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appears to me, important extension, by utilising an additional artifice already employed by Messrs. Hardy and Littlewood.

§ 12. We may similarly, when desirable, instead of the connected result referred to in § 10, use a property of an integral of a function of given kind of summability, such as follows from considerations exposed in my paper on "Summable Functions and their Fourier Series."*

On a Cassegrain Reflector with Corrected Field.

By R. A. Sampson, F.R.S.

(Received December 28, 1912,—Read February 13, 1913.)

(Abstract.)

The purpose of this memoir is to discover an optical appliance which shall correct in a practical manner the faults in the field of a Cassegrain reflector, while leaving unimpaired its achromatism and the characteristic features of its design, which gives a focal length much greater than the length of the instrument, combined with a convenient position of the observer. The question touches an investigation by Schwarzschild† as to what can be done with two curved mirrors the figures of which are not necessarily spherical. With these he corrects spherical aberration and coma, but in order to secure a flat field he is led to a construction in which the second mirror, which is between the great mirror and its principal focus, is concave, and therefore shortens the effective focal length, in place of increasing it. The deformations from spherical figures are also so great, especially for the great mirror, as to leave it doubtful whether the construction discussed could ever be the model for practicable instruments. If we keep to the Cassegrain form, spherical aberration and coma may equally be corrected by deformations of the mirrors which, though large, are less extreme, but there remains a pronounced curvature of the field. For this reason I am led, in the present memoir, to consider more complicated systems produced by the interposition of systems of lenses. Achromatism can be preserved completely for a single focus if there are three lenses of focal length determined when their position are given, and if all are made of the same glass. One of these lenses, which I

* 'Roy. Soc. Proc.,' 1912.

† 'K. Gesell. d. Wissenschaften zu Göttingen, Math.-Phys.-Classe,' Neue Folge, 1905, vol. 4.

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call the reverser, is silvered at the back and replaces the convex mirror; the other two are placed close together in the way of the outcoming beam, about one-third of the distance from the great mirror to the reverser; the members of this pair, which I call the corrector, are of nearly equal but opposite focal lengths, introducing very little deviation in the ray but an arbitrary amount of aberration, according to the distribution of curvatures between the two faces of each lens. All the surfaces are supposed spherical except that of the great mirror. The essential problem is to bring the necessary work into a form that will allow unknown quantities which express the distribution of curvature between the faces of each lens to be carried forward algebraically. The methods employed are those of a recent memoir by the author,* and a part of the paper is occupied in working out expressions to which this theory leads, for thin lenses, systems of thin lenses, mirrors, reversers and the like, and it may be regarded as an expansion and working illustration of that memoir. This part does not lend itself to summary. When the expressions are obtained the solution proceeds in a straightforward manner, by approximation, which is somewhat complicated owing to the number of considerations which it is necessary to keep in view, but is not otherwise difficult. The solution is completed at the stage where the unextinguished aberrations are considered negligible.

Before determining the aberrations the system was made achromatic in respect to the normal, that is to say, the linear or ideal scheme, both in respect to position and magnification of the image. Hence, at the end of the above steps, there might remain chromatic differences of the various aberrations, and it is a necessary condition that these also should prove inconsiderable. When the curves of the lenses were given by a final solution the aberrations were calculated, to the third order without approximations, for the whole system for two different refractive indices, viz., $\mu = 1{\cdot}5200$ and $\mu = 1{\cdot}535200$.

The object aimed at was to extinguish spherical aberration, coma, and curvature of the field, and to keep astigmatism down to very narrow limits. The field whose curvature is contemplated is the field which passes through the circular images that lie midway between the two focal lines in a system which is free from coma. Hence, in the system sought, the images of all points at the focal plane would be strictly circles, which increased in diameter with the square of the angular breadth of the field.

Distortion comes into the system with the separated lenses, but this does not impair the images of points, and so long as it does not reach an

* "A New Treatment of Optical Aberrations," 'Phil. Trans.,' vol. 212, pp. 149–185.

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unmanageable amount it may be dealt with as a correction, calculated and applied to any measures that are made.

