rest mass of tachyons.

Arguing that we have no experimental knowledge of tachyons, so that it is worthwhile searching for alternative dynamics, Cox and Hill’s paper explores the consequences of

$$m_D = \frac{\mu_R + \mu_\infty \cdot v_x/c}{\sqrt{(v_x/c)^2 - 1}}, \tag{3.14}$$

which ensures a finite limit

$$m_D \to \mu_\infty \tag{3.15}$$

for the so-called transcendental tachyons ($v_x \to +\infty$). Equation (3.14) generates an expression of energy which bears little or no resemblance to the well-known expression of SR.

Cox and Hill hint that many other possibilities for $m_D$ exist. For all the value of Cox and Hill’s suggestion to explore alternative dynamics for tachyons, an easy objection to their line of reasoning is that we are unlikely to reach a realistic theory of tachyons only by guesswork.

The situation reminds us of the caricature of a thoughtful Einstein standing in front of the blackboard, where he has just written a list of formulae: \('E = mc', \ 'E = mc^2', \ 'E = mc^3', \ 'E = mc^4'\) and so on. Which one is the right one?

Clearly, Einstein did not get to SR this way. His effort was spurred by the incompatibility of pre-existing theories (Newtonian mechanics and Maxwell’s electromagnetism). Given the unanimous acceptance of SR and its rock-solid experimental support in the subluminal realm so far (provided that PR is believed to be compatible with all known data), a radical departure from SR in the superluminal range of velocities must be justified by stringent theoretical necessity.
In our opinion, there is no need for dramatic changes such as (3.14) because only a minor correction to the expression of tachyon momentum is enough to achieve the important goal of pointlike localizability of spinless tachyons. Let us get back to Cox and Hill’s paper. Energy balance, written as (3.7) or as
\[ \frac{dE}{dv_x} = \frac{dp_x}{dv_x} \cdot v_x, \] (3.16)
is insensitive to additive constants of energy and momentum, thanks to derivatives.

Actually, energy in SR already contains the additive constant \( E = mc^2 \) (the energy equivalent of the rest mass), that is to say, the energy when \( v_x = 0 \). It is worthwhile exploring the addition of a finite constant to tachyon momentum. The discontinuity across the speed of light barrier is concealed beneath the fact that both bradyon momentum,
\[ p_x = \frac{m_R v_x}{\sqrt{1 - (v_x/c)^2}}, \] (3.17)
and tachyon momentum,
\[ p_x = \frac{\mu_R v_x}{\sqrt{(v_x/c)^2 - 1}}, \] (3.18)
(in the usual extension of SR), tend to infinity when \( v_x \) tends to \( c \) so that momentum is discontinuous by itself across the \( c \) barrier. The addition of a finite constant on the superluminal side does not alter the nature of the discontinuity, because infinity remains infinity when added to a finite constant. This relatively minor modification to the (hypothetical) expression of tachyon momentum will be explored in subsequent sections.

4. The Klein–Gordon PDE for tachyons and the localization problem

The Klein–Gordon PDE (KG) was found out independently by many physicists and nowadays its heuristic deduction is an easy exercise for undergraduates in QM [31]. It was found out by Erwin Schrödinger even before the PDE that bears his name and later rejected because it did not agree with spectroscopical data for hydrogen; moreover, its physical interpretation was hindered by the appearance of negative probabilities [85]. It was recovered and interpreted at the dawn of QFT. The KG arises from a fundamental dynamical identity of SR
\[ E^2 - c^2 p^2 = m_R^2 c^4, \] (4.1)
when both members are interpreted as operators and applied to the wave function \( \phi \). In SI units, the operators associated with energy and momentum are
\[ \hat{E} = \hbar \frac{\partial}{\partial t} \] (4.2)
and
\[ \hat{p} = -\hbar \frac{\partial}{\partial x}. \] (4.3)

Their substitution in (4.1) gives
\[ \frac{\partial^2 \phi}{\partial t^2} - c^2 \frac{\partial^2 \phi}{\partial x^2} + \left( \frac{m_R c^2}{\hbar} \right)^2 \phi = 0, \] (4.4)
which is the unidimensional version of the KG.

As noted in §1, a tachyon possesses an imaginary rest mass
\[ m_R = i\mu_R, \] (4.5)
where \( \mu_R \) is the fictitious rest mass which is real and positive.
Substitution of (4.5) into (4.4) gives

\[
\frac{\partial^2 \phi}{\partial t^2} - c^2 \frac{\partial^2 \phi}{\partial x^2} - \left( \frac{\mu R c^2}{\hbar} \right)^2 \phi = 0. \tag{4.6}
\]

In spite of the wave-particle duality, subatomic particles are always detected as pointlike objects. Electrons and the other leptons, believed to be truly elementary, are considered pointlike particles without internal structure.

