the photo-electric action, on the under side of the gauze B, of the radiation from the helium atoms, may ionise the gas between the gauzes B and C. Such high-speed electrons, since they would be directed downwards, would not be detected in the velocity distribution curves. That the ionisation produced at 25·6 volts is not due to this cause, was proved by making a series of experiments, using the method first described in this paper. In the first experiment of this series, the excess of the retarding potential V_3 over the accelerating potential (V_1+V_2) was 1·4 volt. In the succeeding experiments larger values of this excess were used up to a maximum of 32 volts, but the other conditions under which the experiments were conducted remained the same throughout the series. With the larger values it is clear that photo-electrons from the gauze B would be able to ionise the gas when the accelerating potential was large enough to produce radiation from the helium atoms, but in no case was ionisation detected until the velocity of the electrons from the glowing cathode was raised to about 25·6 volts. The ionisation which is suddenly detected at this point, is thus evidently not due to the action of photo-electrons produced by the 20·4-volt radiation.

Note on the Elasticity of Metals as Affected by Temperature.

By A. Mallock, F.R.S.

(Received January 1, 1919.)

When two solids or structures, identical in form and differing only in the material of which they are composed, are subjected to stress, their strengths depend on the limits of the strains which those particular materials will bear without permanent set or rupture, but their stiffnesses are directly proportional to the elastic constants.

It is with the strength of structures that engineers are mostly concerned, and a great deal of work has been done on the strength of materials for various classes of strains, of which some, though not much, relates to the variation of strength with temperature.

As regards the elastic constants themselves, however, there has been no systematic investigation of their temperature variation, though a few measures of the kind have been made in Switzerland and America.

The subject is not without interest both from a purely scientific and a practical point of view; for, as will be seen from this note, there is a relation

between the melting points of various metals and the rate at which their rigidities alter with the temperature, and rigidity itself is the determining factor in the expressions for the natural periods of similar structures.

On examining the descriptions of all the known metallic elements which are solid at ordinary temperatures, it appeared that there were 30 whose elastic properties might be made the subject of direct experiment, and could in one way or another be formed into thin plates or rods suitable for elastic tests. These are given in the following lists :—

1. Iridium.	11. Chromium.	21. Bismuth.
2. Rhodium.	12. Uranium.	22. Cadmium.
3. Platinum.	13. Vanadium.	23. Tin.
4. Palladium.	14. Molybdenum.	24. Cerium.
5. Iron.	15. Tantalum.	25. Zinc.
6. Gold.	16. Tungsten.	26. Beryllium.
7. Silver.	17. Thallium.	27. Strontium.
8. Nickel.	18. Lead.	28. Calcium.
9. Cobalt.	19. Aluminium.	29. Lithium.
10. Copper.	20. Magnesium.	30. Arsenic.

For the preliminary experiments the following 15 were chosen on account of the ease with which specimens could be obtained, viz. :—

1. Rhodium.	6. Copper.	11. Zinc.
2. Platinum.	7. Gold.	12. Lead.
3. Iron.	8. Silver.	13. Cadmium.
4. Palladium.	9. Magnesium.	14. Bismuth.
5. Nickel.	10. Aluminium.	15. Tin.

It was proposed to measure the ratios of Young's Modulus for these metals between the temperatures of liquid air and 100° C.

Although the object of the experiments was the determination of the variation of the modulus, and not the modulus itself, direct measures of the latter were made which were in substantial agreement with those already given by other methods for such metals as have been tested.

The method used for determining the variation of the modulus depended on noting the frequencies of a vibrating "system" in which the potential energy was confined to the test piece and the kinetic energy to a rigid mass attached to it.

This had to be put in such a shape that the temperature alterations only affected the test piece.

In the first trials a "tuning-fork" was made of two stiff rods of hard wood, with two equal test pieces attached at one of the ends of each, both being clamped to a single metal support; but, owing to the limitation of the possible amplitude of vibration, and for other reasons, this plan was

abandoned in favour of a single rod and test piece, mounted as shown in fig. 1.

This arrangement was convenient, as it allowed the test piece to be immersed in fluid without the latter coming in contact with the vibrating rod, B. The cooling or heating fluid was contained in a silvered vacuum vessel about $1\frac{1}{2}$ inch in diameter. The test pieces were made in the form of plates about 1·25 inch long, 0·25 to 0·5 broad and 0·01 inch thick.