In the following particulars of the construction, together with its outstanding defects, the notation, though not exactly in the standard form, almost explains itself; thus all the quantities with suffix 2, for example, relate to the reverser, a_2 being its semi-aperture, t_2 its thickness, R_2, R_2' the radii of its anterior and posterior surfaces. Similarly, the suffixes 4, 6, relate to the first and second lenses of the corrector, while the quantities d are the distances from surface to surface. The unit employed is 1 inch. The quantity ϵ gives the departure of the great mirror from a parabolic figure, on such a scale that $\epsilon_0 = 1$ would give a sphere. Hence the figure is five-sixths of the way from a sphere to a paraboloid. The greatest angle between the ray and the normal to any surface is 11°, at emergence from the second surface of the first lens of the corrector.

For comparison the particulars of a Newtonian of equal focal length and aperture and paraboloidal mirror are also given.

It may be remarked that if the solution had been made originally for the greater refractive index in place of the smaller, the residual aberrations shown by the other would apparently have been less in place of greater, so that those for which the solution was made are to be regarded as the significant ones.

Great mirror—	
Aperture	$2a_0 = 40{\cdot}0$
Radius of curvature	$R_0 = -400{\cdot}000$
Figure	$\epsilon_0 = +0{\cdot}16468$
	$d_1 = +132{\cdot}013$
Reverser—	
Aperture	$2a_2 = 16{\cdot}2$
First surface	$R_2 = +211{\cdot}603$
Silvered surface	$R_2' = +221{\cdot}289$
Thickness	$t_2 = 2{\cdot}000$
	$d_3 = +90{\cdot}676$
Corrector, first lens—	
Aperture	$2a_4 = 12{\cdot}2$
First surface	$R_4 = -144{\cdot}298$
Second surface	$R_4' = +48{\cdot}824$
Thickness	$t_4 = 1{\cdot}250$
	$d_5 = +0{\cdot}500$
Ditto, second lens—	
Aperture	$2a_6 = 12{\cdot}2$
First surface	$R_6 = -4138{\cdot}559$
Second surface	$R_6' = -38{\cdot}285$
Thickness	$t_6 = 1{\cdot}500$
Distance to principal focus	$d_7 = +71{\cdot}377$
Focal length	$f_7' = +508{\cdot}802$
Distance of principal focus beyond surface of great mirror	$+\ 33{\cdot}290$
Whole length of instrument	$167{\cdot}3$

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Specification of Field at Angular Radius 34·4′ = $\tan^{-1} 0\cdot01$.

	μ = 1·5200.	μ = 1·5352.	[Newtonian.]
Radius of least circle of aberration	0·000″	−0·007″	0·00″
„ comatic circle	−0·005	+0·069	+0·80
„ focal circle	+0·370	+0·566	−0·41
Distortional displacement	+6·75″	+7·13″	0·00
Curvature of field	−1/16282	−1/542·3	−1/508·8

On a New Analytical Expression for the Representation of the Components of the Diurnal Variation of Terrestrial Magnetism.

By George W. Walker, M.A., A.R.C.Sc.

(Communicated by Prof. J. H. Poynting, F.R.S. Received January 8,—Read January 30, 1913.)

In any enquiry as to the cause or causes that contribute to daily or seasonal change of a periodic character in any observational quantity, the primary step is the determination of a simple and comprehensive expression for the dominant features of the phenomenon.

The periodic character of the variations of an element of terrestrial magnetic effect, such as declination, horizontal force, or the equivalent geographical components of force, is evident on almost every daily record obtained. When the hourly values are set out and properly cleared from non-periodic change (a problem of considerable subtlety), the historic method is to compute the Fourier harmonic components. Another method that appears to possess great power is that so successfully carried out by Dr. W. N. Shaw in representing the daily and seasonal changes of meteorological elements by means of "isopleths."

If a Fourier analysis reveals the existence of a limited number of dominant terms, that is so far satisfactory and provides definite material on which the theorist may work. But, in the case of the terrestrial magnetic elements, there is abundant evidence to show that even four Fourier terms give a very inadequate representation of the facts, and, with the exception of Dr. Schuster's valuable memoirs based on consideration of the first two terms in the diurnal variation, the subject is in rather a dismal state.

The problem has attracted my attention for several years, and the following