distribution is reached in the rigid phase, and the new phase, as Prof. Trouton has shown, must be considered as bivariant till the film has attained a certain limiting thickness. It would be interesting to speculate as to the subsequent course of Prof. Trouton's curves, and I hope that his further researches will throw light on the subject.]

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*Effects of Self-induction in an Iron Cylinder.**

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If a solid cylindrical conductor be divided into imaginary concentric tubular conductors, the ordinary self and mutual induction theory shows that when the conductor is subjected to an alternating potential difference, the interior shells carry electric currents of smaller density than the exterior ones, and the currents suffer a phase displacement greater the nearer the centre of the cylinder. The theory shows that the permeability and conductivity of the material play an important part, the effects above mentioned being increased in magnitude with increasing permeability and conductivity. When, however, the material of the conductor has variable permeability the problem becomes more complicated, and it is the object of this paper to examine more closely what goes on in an iron cylinder when electric currents are reversed in it, and maintained steady after reversal. A second part of this research will deal with alternating currents of varying frequency and wave-form.

The cylinder employed is of mild steel and has a diameter and length each equal to 10 inches (25·4 cm.). It is provided with holes drilled in a plane containing its axis of figure in such a manner that exploring coils can be threaded to inclose certain portions of that plane. The exploring coils are three in number. They are each 2 inches wide in a direction parallel with the axis of figure and midway between the ends of the cylinder. Their depths in a radial direction are 1, 2, and 2 inches, and their average radii are 0·5, 2 and 4 inches respectively. These coils are referred to as Coils Nos. 1, 2, and 3, No. 1 being near the centre of the cylinder. The cylinder has been already described,† but for the purpose of

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† 'Roy. Soc. Proc.,' vol. 69, p. 440.

the present research has had a hole $\frac{1}{4}$-inch diameter drilled through it coinciding with the axis of figure. At each end of the cylinder are two massive gunmetal projections, which in the present research serve to conduct the electric current into the cylinder. The conductors attached to these projections are connected to a reversing switch so constructed that at its mid position it short-circuits the circuit of the cylinder and its conductors, which were arranged in the form of a circle about 6 feet diameter. The electric current was supplied by storage cells through an adjustable resistance and the shunt of an ampère-meter. The potential difference of the cells and the adjustable resistance were such that on reversing the current in the cylinder its value in the main circuit remained constant. In fact, the reversal of the main current was practically instantaneous. The epoch of reversal was noted on a seconds clock and at two-second intervals after reversal the deflections of the dead-beat galvanometers in circuit with the exploring coils were noted. The deflections have been reduced to volts per turn per square centimetre of the coils from which they were obtained, and plotted in terms of the time. Figs. 1, 2, and 3 give the results obtained from Coils Nos. 1, 2, and 3 respectively, and each curve is numbered to correspond with the total ampères reversed when it was observed. The curves have been integrated in order to find the maximum average value of the induction density B for the respective coils. The average values are set out in Table I.

Current Density.

