Some Problems connected with Evaporation from Large Expanses of Water.

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Introductory.

1. The water which falls as rain is originally evaporated either from land surfaces, forests, etc., or from the oceans, in particular from the latter. It is therefore of importance to ascertain how the evaporated water distributes itself in an atmospheric current during its progress across a water surface, and, further, to investigate the relation between the length of the path over the water surface and the amount of water thereby rendered available. In this paper, these problems are treated for the case of a current of air of uniform speed moving over a water surface of uniform temperature. An empirical formula is employed to represent the rate of evaporation from each element of the water surface, and account is taken of the stirring upwards of the evaporated water by the agency of turbulence, assuming the latter to be uniformly distributed throughout the current of air. The results obtained would find application, for example, in discussing problems connected with evaporation in the trade wind zones, or from inland seas and lakes, or, in particular, from the North Sea, winds from which frequently bring much cloud and quite appreciable rainfall to the eastern coasts of Great Britain and the northern coasts of France. All the constants occurring in the formulæ obtained are known, either accurately or approximately, and, accordingly, the order of magnitude of the evaporation from a given stretch of water, under assigned conditions, can be estimated. What is also of importance, is the comparison of the amount of water evaporated from a given area or of the distribution of water vapour in the air above it, under one set of conditions, with those under a different set. Of particular interest is the effect of varying the speed of the air.

2. The suggestion which led to the present investigation is contained in a paper on "The Meteorological Conditions of an Ice Sheet and their bearing on the Desiccation of the Globe."* The author of that paper, in considering the evaporation in tropical and sub-tropical regions during the Quaternary Ice Age as compared with that at the present time, has occasion to compare the rate of evaporation from *large* expanses of water under different

* C. E. P. Brooks, 'Quarterly J. Roy. Meteorolog. Soc.,' vol. 40 (1914).

conditions as to wind and surface temperature, and Lt.-Col. E. Gold, in the discussion, emphasises the fact that this is in reality a very complex problem for the formula used (quoted below, p. 474) for the rate of evaporation from an element of the surface, involves the vapour pressure at the dew point of the air "near" the surface. Now this vapour pressure varies from point to point along the current of air which is traversing the water, so that an integration is necessary in order to obtain the rate of evaporation from a large stretch of water, and, further, the vapour pressure at any point depends on the speed with which the air has reached that point, which complicates the problem of comparing the evaporation under different wind conditions.

3. Before proceeding further, it should be mentioned that the quantity described as the vapour pressure "near" the surface is not so indefinite as might at first sight appear to be the case. It is known that, though the air flowing over a water surface is in general in a turbulent condition, yet, next to the water, there is a thin layer, consisting of mixed air and water vapour, the molecules of which are only very slowly replaced by those from the turbulent region above, and which probably remains unbroken by waves, unless conditions are such as to produce spray. In this layer the diffusion is purely molecular, but, beyond it, turbulence endows the air with a greatly increased diffusivity for water vapour. The consequence is that the humidity in general decreases very rapidly from saturation actually at the water surface to some lower value at the point where the turbulent *régime* is entered, after which the change, with increasing distance from the surface, is comparatively slow. Jeffreys* estimates the thickness of this surface layer as of the order of 1 mm. or more, while Bigelow†, in the account of his practical researches in evaporation, refers to a thin film, next to the water surface, in which the humidity passes rapidly from saturation to its value near the surface where the tubes of flow of water vapour are "distorted, broken, carried away in the wind, and converted into a heterogeneous mass of shattered tubes." He states that the gradient of vapour pressure is rapid in the first few millimetres and then slow up to great heights. Marvin,‡ too, states that, close down to the free water surface there must be a thin layer of air, heavily charged with water vapour, which, from our knowledge of viscosity and the flow of fluids, is, in spite of moderate wind action, changed and renewed by the wind only with relative slowness. He points out that the dense vapour sheet must be

* H. Jeffreys, "Some Problems of Evaporation," 'Phil. Mag.,' p. 35, March, 1918.

† F. H. Bigelow, "Studies of the Phenomena of the Evaporation of Water over Lakes and Reservoirs," 'Monthly Weather Rev.,' 1907, 1908, 1910; also 'Bulletin of the Argentine Meteorological Office,' No. 2, Part I, March, 1911: Part II, July, 1912.

