On Mathematical Concepts of the Material World.

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(Abstract.)

The object of this memoir is to initiate the mathematical investigation of various possible ways of conceiving the nature of the Material World. In so far as its results are worked out in precise mathematical detail, the memoir is concerned with the possible relations to space of the ultimate entities which (in ordinary language) constitute the "stuff" in space. An abstract logical statement of this limited problem, in the form in which it is here conceived, is as follows:—Given a set of entities which form the field of a certain polyadic (*i.e.*, many-termed) relation R. What "axioms" satisfied by R have as their consequence that the theorems of Euclidean Geometry are the expression of certain properties of the field of R? If the set of entities are themselves to be the set of points of the Euclidean Space, the problem, thus set, narrows itself down to the problem of the axioms of Euclidean Geometry. The solution of this narrower problem of the axioms of geometry is assumed (*cf.* Part II, Concept I) without proof in the form most convenient for this wider investigation.

Poincaré* has used language which might imply the belief that, with the proper definitions, Euclidean Geometry can be applied to express properties of the field of any polyadic relation whatever. His context, however, suggests that his thesis is, that in a certain sense (obvious to mathematicians) the Euclidean and certain other geometries are interchangeable, so that, if one can be applied, then each of the others can also be applied. Be that as it may, the problem here discussed is to find various formulations of axioms concerning R, from which, with appropriate definitions, the Euclidean Geometry issues as expressing properties of the field of R. In view of the existence of change in the Material World, the investigation has to be so conducted as to introduce, in its abstract form, the idea of time, and to provide for the definition of velocity and acceleration.

The general problem is here discussed purely for the sake of its logical (*i.e.*, mathematical) interest. It has an indirect bearing on Philosophy by disentangling the essentials of the idea of a Material World from the

* *Cf.* 'La Science et l'Hypothèse,' Chapter III, at the end.

accidents of one particular concept. The problem might, in the future, have a direct bearing upon Physical Science, if a concept widely different from the prevailing concept could be elaborated, which allowed of a simpler enunciation of physical laws. But in physical research so much depends upon a trained imaginative intuition, that it seems most unlikely that existing physicists would in general gain any advantage from deserting familiar habits of thought.

Part I (i) consists of general considerations upon the nature of the problem and the method of procedure. Part I (ii) contains a short explanation of the symbols used. Part II is devoted to the consideration of three concepts, which embody the ordinary prevailing ideas upon the subject and slight variants from them. The present investigation has, as a matter of fact, grown out of the *Theory of Interpoints*, which is presented in Part III (ii), and of the *Theory of Dimensions* of Part IV (i). These contain two separate answers to the question: How can a point be defined in terms of lines? The well-known definition of the Projective Point, as a bundle of lines, assumes the Descriptive Point. The problem is to define it without any such assumption. By the aid of these answers, two concepts, IV and V, differing very widely from the current concepts, have been elaborated. Concept V, in particular, appears to have great physical possibilities. Indeed, its chief difficulty is the bewildering variety of material which it yields for use in shaping explanations of physical laws. It requires, however, the discovery of some appropriate laws of motion, before it can be applied to the ordinary service of physical science.

The geometry throughout is taken to be three-dimensional and Euclidean. In Concept V the definition of parallel lines and the "Euclidean" axiom receive new forms; also the "points at infinity" are found to have an intimate connection with the theory of the order of points on any straight line. The *Theory of Dimensions* is based on a new definition of the dimensions of a space.

The main object of the memoir is the development of the *Theory of Interpoints*, of the *Theory of Dimensions*, and of *Concept V.* The other parts are explanatory and preparatory to these, though it is hoped that they will be found to have some independent value.