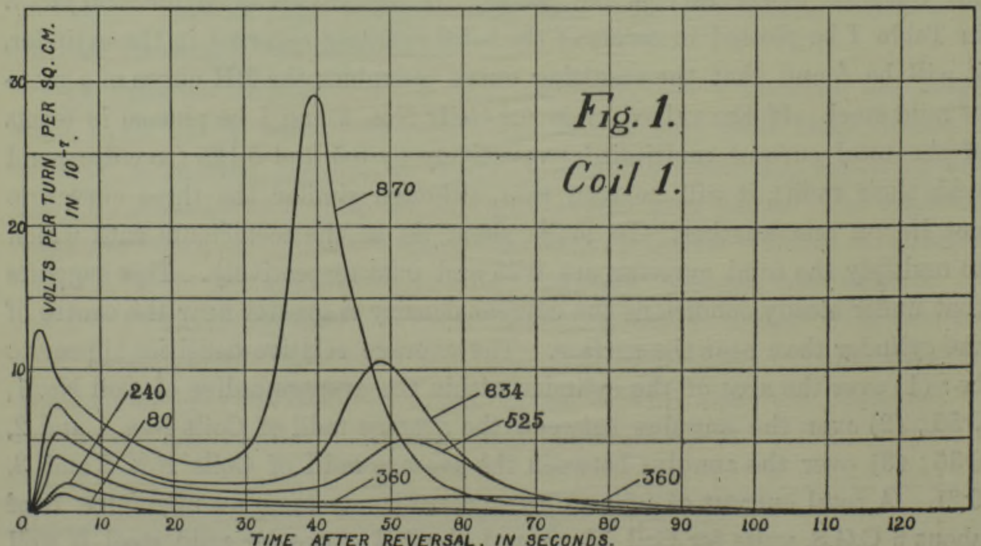
If the current density over the cross-section of the cylinder was constant, the force H would vary as the radius. If the values of B for Coil No. 3 in Table I be plotted in terms of the total currents reversed in the cylinder, it will be found that the resulting curve resembles the BH curve of a piece of mild steel. If the values of B for Coils Nos. 2 and 1 be plotted in terms of the total current multiplied respectively by 0·5 and 0·125 (to correspond with their radii), it will be seen that, although similar, the three curves do not lie on one another. To make them do so the coefficients with which to multiply the total currents are 0·75 and 0·28 respectively. This suggests that under steady conditions the current density is greater near the centre of the cylinder than near the surface. The average relative densities appear to be: (1) over the area of the cylinder within the average radius of Coil No. 1, 0·56; (2) over the annulus between the average radii of Coils Nos. 1 and 2, 0·36; (3) over the annulus between the average radii of Coils Nos. 2 and 3, 0·21. A total current of 950 ampères corresponds to an average force H of about 6 C.G.S. units for Coil No. 3, and the BH curve for mild steel is well represented by the results of the experiments. The relation between the average value of H for each of the coils and the total current is given in Table I.

Table I.

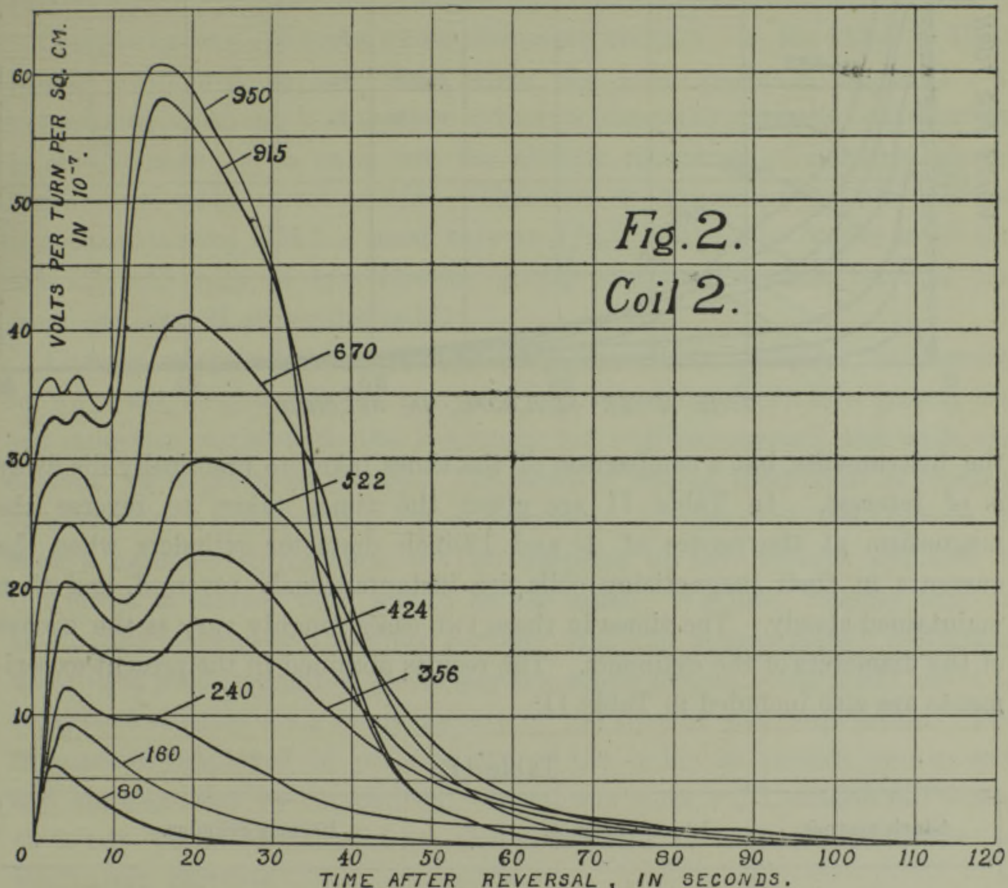
Total current in cylinder.	Coil No. 1.		Coil No. 2.		Coil No. 3.	
	Av. H.	Max. av. B.	Av. H.	Max. av. B.	Av. H.	Max. av. B.
950	1·68	2640	4·5	8950	6·0	9640
670	1·18	1620	3·15	7300	4·2	8200
522	0·924	1040	2·48	6000	3·3	7080
424	0·75	660	2·01	4610	2·68	6210
356	0·627	540	1·69	3560	2·25	5420
240	0·426	200	1·14	1840	1·52	3420
210	0·372	—	0·998	1350	1·33	2520
160	0·283	100	0·758	773	1·01	1400
123	0·218	68	0·583	473	0·777	739
80	0·141	40	0·379	247	0·505	415
45	0·079	—	0·213	136	0·284	180