‡ C. F. Marvin, "A Proposed New Formula," 'Monthly Weather Rev.,' February, 1909.

very thin, for observations at Reno* showed that the humidity one half-inch above the water surface differed little from the value two or three feet higher. It is the region just outside the thin superficial layer which is referred to as "near" the surface.

4. Reference to the annotated bibliography of evaporation prepared by Mrs. Grace J. Livingston† shows that a number of empirical formulæ have, from time to time, been given to represent the rate of evaporation from an exposed water surface. In evaluating the constants in these formulæ, observations of the evaporation from water in pans have generally been used, and there is abundant evidence to show that the results so obtained cannot be transferred to evaporation from the surface of a large expanse of water without correction. A few investigators have, however, taken account of this aspect of the problem, so that there are empirical formulæ available which are designed to apply to the evaporation from an element of a large water surface.

Of the various empirical formulæ the one commonly known as Dalton's has received a great deal of support. Its form is

$$E = A(P_W - p_0)(1 + cv),$$

where E is the rate of evaporation per unit area, P_W the maximum water vapour pressure corresponding to the temperature of the water surface, p_0 the actual water vapour pressure in the air "near" the surface, v the speed of the air "near" the surface, and A and c constants. The constants A and c have been evaluated by a number of investigators, but there is lack of agreement between the results. This is partly due to the use of evaporating pans of different sizes and exposure. Fitzgerald‡ and Carpenter,‡ however, working independently with various exposures, including tanks or pans floating in water, obtained values of A in very good agreement, but widely different values were obtained for the wind coefficient c. This may be partly explained by the fact that while Fitzgerald measured the wind near the surface of the water, Carpenter used records from an anemometer on the roof of a building away from the tanks. The most comprehensive series of experiments in this field of research is that of Bigelow (*loc. cit.*) who, however, was lead to a different formula.§ Nevertheless he recognises Dalton's formula as approximate and

* F. H. Bigelow, "Studies of the Phenomena of the Evaporation of Water over Lakes and Reservoirs," 'Monthly Weather Rev.,' February, 1908.

† 'Monthly Weather Rev.,' June, September, November, 1908; February, March, April, May, June, 1909.

‡ "Annotated Bibliography of Evaporation," Mrs. Grace J. Livingston, 'Monthly Weather Rev.,' March, 1909.

§ Bigelow's formula is $$E = B \frac{P_W}{p_0} \left(\frac{dP}{dT}\right)_W (1 + cv),$$

where P^W, p_0, v and c, have the same significance as in Dalton's formula, B is a constant

gives values for the constants in it, applicable to open water and based on his extensive observations in places with climates sufficiently different to ensure a wide range of meteorological conditions. The remaining formulæ need not be discussed in detail, for none, up to the present, seems to have been established as giving a better representation than the above, of the rate of evaporation from an element of the surface of a large expanse of water. Dalton's formula leads to a readily integrable case and will, therefore, be adopted. For numerical examples Bigelow's estimate of the coefficients in it must be accepted as the best available.

5. The problem is that of the upward diffusion of water vapour in a current of air of uniform speed, with a boundary condition at the surface given by the above empirical formula. The upward flux of water vapour per unit area at any level is represented by the expression $-K\rho\, \partial\mu/\partial z$, where μ is the mass of water vapour per unit mass of air, z the height above the surface, ρ the density of the air and K the coefficient of eddy-diffusivity. Evidence is gradually being collected as to the values which K may assume, and measurements by Richardson* indicate an increase with height, at least in the first 1000 metres above the surface. However, in the present instance, only a first approximation will be made and K will be assumed independent of height z.

Notation.

The system is referred to rectangular axes with the origin at a fixed point "near" the water surface, and the z-axis vertical, while the air is supposed moving horizontally in the direction of the y-axis. The motion is turbulent, but the mean† conditions at any point (y, z) are supposed constant and the following notation is used to represent them:—

ρ, the density of the air.

μ, the mass of water vapour per unit mass of air.

W, the absolute humidity $(= \mu\rho)$.

H, the relative humidity.

T, the absolute temperature.

p, the actual pressure of water vapour.

and $\left(\frac{dP}{dT}\right)_W$ is the rate of change of maximum water vapour pressure with temperature at the temperature of the water surface.