The vibration was maintained by the mechanism of an electric clock which I made about 40 years ago, and which I need not describe in detail at present (see Appendix), but whose action was to give the pendulum a light blow in the direction of motion very nearly at the instant of its passing the vertical.

With this mechanism the vibrations were satisfactorily isochronous, even when the amplitudes were made to vary 20 or 30 per cent.

The frequencies of the vibrations were recorded by connecting the maintaining mechanism with a chronograph, on which a clock simultaneously marked seconds. Fig. 2 gives a general view of the arrangement. Observa-

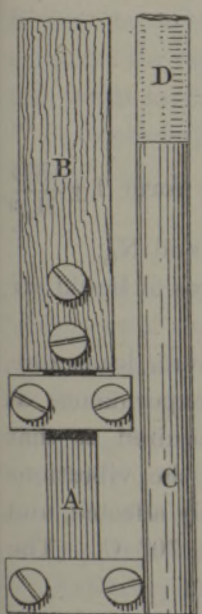

Fig. 1.

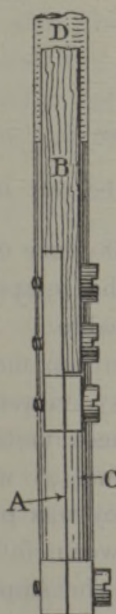

Fig. 1a.

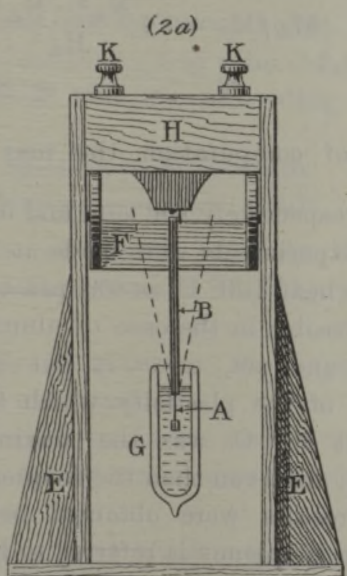

Fig. 2.

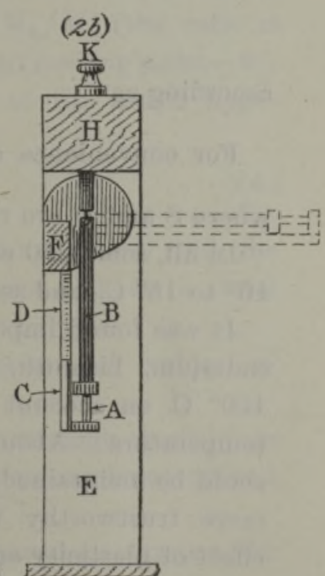

Fig. 2a.

Figs. 1 and 1a.—A. Specimen of metal to be tested. B. Vibrating rod of hard wood (dimensions 8 inches × 0·5 × 0·25). C. Brass rod support. D. Glass rod.

Figs. 2 and 2a.—A, B, C and D as in fig. 1. E. Wooden frame. F. Swinging holder for D, allowing the vibrator, B, to be brought into the horizontal position shown by the dotted lines in 2*a*. G. Silvered vacuum vessel to contain liquid air or hot water. H. Maintaining mechanism. K, K. Terminals for connections to battery and chronograph.

tions were taken at the ordinary room temperature (10° to 15° C.) both when the rod was horizontal and the vibration in a horizontal plane, and when the rod was in the upright position, in which the other temperature tests were carried out.

By means of the horizontal (H_{θ_1}) and vertical (N_{θ_1}) frequencies thus found at temperature θ_1, together with the vertical frequency N_{θ_2}, at any other temperature, θ_2, the ratio of Young's Modulus at the two temperatures ($M_{\theta_2}/M_{\theta_1}$) may be found as follows:—

When the plane of vibration is horizontal, the only accelerating force is the flexural elasticity of the test piece, and the frequency H_{θ_1} may be stated as

$$1/H_{\theta_1} = 2\pi\sqrt{(L/AM_{\theta_1})}.$$

In the same way the frequency of the vertical vibration is

$$1/N_{\theta_1} = 2\pi\sqrt{[L/(AM_{\theta_2} - g)]} \text{ at temperature } \theta_1,$$

and

$$1/N_{\theta_2} = 2\pi\sqrt{[L/(AM_{\theta_2} - g)]} \text{ at temperature } \theta_2.$$

Eliminating L, A, and g, it will appear that

$$M_{\theta_2}/M_{\theta_1} = 1 \pm \frac{N_{\theta_2}{}^2 - N_{\theta_1}{}^2}{H_{\theta_1}{}^2},$$

according as $N_{\theta_2} > \text{ or } < N_{\theta_1}$.