Comparison of E.M.F. Curves.

The E.M.F. curves are given in figs. 1, 2, 3, to which reference will be made. Coil No. 3 experiences its maximum rate of change at once, although after reversal of about 400 ampères there is slight evidence of a second maximum at about 20 seconds after reversal. After about 400 ampères it will be seen that as the total current increases the curves cross one another at shorter intervals, indicating that the effects penetrate more rapidly after the maximum average induction density has passed the value at which maximum permeability occurs.



Coil No. 2 shows the effect of a second maximum more markedly. When the total current in the cylinder is small there is no second maximum. At about 200 ampères the second maximum shows signs of being developed, and with larger currents its development is such as to make it the most important feature of the curves. As in the case of Coil No. 3, the E.M.F. curves cross one another at earlier intervals after about 420 ampères has been reached.



Coil No. 1 shows similar effects, but its E.M.F.'s have a still later maximum for a given current. Moreover, the time between the first and second maxima is greater than in the case of Coil No. 2. These results are in keeping with what has already been observed in the case of the reversal of currents in a copper coil surrounding cylinders of 4 and 12 inches diameter,* and in the present cylinder when rotated in a magnetic field.† It is difficult to say exactly how long the effects take to die away owing to the ultimate want of sensibility of

* 'Phil. Trans.,' A, vol. 186 (1895), pp. 93—121; also 'Journ. Inst. Elec. Eng.,' Part 116, vol. 24, p. 194.

† 'Roy. Soc. Proc.,' vol. 69, p. 435, and vol. 70, p. 359.

