On the Approximate Arithmetical Solution by Finite Differences of Physical Problems involving Differential Equations, with an Application to the Stresses in a Masonry Dam.

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(Abstract.)

In order to deal with irregular boundaries, analysis is replaced by arithmetic, continuous functions are represented by tables of numbers, differentials by central differences. Problems then fall into two classes:—

(A) The relation between the equation obtaining throughout the body, and the boundary condition is such that the integral can be stepped out from a boundary. This class includes equations of all orders and degrees. It has been treated by arithmetical differences by Runge, W. F. Sheppard, Karl Heun, W. Kutta, and Richard Ganz. Examples of a specially simple method are given.

(B) The integral must be determined with reference to the boundary as a whole, as in Dirichlet's problem. The method given has only been worked out for a limited group of linear equations,—namely for those in connection with which a function analogous to potential energy exists, which is a complete minimum when, and only when, the difference equations are satisfied. Under this condition the difference between the finite-difference-integral ϕ_u and a function ϕ_1 of the independents, having the correct boundary conditions but otherwise arbitrary, can be expanded in the form $\phi_1 - \phi_u = \sum A_k P_k$, where the $A_1 \ldots A_n$ are constants and $P_1 \ldots P_n$ are "principal modes of oscillation" defined by $\mathfrak{D}' P_k = \lambda_k^2 P_k$, where $\mathfrak{D}' \phi_u = 0$ is the difference equation to be integrated and λ^2 is a constant. Now we start with the table of numbers ϕ_1 and calculate $\mathfrak{D}' \phi_1$. Then as $\mathfrak{D}' \phi_u = 0$, we have

$$\mathfrak{D}' \phi_1 = \mathfrak{D}' (\phi_1 - \phi_u) = \sum A_k \lambda_k^2 P_k.$$

Multiplying both sides by some number α_1^{-1} and subtracting from ϕ_1, and altering the boundary numbers so that the boundary condition is still satisfied, we have a new table which may be called ϕ_2; and

$$\phi_2 - \phi_u = \sum A_k (1 - \alpha_1^{-1} \lambda_k^2) P_k.$$

Repeating the process with $\alpha_2 \ldots \alpha_m$, we get

$$\phi_{m+1} - \phi_u = \sum A_k (1 - \alpha_1^{-1} \lambda_k^2)(1 - \alpha_2^{-1} \lambda_k^2) \ldots (1 - \alpha_m^{-1} \lambda_k^2) P_k.$$

Now a function I exists such that $\mathfrak{S}\mathrm{IP}_k^2 = 1$, $\mathfrak{S}\mathrm{IP}_l\mathrm{P}_k = 0$, where $\mathfrak{S}$ denotes a summation throughout the region. Therefore

$$\mathfrak{S}\,\mathrm{I}\,(\phi_{m+1}-\phi_u)^2 = \sum[\mathrm{A}_k(1-\alpha_1^{-1}\lambda_k^2)\ldots(1-\alpha_m^{-1}\lambda_k^2)]^2.$$

Now by a sufficient number of suitably chosen α's the polynomial in λ^2 on the right can be made small throughout the range from λ_1^2 to λ_n^2. Therefore the error of ϕ_{m+1} can be made small; for, since I is one-signed, it is measured by the L.H.S. The process is arithmetical.

Under certain conditions the error due to finite central differences is of the form $e_2h^2+e_4h^4+e_6h^6+$etc., where h is the co-ordinate difference and the e's are functions of position independent of h. If the integral has been found for two or more sizes of h, more exact values of it can be extrapolated by this formula.

These methods have been applied in the paper to calculate the stress-function in a masonry dam.

The Initial Accelerated Motion of Electrified Systems of Finite Extent, and the Reaction produced by the Resulting Radiation.

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(Abstract.)

When the exact equations of motion of a dynamical system are known, it is in general possible, by a well-known process, to determine the initial mode of change from a steady to a variable state, even when the primary equations cannot be completely integrated in the general case. The primary object of this investigation was to show that the same process is applicable to the equations of motion of finite electrified systems. That the results would be applicable to the question of the electric inertia of "electrons" was constantly kept in view; and it was felt to be undesirable to make any approximation depending on extreme smallness of the electrified system.

The various expressions hitherto used for the electric inertia of "electrons," *at speeds comparable with that of radiation*, depend, I think, without exception, on consideration of the energy of a steady state. The "quasi-stationary" principle assumes that, when the energy in a steady state is known, it is