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A collection on 'Climate dynamics: multiple scales and memory effects'

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Introduction

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Climate dynamics is attracting increasing attention from the scientific community because of its intrinsic beauty and complexity, but also because of the socio-economic implications of anthropogenic climate change. Much of this complexity is due to the multiple space and time scales that characterize the processes active in the climate system and the phenomena they give rise to—from raindrops to major hurricanes and on to the oceans' overturning circulation—and to the nonlinearity of the interactions among these processes.

Because of this complexity and nonlinearity, the climate sciences have developed a modelling approach based on a full hierarchy of models, from highly idealized 'toy' models to highly detailed global climate models (GCMs), passing through many intermediate types of more-or-less highly resolved ones. Within this hierarchy, the use of delays to simplify the modelling of certain process interactions via memory effects has become increasingly popular. The present collection of six articles is a modest contribution to this rapidly expanding field.

Energy balance models (EBMs) are highly simplified climate models that emphasize the balance between incoming, solar and outgoing, terrestrial radiation as the main mechanism that determines planetary temperature. Bhattacharya *et al.* [1] were the first to introduce a delay into such an EBM to represent the memory effect of slow growth and decay of ice sheets on reflecting solar radiation. In this collection, Díaz *et al.* [2] study, mathematically and numerically, the multiple solutions of such an EBM with memory effects (EBMMs) coupled to a stationary ocean model.

'One contribution to a special feature on 'Climate dynamics: multiple scales and memory effects'.

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Roques *et al*. [3] examine here the problem of inferring the parameters of an EBMM in a palaeoclimatic context, in which partial observations have errors in their dates, as well as in their values. It turns out that a surprisingly small number of observations suffice to determine a key model parameter in certain cases.

Stinis [4] investigates here the general problem of model reduction in the absence of scale separation via the Mori–Zwanzig formalism of statistical physics. The scale separation requires introducing delayed effects of the slow, resolved variables on the evolution of the fast, unresolved ones. The proposed approach to obtain a relatively accurate and still computable reduced model is a time-dependent analogue of the renormalization process used in high-energy and condensed matter physics. Kondrashov *et al*. [5], for instance, propose a complementary approach by constructing multilayer stochastic models.

El Niño–Southern Oscillation (ENSO) is a dominant phenomenon of seasonal-to-interannual climate variability. ENSO modelling is a particularly active area for the use of delay differential equations (DDEs). Krauskopf & Sieber [6] carry out here a bifurcation analysis of Tziperman *et al*.'s [7] DDE model of ENSO, as well as providing a review of the specifics of numerical analysis issues in this context. These issues have to do with the infinite-dimensional character of DDEs.

MacMartin & Tziperman [8] apply here transfer function methodology to help understand ENSO dynamics, compare data with models and identify systematic model errors. Transfer functions represent the frequency-dependent input–output relationship between a pair of variables. The authors use time series of observational data and of high-end GCMs to derive such transfer functions and describe the dynamics of individual subsystem processes relevant to ENSO, as well as to estimate process parameters.

Finally, Hsia *et al*. [9] study here the effects of moisture on the circulation of the tropical atmosphere that is a key component of the ENSO phenomenon. They carry out a rigorous mathematical analysis of a Boussinesq model of this atmosphere in terms of a generalization of bifurcation theory due to Ma & Wang [10], called dynamic transition theory. In particular, they obtain mixed transitions, also known as random transitions, in this model.

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