Pointlike behaviour is described mathematically by P. A. M. Dirac’s \( \delta \), whose integral representation is

\[
\delta(x) = \frac{1}{2\pi} \int_{k=-\infty}^{+\infty} \exp(ikx) \, dk, \tag{4.7}
\]

in complex terms, and

\[
\delta(x) = \frac{1}{\pi} \int_{k=0}^{+\infty} \cos(kx) \, dk, \tag{4.8}
\]

in real terms [86]. Equation (4.8) shows that perfect pointlike localization needs all wavenumbers \( k \) from zero to infinity or, reciprocally, all wavelengths from infinity to zero.

Unfortunately, it is easy to prove that (4.5) admits no sinusoidal wave solution having a wavelength greater than

\[
\frac{\hbar}{\mu R c}. \tag{4.9}
\]

Lack of the longest wavelengths prevents pointlike localization. The difficulty was addressed by Gerald Feinberg in his paper on the QFT of tachyons [30]. Although pointlike localization is unreachable within orthodox relativistic QM, he showed that a power-law localization is feasible.

5. A possible solution to the localization problem

Suppose that (4.2) remains unaltered, whereas (4.3) is replaced by

\[
\hat{p}_x = -\hbar \frac{\partial}{\partial x} - a_C, \tag{5.1}
\]

for tachyons (not for bradyons). \(-a_C\) is an additive constant which cannot be eliminated by a gauge transformation because the constant is different for tachyons and bradyons. The modified Klein–Gordon PDE (MKG) becomes

\[
\left( \frac{\hbar}{\mu R c} \frac{\partial}{\partial t} \right)^2 \phi - c^2 \left( -\hbar \frac{\partial}{\partial x} - a_C \right)^2 \phi = -\mu R c^4, \tag{5.2}
\]

which can be expanded into a form similar to (4.6).

Let us search for sinusoidal solutions, which (apart from a constant coefficient) take the form

\[
\phi(t, x) = \cos(k_x \cdot x - \omega_0 \cdot t + \alpha_F) \tag{5.3}
\]

in the real domain. \( k_x \) is the wavenumber, \( \omega_0 \) is the angular frequency and \( \alpha_F \) is the phase. Any other solution is obtained as a linear combination of solutions of type (5.3) with complex coefficients. Substitution of (5.3) into (5.2) gives, after the elimination of \( \phi \) and a few algebraic passages,

\[
k_x = \frac{a_C}{\hbar} + \sqrt{\left( \frac{\omega_0}{c} \right)^2 + \left( \frac{\mu R c}{\hbar} \right)^2}. \tag{5.4}
\]

When \( a_C = 0 \) as in the standard KG applied to tachyons, (5.4) confirms that for any real \( \omega_0 \) the minimum value of \( k_x \), associated with the wavelength (4.9), is \( \mu R c / \hbar \). In order to get the full range of \( k_x \) from 0 to \( +\infty \), one must choose

\[
a_C = -\mu R c. \tag{5.5}
\]

This choice allows for the construction of an instantaneously localized solution of the MKG for tachyons in the form of Dirac’s \( \delta \) (4.8).
After the choice (5.5), (5.1) becomes
\[ \hat{p}_x = -\hbar \frac{\partial}{\partial x} + \mu Rc, \] (5.6)
and (4.2) remains unchanged.

Now, we need to show that this modification of the basic dynamical operators is compatible with PR because we want to reach pointlike localizability without sacrificing the PR solution of the other classical paradoxes (first of all R. C. Tolman’s anti-telephone) which afflict the theory of tachyons. As mentioned before, the switching principle by itself is unable to solve the anti-telephone paradox for tachyons whose emission is controllable in the framework of SR. Only the absolute time ordering of events provided by PR avoids violations of retarded causality by superluminal signals.

A little step backwards is beneficial to clarity. Equations (4.2) and (4.3) are consequences of Louis De Broglie’s wave mechanics, which was the immediate antecedent of Erwin Schrödinger’s QM [87].

De Broglie supposed that the wave associated with a particle oscillates synchronously in the rest frame of the particle, that is to say that (apart from a constant coefficient) the wave takes the form
\[ \phi(t_Q) = \cos(\omega_{OQ} t_Q), \] (5.7)
in the real domain, or
\[ \phi(t_Q) = \exp(i\omega_{OQ} t_Q), \] (5.8)
in the complex domain.