* L. F. Richardson, "Some Measurements of Atmospheric Turbulence," 'Phil. Trans.,' A, vol. 221 (1920).

† Time average over an interval centred at any particular instant and sufficiently long to smooth out the minor fluctuations due to turbulence. The turbulence is assumed uniformly distributed, so that the interval would be the same for all parts of the air current. A more rigorous definition hardly seems necessary here.

P, the maximum water vapour pressure at temperature T.

v, the velocity of the air,

K, the coefficient of eddy-diffusivity in a vertical direction.

t, the time taken by a sample of air to travel a distance y from origin $(= y/v)$.

A suffix 0 denotes conditions at the level $z = 0$, while a dash denotes conditions at $y = 0$. T_W denotes the temperature of the water surface and P_W the corresponding maximum vapour pressure.

Mathematical Analysis.

6. Let D/Dt denote $\partial/\partial t + v\partial/\partial y$, then if the vertical flux of water vapour at any point is $-K\rho\,\partial\mu/\partial z$ and the horizontal flux $-K_y\rho\,\partial\mu/\partial y$, where K_y is used for the moment to denote the coefficient of eddy-diffusivity in a horizontal direction, the "equation of continuity" gives,

$$\frac{D}{Dt}(\rho\mu) = \frac{\partial}{\partial y}\left(K_y\rho\frac{\partial\mu}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\rho\frac{\partial\mu}{\partial z}\right).$$

Now K_y may or may not be equal to K, but in any case the first member on the right is negligible in comparison with the second, except perhaps in the immediate vicinity of the origin $y = 0$, in consequence of the very slow horizontal variations of the meteorological elements in this problem as compared with the vertical variations. If, further, the state is steady, so that $\frac{\partial}{\partial t}(\rho\mu) = 0$, then

$$v\frac{\partial}{\partial y}(\rho\mu) = \frac{\partial}{\partial z}\left(K\rho\frac{\partial\mu}{\partial z}\right).$$

K is to be taken independent of z and thus, neglecting $\partial\rho/\partial z$

$$\frac{\partial\mu}{\partial y} = \frac{K}{v}\frac{\partial^2\mu}{\partial z^2}.$$

The rate of evaporation per unit area, $A(P_W - p_0)(1 + cv)$ must equal the value of $-K\rho\,\partial\mu/\partial z$, the vertical flux, at $z = 0$, so that the boundary condition to be satisfied there is

$$(-K\rho\,\partial\mu/\partial z)_0 = A(P_W - p_0)(1 + cv).$$

Now $p = \alpha TW = \alpha T\rho\mu$, where $\alpha = 4{\cdot}616 \times 10^6$ when C.G.S. units are used,* and thus the boundary condition becomes

$$(\partial\mu/\partial z)_0 = -h(1 - \beta\mu_0)/\beta,$$

where $$\beta = \alpha T_0 \rho_0 / P_W$$

and $$h = A\alpha T_0(1 + cv)/K,$$

* 'Computer's Handbook,' Meteorological Office, 223, Introduction. London, 1916.

We require, therefore, a solution of

$$\frac{\partial \mu}{\partial y} = \frac{K}{v}\frac{\partial^2 \mu}{\partial z_2}$$

with conditions $$(\partial\mu/\partial z)_0 = -h(1-\beta\mu_0)/\beta$$

and $$\mu' = f(z),$$

the latter denoting the arbitrary distribution of water substance above the origin.

Write $$\Omega = 1-\beta\mu.$$

Then $$\frac{\partial \Omega}{\partial y} = \frac{K}{v}\frac{\partial^2 \Omega}{\partial z^2}$$

with conditions $$(\partial\Omega/\partial z)_0 = h\Omega_0$$

and $$\Omega' = f_1(z)$$

where $$f_1(z) = 1-\beta f(z).$$

If v and T_0 are supposed independent of y and if vertical variations of v are neglected, the solution* is

$$\Omega = h\int_0^\infty \phi(z+\zeta, t)\, e^{-h\zeta}\, d\zeta,$$

where $$\phi(z, t) = \frac{1}{2\sqrt{(\pi K t)}}\int_0^\infty F(z')\left[e^{-(z-z')^2/4Kt} - e^{-(z+z')^2/4Kt}\right] dz',$$