For convenience of computation, this may be put in the form $1 \pm \frac{SD}{H_{\theta_1}{}^2}$, where S and D are respectively the sum and difference of N_{θ_2} and N_{θ_1}.

In all, some 250 experiments were made at the temperatures of liquid air, 10° to 15° C., and as near 100° C. as was practicable.

It was found impossible in the case of aluminium, magnesium, lead, zinc, cadmium, bismuth, and tin, to carry out any vibration experiments at 100° C. on account of the plasticity which these metals acquired at that temperature. About 80° C. was the maximum at which the vibrations could be maintained, and even then the frequency was probably affected, and more trustworthy results were obtained between 50° and 70° C. (The effect of plasticity on frequency is referred to in the Appendix.)

The collective results of all the experiments are given in the Table on p. 433.

It will be seen at a glance that the more infusible metals are least affected by temperature, and this suggests that there may be a relation between the melting point and the variation of the elastic constants.

That the modulus itself does not bear any simple relation to the melting point is clear from an inspection of tables of the latter, but this does not prove that its variation may not do so.

No.	Metal.	Melting points, absolute temperature.	Ratio of Young's Modulus at 0 Abs. to Young's Modulus at 0 Centigrade.
1	Rhodium	2270°	1·18
2	Platinum	2045	1·27
3	Iron	1870	1·27
4	Palladium	1770	1·27
5	Nickel	1720	1·12
6	Copper	1324	1·37
7	Gold	1315	1·32
8	Silver	1224	1·37
9	Magnesium	903	1·57
10	Aluminium	870	1·44
11	Zinc	690	2·00
12	Lead	595	1·80
13	Cadmium	585	2·50
14	Bismuth	530	1·85
15	Tin	500	2·22

As a tentative hypothesis with which to compare the results of experiment, it may be assumed that the variation of the modulus with temperature is constant for each metal, and that the variation makes $M_{\theta_2}/M_{\theta_1}$ (the ratio at temperatures θ_2 and θ_1) proportional to (melting point $-\theta_2$)/(melting point $-\theta_1$). If θ_2 = absolute zero and $\theta_1 = 0°$ C.—then, according to the above hypothesis,

$$\frac{M_{\theta_2}}{M_{\theta_1}} = \frac{\text{Melting point absolute}}{\text{Melting point centigrade}}. \qquad \text{(A)}$$

In the diagram (fig. 3) the abscissa is absolute temperature, and the ordinates of the full curve are the values of $M_{\theta_2}/M_{\theta_1}$ from (A), for a metal whose melting point is θ.

In the same diagram the spots show the experimental values of $M_{\theta_2}/M_{\theta_1}$ for the various metals examined.

There is a distinct resemblance between the mean line through these spots and the full curve, though the discrepancies are more than can be accounted for by experimental errors.

Arbitrary formulæ of very various types might be used to express the results more closely, but I will not mention any of these at present.

It must be remembered that Young's Modulus is a complex quantity involving both rigidity and volume compressibility, and I hope shortly to carry out further trials on the variation of rigidity alone. It is quite possible that these would give results which are somewhat different, and perhaps more congruent with the simple temperature relation suggested in (A).