\( \omega_{OQ} \) is connected to the energy \( E_Q \) of the particle in the Q inertial frame (QIF) by the relation
\[ \omega_{OQ} = \frac{E_Q}{\hbar}. \] (5.9)
De Broglie passed from the QIF rest frame to another inertial frame P (PIF) moving with respect to the other. Using both the kinematic and dynamical Lorentz transformations of SR, he found that (5.9) remains true in the new inertial frame and it is supplemented by
\[ k_x = \frac{p_x}{\hbar}, \] (5.10)
in any IF.

Hence, the wave function (5.3), written in the complex domain, becomes
\[ \phi(t, x) = \exp \left[ \frac{i}{\hbar} \cdot (p_x \cdot x - E \cdot t) \right]. \] (5.11)

Equations (4.2) and (4.3) are deduced readily from the interpretation of partial derivatives with respect to \( t \) and \( x \) as operators. Of course, these manipulations would be no more than analytical sleight of hand without the decisive experimental support provided by C. J. Davisson and L. Germer’s celebrated experiment on electron diffraction just after De Broglie’s paper [88]. When trying to justify heuristically (4.2) and (5.6) in the framework of PR instead of SR, a difficulty is encountered from the start, even for bradyons and in absence of the momentum correction \( c_A \).

Suppose that inertial frame Q (QIF) is the rest frame of the particle. In PR, time ordering is absolute (even for space-like intervals), so the synchronous oscillations described by (5.8) would remain synchronous in any other IRF. As the kinematic transformation (2.1) for PR does not mix space and time, as already noted in §2, the shift from frame Q to another frame P does not provide any more the wave function (5.11) with the missing spatial dependence.
Assuming that, just as in the case of De Broglie’s reasoning, any analytical deduction possesses at most only a heuristic value in the absence of experimental support, we apply the seemingly trivial artifice of writing

$$b + (1 - b) = 1,$$

(5.12)

in the right-hand side of (5.8), which becomes

$$\phi(t_Q) = \exp[ib\omega_{OQ}t_Q + i(1 - b)\omega_{OQ}t_Q]$$

(5.13)

where $b$ is any real number.

Now we have two instances of time $t_Q$ in the right-hand side. Let $v_{PQX}$ be the velocity of QIF (the tachyon rest frame) with respect to PIF (the privileged IF of PR). For the sake of brevity, we define

$$S_R = \sqrt{(v_{PQX}/c)^2 - 1}$$

(5.14)

(SR stands now for ‘square root’ instead of SR, of course). The kinematic transformations of PR (see §2) become, in finite form,

$$t_p = \frac{t_Q}{S_R}$$

(5.15)

and

$$x_p = S_R \cdot x_Q + \frac{v_{PQX}}{S_R} \cdot t_Q.$$  

(5.16)

They provide two expressions of $t_Q$, which are

$$t_Q = S_R t_p$$

(5.17)

and

$$t_Q = \frac{S_R x_p - S_R^2 x_Q}{v_{PQX}}.$$  

(5.18)

Remember that a tachyon in PR is defined unambiguously as a particle endowed with a superluminal speed with respect to the PIF, so there is no risk of division by zero because $v_{PQX} \neq 0$: a tachyon cannot be brought to rest in the PIF.

As QIF is the rest frame of the tachyon, it is always possible to choose QIF so that

$$x_Q = 0,$$

(5.19)

at any time.

Substitution of (5.17) and (5.18) into the right-hand side of (5.13) gives

$$\phi(t_p, x_p) = \exp\left[i b \omega_{OQ} \cdot \frac{S_R}{v_{PQX}} \cdot x_p + i (1 - b) \omega_{OQ} S_R t_p\right].$$

(5.20)

Now we write the dynamical transformations of PR (see §2) for tachyons under the assumption that tachyon momentum has a discontinuity $c_Ax$ when passing from QIF to PIF (see §3).

As a tachyon, unlike a bradyon, can never be brought to rest in the PIF (remember Sudarshan’s solution to the dynamical objection in §1), no contradiction arises because the rest frame QIF of a tachyon is always distinct from the PIF. Apart from logical contradictions, the acceptability of the discontinuity assumption depends as always upon its physical consequences.