$$F(z) = f_1(z) - f_1'(z)/h \qquad \text{and} \qquad t = y/v.$$

This solution holds for the arbitrary distribution of water vapour at $y = 0$ given by $\mu' = f(z)$. The expression may be converted into one containing known functions only if $f(z)$ is taken as a constant, which describes the case of air which has descended or passed off dry land where it has been well stirred up. Then introducing the notation $\sigma = zh/2$ and $\tau = h\sqrt{(Kt)}$, and omitting the analysis, the solution becomes

$$\Omega/\Omega' = \Theta(\sigma/\tau) + e^{-(\sigma/\tau)^2}\psi(\sigma/\tau+\tau),$$

$$= \chi(\sigma, \tau),$$

where $$\Theta(x) = \frac{2}{\sqrt{(\pi)}}\int_0^x e^{-u^2}\, du,$$

and $$\psi(x) = \frac{2}{\sqrt{(\pi)}}\int_x^\infty e^{x^2-u^2}\, du.$$

Note that $$\psi(x) = \chi(0, x).$$

* See Carslaw, 'Introduction to Fourier Series,' Macmillan.

Relative humidity may now be introduced if we note that

$$\Omega = 1-\beta\mu = 1-\frac{\alpha T_0\rho_0 u}{P_W} = 1-\frac{PT_0\rho_0}{P_W T\rho}\cdot\frac{\alpha T\rho\mu}{P}$$

$$= 1-\frac{PT_0\rho_0}{P_W T\rho}\cdot\frac{p}{P} = 1-\frac{H}{\lambda},$$

where $\lambda = P_W T\rho/PT_0\rho_0$.

Also

$$\Omega' = 1-\beta\mu' = 1-\frac{\alpha T_0\rho_0\mu'}{P_W} = 1-\frac{P_0}{P_W}\frac{p_0'}{P_0} = 1-\frac{P_0}{P_W}H_0'.$$

While it is not essential, we shall take the surface air temperature equal to that of the surface water, so that $T_0 = T_W$ and $P_0 = P_W$.

Then $\Omega/\Omega' = (1-H/\lambda)/(1-H_0')$, but $\Omega/\Omega' = \chi(\sigma, \tau)$ and hence the relative humidity at any point is given by

$$H = \lambda\{1-(1-H_0')\chi(\sigma, \tau)\}.$$

If $z = 0$, then $\sigma = 0$ and $\lambda = 1$, and the expression for the relative humidity "near" the surface is

$$H_0 = 1-(1-H_0')\psi(\tau).$$

The rate of evaporation per unit area at any point of the surface, given by Dalton's formula, $A(P_W-p_0)(1+cv)$, may be written $AP_0(1-H_0)(1+cv)$, which on substituting for H_0 becomes $E'\psi(\tau)$, where $E' = AP_0(1-H_0')(1+cv)$, the rate of evaporation per unit area at the origin.

The mass of water evaporated per unit time from a strip of unit breadth, extending from the origin a distance y along the y-axis is accordingly given by

$$E'\int_0^y \psi(\tau)\,dy \quad \text{where} \quad \tau = h\sqrt{(Ky/v)}.$$

Performing the integration, the result is

$$E'y\nu(\tau) \quad \text{where} \quad \nu(\tau) = [2/\tau\sqrt{(\pi)}-\{1-\psi(\tau)\}/\tau^2].$$

The average rate over the strip, or the mean depth evaporated from the strip per unit time, is thus $E'\nu(\tau)$. Again, the rate of evaporation from the strip of length y must equal the difference between the rate at which water is carried across a vertical plane normal to the current at distance y from the origin and the rate at which it is carried across a similar plane at the origin, since a steady state is considered and there is no accumulation of water vapour in any region. Hence, if all the water gained by a column of air on a base of unit area in passing a distance y over the water surface could be precipitated instantaneously, the depth of rain produced would be

$$E'y\nu(\tau)/v \qquad \text{or} \qquad E't\nu(\tau).$$

7. Collecting the more important formulæ, we find that the relative humidity at any point, under the conditions laid down in the course of the preceding work is given by

$$H = \lambda\{1-(1-H_0')\chi(\sigma, \tau)\}, \tag{1}$$

where
$$\sigma = zh/2; \qquad \tau = h\sqrt{(Ky/v)},$$

$$h = A\alpha T_0(1+cv)/K; \qquad \lambda = P_0T\rho/PT_0\rho_0.$$

"Near" the surface the relative humidity is,

$$H_0 = 1-(1-H_0')\psi(\tau). \tag{2}$$

The rate of evaporation, per unit area, at any point of the surface is

$$E'\psi(\tau), \tag{3}$$

while the mean depth of evaporation, per unit time, from a strip of length y, extending from the origin, is