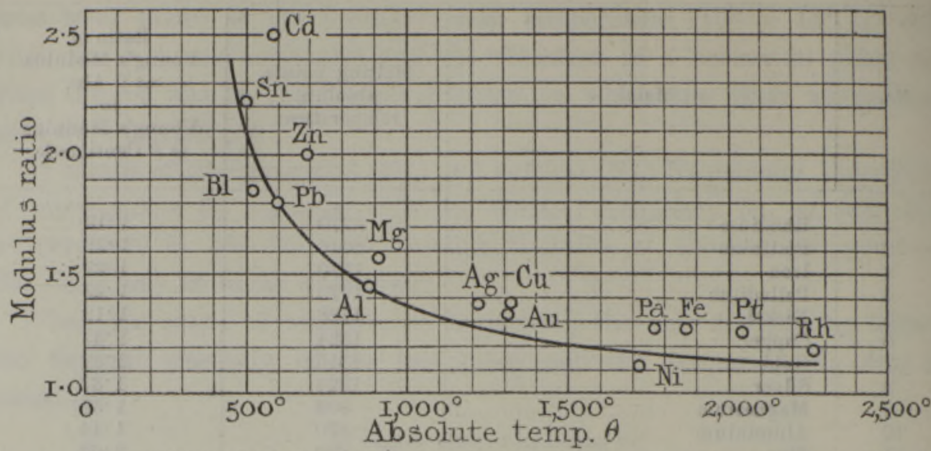

FIG. 3.—The ordinates of the full curve show (on the assumptions made in forming the expression A) the ratio of Young's Modulus at Absolute zero to that at zero Centigrade (or at any other two temperatures having the same difference), for a metal whose melting point is given by the abscissa θ. The circles show the values of this ratio as found by experiment on the different selected metals.

APPENDIX.

(1) *The Maintaining Mechanism, and its Effect in Altering the Natural Period of the Vibrator.*

Since the value of this series of experiments depends in a great measure on the maintenance of isochronous vibrations, a description of the mechanism used for that purpose is here given. A small crank shaft (fig. 4), weighing a few grains, is provided with two arms, A and B, whose motion is limited by the stops C_A, C_B. Silk threads, D_A, D_B, are attached at one end to the springs S_A, S_B, and at the other to the cranks. The mechanism, which is actuated by electromagnets, causes D_A and D_B to be put into tension by S_A and S_B alternately, at the times when A or B are in contact with C_A or C_B. Thus the crank shaft is naturally in unstable equilibrium unless A or B are displaced. A very small displacement of either causes the crank to rotate through 90°, with what velocity the acting spring can give it. Having done so the acting spring is released and the other thread is brought into tension. In the clock referred to, the pendulum, in passing through the vertical, displaced A and B, and received a blow from B or A in the direction of motion. For this purpose the end of the pendulum was provided with a tripping piece, E, whose dimension, e, in the direction of motion, was rather greater than $A'B\,(=b)$. The velocity of the pendulum, when passing the vertical, is $2\pi a/\tau$ (where a is the amplitude and τ the period), and is ultimately such that, if the energy which could be supplied by the springs is

greater than that lost by the pendulum in each swing, the impacts of A and B are divided between E and the stop, until $2\pi a\Delta t/\tau = b-e$, where Δt is the small time which the crank takes to make a quarter revolution. Thus any desired amplitude, a, may be maintained by giving a suitable value to e.

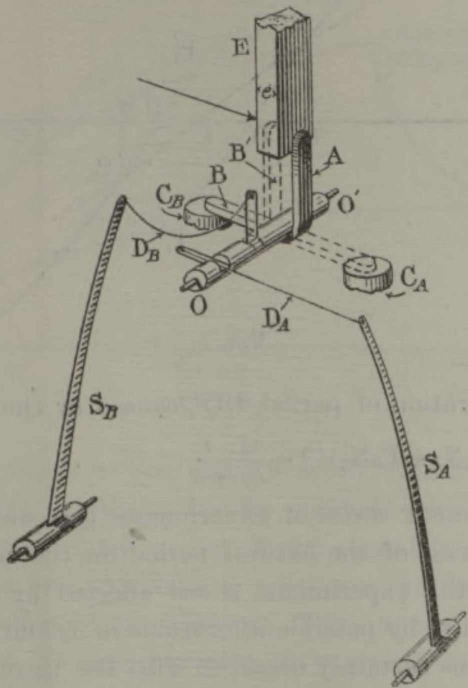

FIG. 4.—Diagrammatic sketch of the maintaining mechanism. O O′. Crank shaft. A, B. Arms on the shaft which give the vibrator, E, a blow in its passage through the equilibrium position. C_A, C_B. Stops limiting the motion of A and B. D_A, D_B. Silk threads from the cranks to the actuating springs S_A, S_B.

To examine the effect of the maintaining mechanism on the period, suppose that, immediately after an impact, the virtual amplitude is a_0 (that is, that in the absence of resistance, the displacement of the pendulum from the vertical is $a_1 \sin 2\pi C/\tau$). Before the end of the first half period, the virtual amplitude will have been reduced by resistance to a_1, at which time it will be further reduced to a_2 by setting in motion A or B. After the interval Δt, E receives the blow which restores the virtual amplitude to a_0.