The dynamical transformations of PR (see §2) for tachyons become

$$p_{Fx} + c_Ax = S_R \cdot p_{QX} + \frac{v_{PQX}}{S_R} \cdot \frac{E_Q}{c^2}.$$  

(5.21)
and

\[ E_P = \frac{E_Q}{S_R}. \] (5.22)

The above equation (5.21) reduces to

\[ p_{Px} + c_{Ax} = \frac{v_{PQx}}{S_R} \cdot \frac{E_Q}{c^2}, \] (5.23)

where \( c_{Ax} \) is an additive constant as the tachyon is at rest in QIF. Moreover, suppose that (5.9) remains true for tachyons. It is easy to obtain two expression of \( \omega_{QQ} \): one containing \( E_P \), which is

\[ \omega_{QQ} = \frac{E_P S_R}{\hbar}, \] (5.24)

the other containing \( p_{Px} \), which is

\[ \omega_{QQ} = \frac{S_R c^2 (p_{ Px} + c_{Ax})}{\hbar v_{PQx}}. \] (5.25)

From the definition (5.14) of \( S_R \), we get

\[ v_{PQx}^2 = (S_R^2 + 1)c^2. \] (5.26)

Up to now the coefficient \( b \) has remained undetermined. If we choose

\[ b = 1 + \frac{1}{S_R^2}, \] (5.27)

then, after a few algebraic steps, (5.20) becomes

\[ \phi(t_P, x_P) = \exp \left\{ \frac{i}{\hbar} \cdot \left[ (p_{Px} + c_{Ax}) \cdot x_P - E_P t_P \right] \right\}. \] (5.28)

The proof is true for any \( c_{Ax} \); obviously, we shall choose \( c_{Ax} \) to \( \alpha_C \) given by (5.5). Deduction of (5.1) and (4.2) proceeds from (5.28) as in De Broglie’s standard wave mechanics.

### 6. Conclusion

In 1972, a pioneering study at the Massachusetts Institute of Technology, ‘The Limits of Growth’, set insurmountable limits to economic growth based upon natural resources available on Earth [89]. After several centuries (since the Renaissance) of exponential development, which culminated in the exceptional achievements of the twentieth century, we are rapidly approaching some of these limits in the twenty-first century. The ongoing transition to renewable energies may delay the crisis but not avert it. Only the Universe provides the almost limitless material resources for the endless development of civilization. The finite speed of light barrier is perhaps the biggest physical obstacle to the future colonization of the Universe by mankind, although Enrico Fermi in the early 1950s (shortly before his untimely death) devised a scheme of galaxy colonization based upon subluminal speeds [90]. The far-sighted physicist Gerald Feinberg, who coined the word \textit{tachyon}, clearly recognized the link between FTL motion/communication and extraterrestrial civilization in the Universe [91]. More recently Giancarlo Genta of the Technical University of Turin, Director of the Italian Center for SETI (Search for Extraterrestrial Intelligence) Studies, has devoted a chapter to FTL communication/travel in his book [92]. NASA funded a ‘Breakthrough Propulsion Physics’ project from 1996 to 2002 under the direction of Marc G. Millis (http://www.grc.nasa.gov/WWW/bpp/). BPP explored alternatives to standard rocket propulsion suitable for interstellar travel and related problems, including the above-mentioned tritium endpoint anomaly and superluminal signalling. James F. Woodward claims to have experimented with an innovative electric propulsion system which, unlike rockets, does not rely upon the ejection of matter. He has even described a ‘wormhole generator’ for FTL interstellar travel. But, although published in peer-reviewed papers and a book [93], his ideas have not received any independent experimental confirmation until now.
If controllable tachyons exist, an extraterrestrial civilization could use them to coordinate its expansion over stellar or even galactic scale.

According to Wien’s displacement law of blackbody radiation, relatively low-temperature technological activities by such advanced alien civilizations would generate very large-scale infrared sources. The idea was pioneered by Freeman Dyson, one of the leading theorists of quantum electrodynamics together with Richard P. Feynman, Julian Schwinger and Sin-Itiro Tomonaga [94]. Huge ‘Dyson spheres’ of artificial origin and galactic size could be detected from the Earth with existing technology, but their search has been fruitless [95]. In the author’s opinion, the failed search for Dyson spheres is perhaps the most significant (although indirect) evidence against the existence of tachyons.

The study of tachyons, begun with Heaviside [2], is fraught with difficulties and has not found any reliable experimental confirmation up to now. However, in view of the countless scientific and technological rewards that the future discovery of a superluminal loophole may provide, it is worthwhile pursuing the exploration of alternative tachyon dynamics in the spirit of Cox and Hill’s paper [76].

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References