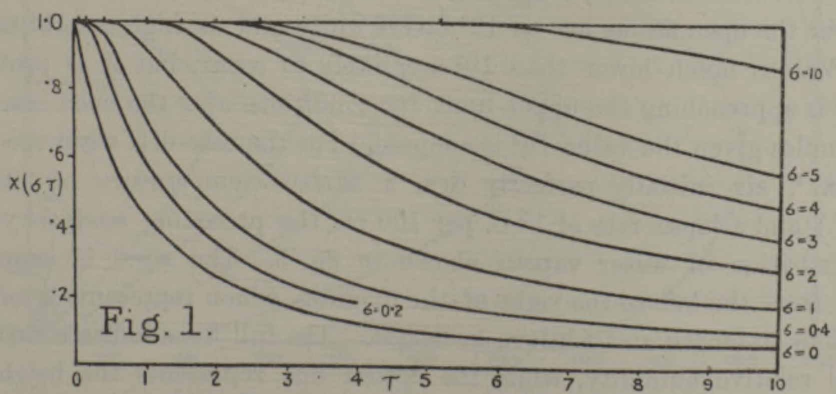
$$E'\nu(\tau), \tag{4}$$

where E' is the rate of evaporation, per unit area, at the origin, and equals $AP_0(1-H_0')(1+cv)$.

Finally, the water gained by a column of air on a base of unit area in passing a distance y over a water surface, would, if it could all be precipitated instantaneously, produce a depth of rain equal to

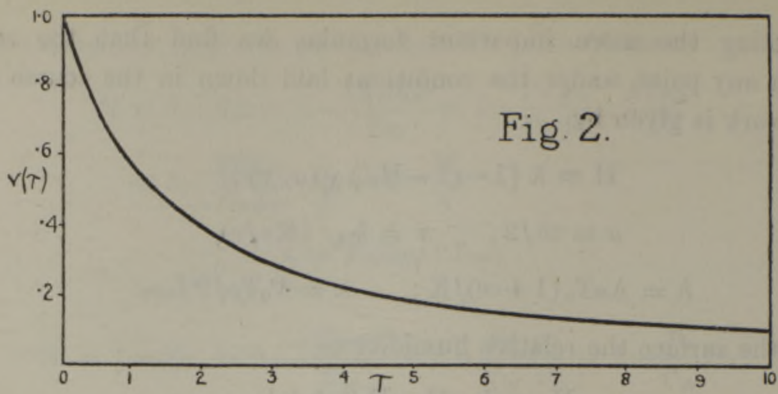
$$E't\nu(\tau). \tag{5}$$

Fig. 1 shows the way in which $\chi(\sigma, \tau)$ depends on σ and τ. Each curve



Representing the functions $\chi(\sigma, \tau)$ and $\psi(\tau)$.

represents it as a function of τ for a given value of σ. The curve for $\sigma = 0$ represents the function $\psi(\tau)$. $\nu(\tau)$ is given in fig. 2.



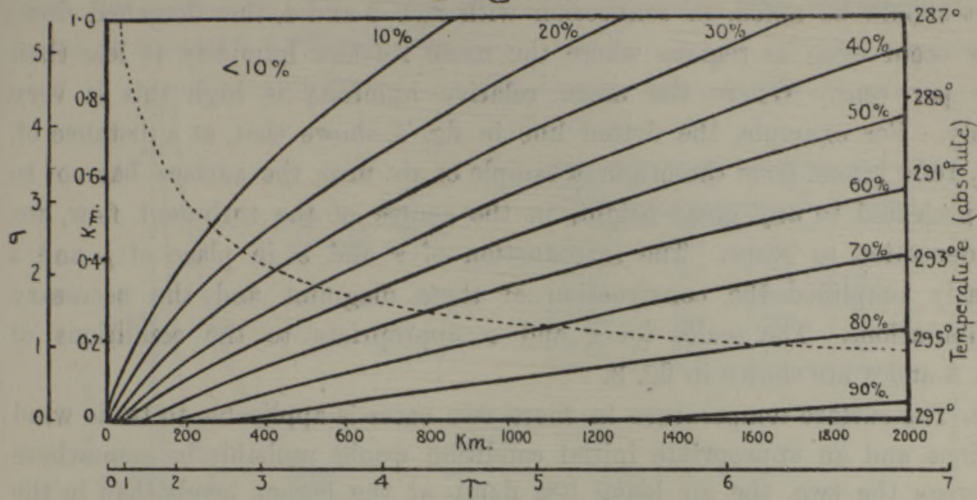
Representing the function $V(\tau)$.