In fig. 5 let P_1P_1' be part of the sine curve of amplitude a_1, and P_1 the point at which E first engages with the maintainer. Let P_1P_2 be the part of curve whose amplitude is a_2, P_2 being the point at which E receives the blow restoring the amplitude to a_0. The curve of amplitude a_0 is indicated by $P_2'P_2$. In general, the difference between a_0 and a_1 may be neglected, for in any escapement or maintaining mechanism by far the greater part of the

work done is in making good the loss occasioned by its own resistances. If, for the present purpose, a_1 and a_0 are treated as being identical, it can be easily

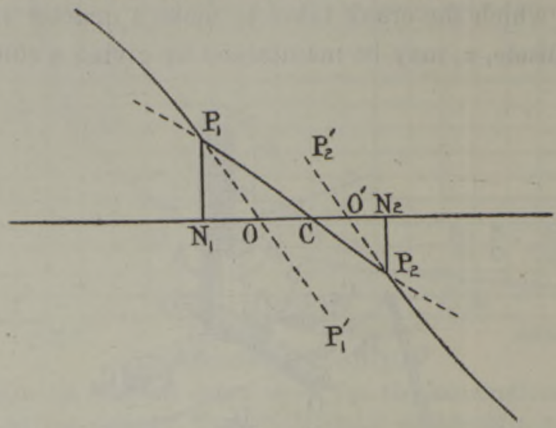

FIG. 5.

shown that the alteration of period, OO′, caused by the action of the maintainer is equal to $(P_1N_1+P_2N_2)\left(1-\frac{a_0}{a_2}\right)\frac{\tau}{\pi a}$.

In most of the present series of experiments this works out at about six parts in 10,000 in excess of the natural period for the more infusible metals.

The usefulness of the experiments is not affected by any constant alteration of period, but only by possible alterations in a_0 and a_1/a_2 with temperature. Such alterations certainly occurred with the more fusible metals, and the deductions as to the change in Young's Modulus are somewhat less exact in their case than for the rest.

(2) *Young's Modulus for Plastic Materials.*

A material such as pitch yields continuously even to the smallest stress, but yet partially recovers its shape when the stress is removed. It is clear, therefore, that in this case the definition of Young's Modulus (viz., the stress which doubles the length of an element of unit cross-section) must be modified.

The easily fusible metals behave in something the same way even at temperatures below 100° C., but with this difference: that they do not continue to yield if the applied stress is small. So long as the stress is confined within certain limits, the strain also gradually approaches a limit as the time for which the stress acts is prolonged.

I may refer to a former paper of mine,* at the end of which the behaviour of zinc under stress is considered at some length. Dia. 10 in that paper will

* "Some Measures of Young's Modulus," 'Roy. Soc. Proc.,' vol. 49 (1891).

show that the strain in terms of time for a given stress may be represented by some such expression as $A(1-e^{-c_1t})+B(1-e^{-c_2t})$, where the first term refers to elastic yielding, and the second to permanent set (fig. 6).

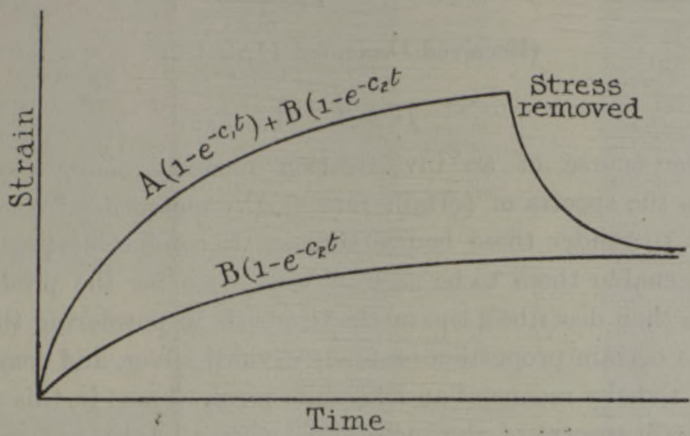

FIG. 6.

If during the application of the stress a small vibration, having a period short compared to $1/c_1$, is set up in the material, the elasticity deduced will depend almost entirely on c_1, but if the period is long, the apparent elasticity will involve c_2 and will indicate a lower value for Young's Modulus, and also a rapid extinction of the vibration. For zinc and other plastic metals the apparent values of Young's Modulus vary greatly with the method of measurement employed.

The loss of amplitude for the n^{th} swing will be of the order $B(e^{-ntc_2}-e^{-(n+1)tc_2})$. If $1/c_2$ is large compared to t (the time for which the stress operates), the permanent set will be given by the straight line Bc_2t, and in this way the expression may be made to represent the yielding of a purely viscous material.