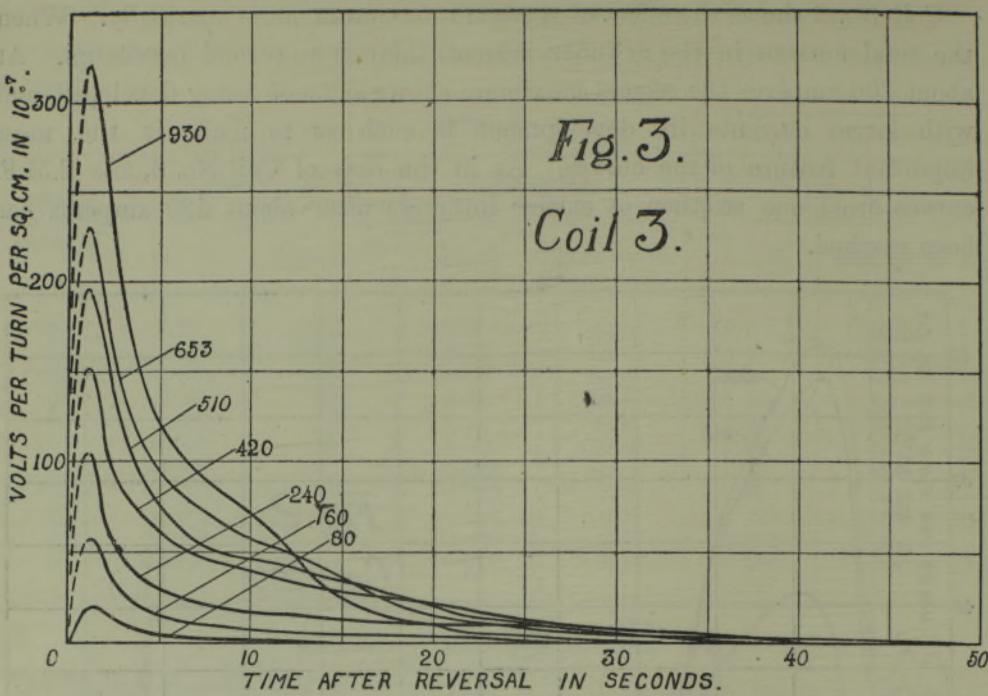


Fig. 3. Coil 3.

the instruments, but a comparison of the times taken to practically die away is of interest. In Table II are given the times taken to reverse the magnetism at the centre of 4- and 12-inch diameter cylinders when the currents in their magnetising coils are instantaneously reversed, and then maintained steady. The times in these two cases roughly vary as the square of the diameters of the cylinders. The results obtained in the present experiments are also included in Table II.

Table II.

4-inch magnet.		12-inch magnet.		10-inch cylinder.			
Duration in seconds.	Max. H.	Duration in seconds.	Max. H.	Duration in seconds.			Total current in cylinder.
				Coil No. 1.	Coil No. 2.	Coil No. 3.	
40	1·7	360	1·2	—	20	—	45
45	3·0	420	2·4	20	28	15	80
33	4·96	180	6·0	30	48	30	160
10	16·0	80	11·0	58	80	50	210
5	37·0	50	24·0	75	100	—	240
				116	112	60	356
				120	120	50	424
				190	90	—	522
				90	75	40	670
				90	65	30	950

Upon integrating the E.M.F. curves it was found that the average magnetic flux, for total currents up to about 240 ampères, was reversed in sign for Coil No. 2 after Coil No. 1. For currents greater than 240 ampères the curves cross the axis of time in the order 3, 2, 1. In all cases, however, the total interior average currents, as obtained from the magnetic hysteresis loops, reversed in the order 3, 2, 1.

Application of Results to other Sections.

Comparing two cylinders whose diameters are as 1 : n, the value of H at similar radii will be the same when the total currents are as 1 : n. Considering unit length of the two cylinders, the total magnetic induction up to similar radii varies as n, and the electric resistance of similarly placed longitudinal paths varies as $1/n^2$. Therefore to induce n times the current in those paths the E.M.F.'s must vary as $1/n$. If the time varies as n^2 the E.M.F.'s will vary as $1/n$, thereby giving rise to the same value of the magnetic force H at similar radii.

A paper recently published* dealt with the self-induction of bull-headed railway rails, weighing 70 lbs. per yard. It was there shown how greatly the self-induction varies with the frequency for a given current, and with the current for a given frequency. The head of one of these rails is roughly equivalent to a cylinder of 2 inches diameter. A current of 100 ampères in such a cylinder corresponds to 500 ampères in our 10-inch cylinder if the forces at similar radii are to be the same. We infer roughly that an alternating current of 8 seconds periodic time and approximately rectilinear wave-form would permit of practically the whole section being made use of as regards conduction for a small fraction of the time of each half period. The frequencies employed in practice are of the order 25 periods per second, and are enormous by comparison. In railway work a ⎍-shaped rail would obviously be more suited from the standpoint of electric conduction of alternating currents.

A current of about 0·5 ampère in an iron wire of 0·1 inch diameter would give rise to a force of about 0·3 near the surface. Changes of magnetism in our 10-inch cylinder were observed 20 seconds after reversal of about 50 ampères. A frequency of 250 would allow of the full section of the wire being made use of during a small fraction of the time of each half period with 0·5 ampère, but not with a current of 5 ampères.

In conclusion, I wish to thank Mr. A. E. O'Dell for his patience and care in working out results, and Mr. H. W. Franks for his assistance in the experimental part of the paper.

* 'The Electrician,' February 23, 1906.