Numerical Examples.

8. However, before discussing these results, it will be well to introduce some numerical values for the constants in them. Taking, as mentioned above, Bigelow's estimate (*loc. cit.*) for the constants in the evaporation formula, and converting them into C.G.S. units, we obtain $A = 3{\cdot}14 \times 10^{-10}$ and $c = 3{\cdot}024 \times 10^{-3}$, when E is measured in cm. sec.$^{-1}$, or g. sec.$^{-1}$, v in cm. sec.$^{-1}$ and the vapour pressures in dynes cm.$^{-2}$. These figures are for fresh water. For salt water it is thought that the evaporation should be somewhat less, but the correction is not known definitely and depends on the salinity. A further complication, of which the effect is unknown, is the formation of spray, which must, when forming, seriously disturb the restraining superficial vapour film referred to in an early paragraph. The value of K is not a constant of the atmosphere, and, indeed, may vary over a very large range. Richardson (*loc. cit.*), by observations on the scattering of smoke from a steamer, has found a value over the open sea as low as 10^2 C.G.S. units and as high as 10^4 C.G.S. units. Values much lower than 10^2 are likely to occur, but it is probable that 10^4 is approaching the upper limit for conditions over the open sea. In the examples given the value 10^4 is adopted. For the case of a wind speed of 5 m. sec.$^{-1}$, air initially perfectly dry, a surface temperature of 297° a. (75·2° F.), and a lapse rate of 1° C. per 100 m., the preceding analysis yields the distribution of water vapour shown in fig. 3. The wind is supposed blowing from the left to the right of the diagram, which represents a section 2000 kilom. in length and 1 kilom. in height. The full lines indicate surfaces of equal relative humidity, while the dotted line represents the height at which cloud would commence to form in a sample of air transferred from the surface directly upwards without loss or gain of heat. In fig. 4 the mass of water vapour per mass of air is supposed initially (*i.e.*, on the left of the diagram) constant at all heights, and such as to give a relative humidity of

50 per cent. at the surface, while the remaining conditions are the same as in fig. 3. This initial condition implies cloud above the height of about

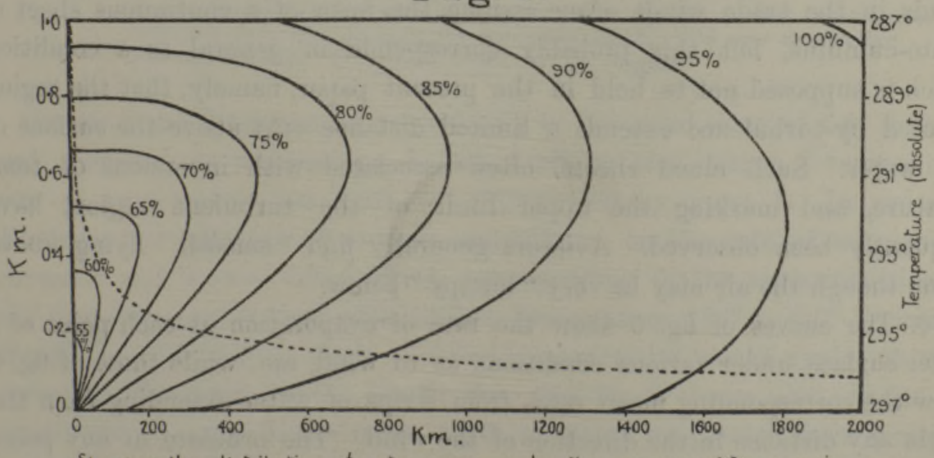
Fig. 3.



