Space described in a Given Time by a Projectile Moving in Air.

By A. Mallock, F.R.S.

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It was pointed out in the paper on "The Resistance of Air," recently read before the Society, that for velocities from 1100 to 3000 feet per second, the resistance curve of a pointed projectile was approximately represented by a straight line which, if produced, would cut the axis of abscissæ at $v = 850$ f.s. The equation of this line is

$$R = 2{\cdot}53\,(v-v'),$$

where R is the resistance in pounds per square foot of cross-sectional area, the velocities being in feet per second, and where $v' = 850$ f.s. The retardation (f) of the shot is given by $f = \frac{2{\cdot}53g}{\rho l}(v-v')$, where ρ is the weight of the unit volume of material, and l the equivalent length of the projectile. (By equivalent length is meant the length of a cylinder of the same weight and density as the projectile.)

The quantity ρl is another, and for many purposes a more convenient, form of the ballistic coefficient, which is usually stated as w/d^2. It represents the mass of the projectile per unit area of cross-section on which the retardation acts.

From equation for f, it is easy to deduce a formula for the space traversed by the projectile in a given time, and in this paper I give a few examples of ranges in terms of time computed from it.

The formula for the distance s, traversed by the shot in time t is*

$$s = v't+\frac{u_0}{a}(1-e^{-at}),$$

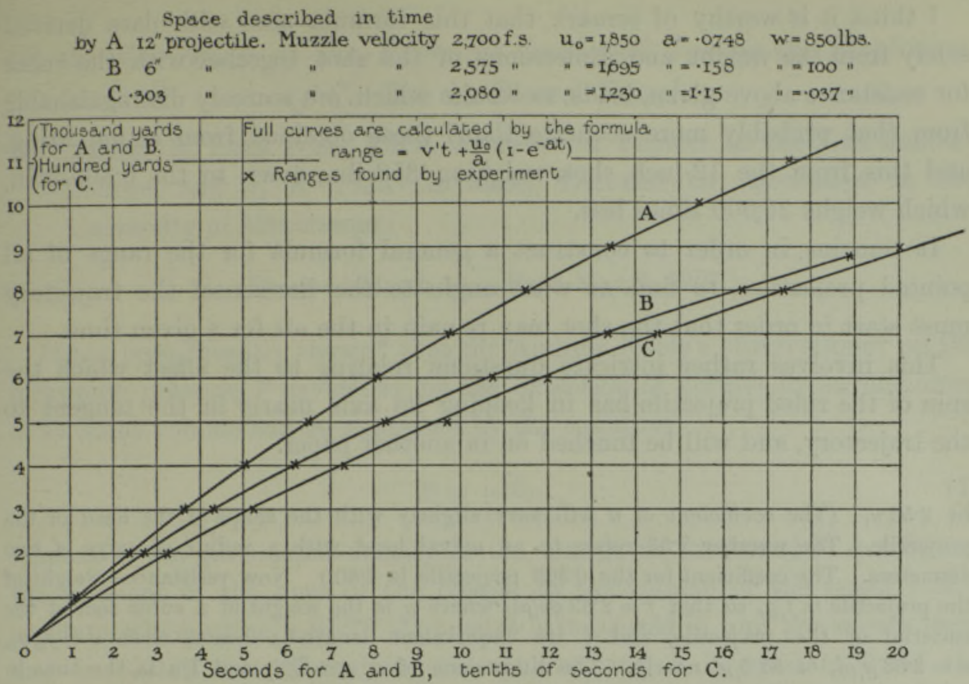
and for the remaining velocity

$$v_1 = v'+u_0e^{-at},$$

where $v' = 850$ f.s. and $v =$ initial velocity $-v'$. It is easy to plot curves of s and v in terms of t from this formula, by making a table for e^{-x} and finding the times which correspond to the various values of x.

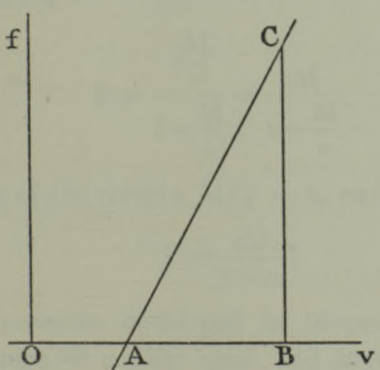
Three examples of such curves are appended in the diagram, viz., for 12-inch, 6-inch, and 0·303-inch projectiles. The results of actual practice

* Let the straight line AC be the diagram of retarding force in terms of velocity, where OB ($= v_0$) is the initial velocity and OA ($= v'$) is 850 f.s.



with the same projectiles are shown by the spots. It will be seen that the results agree very closely with actual practice, so closely indeed as to be within the error of observation.

The equation of AC is $f = -a(v-v')$. Put $v-v' = u$, $v_0 - v' = u_0$, so that $a = f_0/u_0$.



The negative velocity generated in time t by f is $\int f dt$.

Now $f = dv/dt = du/dt = -au$; hence $\log u = -at + c_1$. (1)

When $t = 0$, $u = u_0$, $\therefore c_1 = \log u_0$ and $u/u_0 = e^{-at}$ (1), $\therefore v = v' + u_0 e^{-at}$. The distance s, traversed by the projectile in time t, is $\int v dt$, and $\int v dt = \int (v'+u) dt = v't + \int u dt = v't - \frac{v_0}{a} e^{-at} + c_2$. When $t = 0$, $s = 0$, $\therefore c_2 = u_0/a$, hence $s = v't + \frac{u_0}{a}(1 - e^{-at})$. (2)

Experiment shows that the resistance in lbs. per square foot is nearly represented

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I think it is worthy of remark that this formula used with data derived solely from the weight and dimensions of the shot, together with the value for resistance above given, leads to results which are scarcely distinguishable from (but probably more accurate than) those obtained from actual firing, and this from the 12-inch shot, weighing 850 lbs., down to the 0·303-inch, which weighs 26,000 times less.

It remains, in order to construct a general formula for the range of all pointed projectiles, to find at what angle to the horizontal the trajectory must start in order that the shot may remain in the air for a given time.

This involves rather intricate questions relating to the effect which the spin of the rifled projectile has in keeping its axis nearly in the tangent to the trajectory, and will be touched on in another paper.

by $2·53\,u$. (The coefficient of u will vary slightly with the shape of the head of the projectile. The number 2·53 refers to an ogival head with a radius of ogive of two diameters. The coefficient for the 0·303 projectile is 2·60.) Now resistance ÷ weight of the projectile $= f/g$, so that $f = 2·53\,gv/\rho l$, where ρ is the weight of a cubic foot of the material of the projectile, and l its "equivalent length"; hence, since $a = f_0/u_0$, $a = 2·53\,g/\rho l$, or $81·3/\rho l$ nearly. The dimensions of a_1 are T^{-1}, and $1/a$ is the time in which u is reduced in the ratio of e to 1. If the weight of the shot is given, ρl may be replaced by $4w/\pi d^2$, so that $a = 2·53\,\pi d^2/4w$, or $6·385\,d^2/w$.