Showing the distribution of water vapour in a current of air, initially dry, flowing with uniform speed of 5 m. sec^{-1} over a water surface of temperature 297°a (= 75·2°F), the lapse rate of temperature being 1°C per 100 m and the coefficient of eddy-diffusivity 10^4 C.G.S. units

1350 m., and the diagram shows that this cloud gradually descends until at 2000 kilom. from the origin its base is in the neighbourhood of 1 kilom. above the surface, the 100 per cent. isopleth coming just below this level on

Fig. 4.



Showing the distribution of water vapour under the same conditions as in fig (3) but with the mass of water vapour per mass of air initially (i.e on the left of the diagram) the same at all heights and such as to produce a relative humidity of 50% at the surface.

the right of the diagram.* This is, of course, only an approximate representation, and no account is taken of the thermo-dynamical complications due to latent heat or of precipitation.

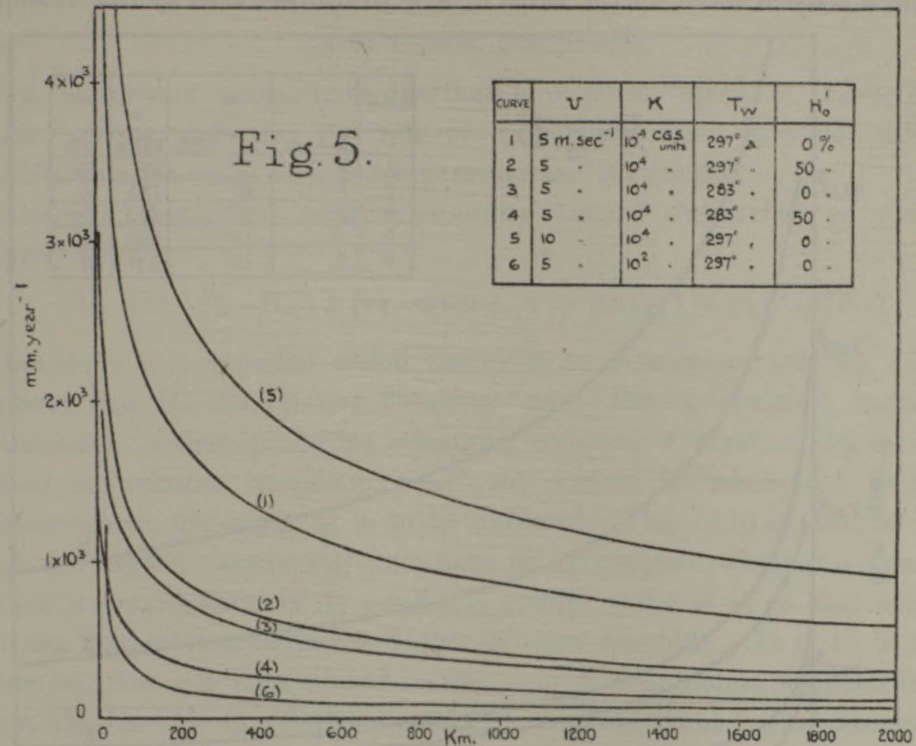
It should be noted, in connection with figs. 3 and 4, that detached cloud may occur even in regions where the mean relative humidity is less than 100 per cent. Where the mean relative humidity is high this is very likely. For example, the dotted line in fig. 4 shows that, at a distance of, say, 1600 kilom. from the origin, a sample of air near the surface has not to be projected to any great height, in the course of the turbulent flow, for condensation to occur. The introduction of τ and σ in place of y and z greatly simplified the construction of these diagrams and the necessary computations. The scales for τ and σ appropriate to the conditions of figs. 3 and 4 are shown in fig. 3.

9. The surface temperature in these two cases is applicable to trade wind regions, and an appropriate initial condition would probably be somewhere between the two, the air being less damp at the higher levels than in the second example. The case represented would be one in which the trade wind clouds are detached cumuli or entirely absent, for a continuous sheet of cloud would imply a mean condition of saturation. The dotted lines in figs. 3 and 4, while marking the condensation level for air rising directly upwards from the surface, would not necessarily mark the level of the bases of such cumuli, for they may be formed by condensation in a rising column of air formed by samples gathered from various levels. They set, however, in these examples, a lower limit to this level. Observations show that the clouds in the trade winds often assume the form of a continuous sheet of strato-cumulus, but this probably corresponds in general to a condition which is supposed not to hold in the present paper, namely, that the region affected by turbulence extends a limited distance only above the surface of the water. Such cloud sheets, often associated with inversions of temperature, and marking the upper limit of the turbulent region, have frequently been observed. Aviators generally find "smooth" flying above them, though the air may be very "bumpy" below.

10. The curves of fig. 5 show the rate of evaporation at each point of a water surface under various conditions as to wind, etc., while those of fig. 6 show the corresponding mean rates from strips of water extending from the origin any distance in the direction of the wind. The ordinate at any point in fig. 6 is accordingly equal to the mean ordinate, between that point and

* The 100 per cent. isopleth is to be distinguished from the dotted line in the diagram. The latter merely marks the condensation level for air rising directly upwards from the surface.

the origin, of the corresponding curve in fig. 5. Thus, under the conditions of curve 1, fig. 5 shows that the rate of evaporation at a point distant 1000 kilom. from the origin is about $8 \cdot 5 \times 10^2$ mm. year^{-1}, while fig. 6 shows that the mean rate from the whole strip, extending from the origin to this point, is about $1 \cdot 4 \times 10^3$ mm. year^{-1}.

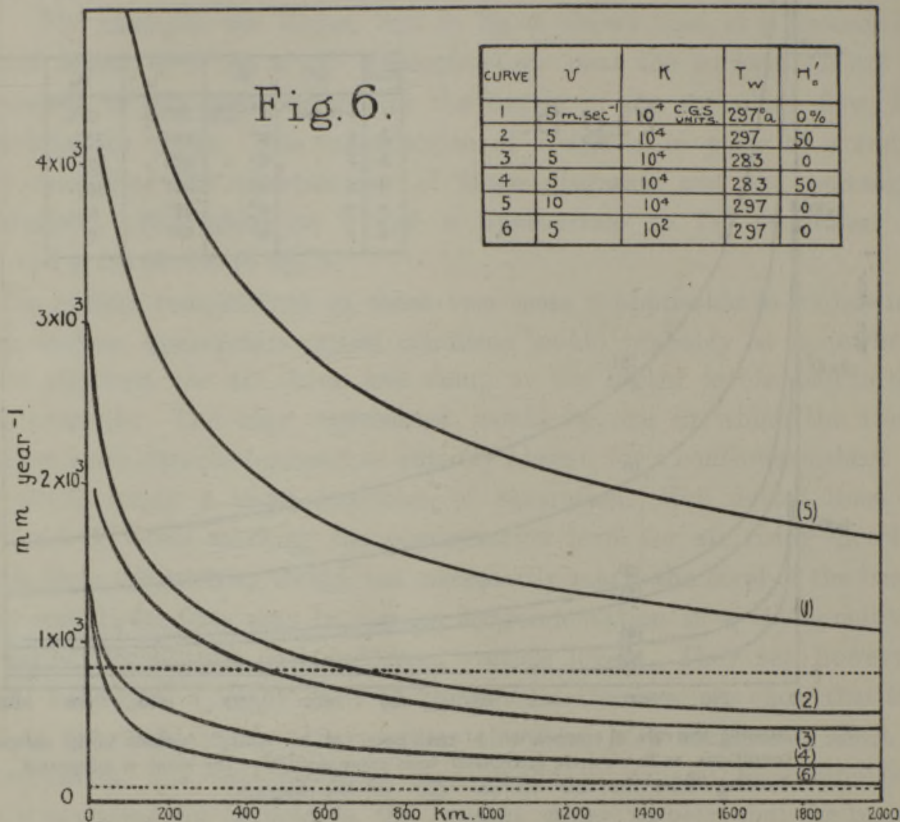


CURVE	v	K	T_w	H_0'
1	5 m. sec^{-1}	10^4 C.G.S. units	297° a	0 %
2	5 "	10^4 "	297° "	50 "
3	5 "	10^4 "	283° "	0 "
4	5 "	10^4 "	283° "	50 "
5	10 "	10^4 "	297° "	0 "
6	5 "	10^2 "	297° "	0 "

Showing the rate of evaporation at each point of a water surface under various conditions as to wind etc [computed from expression (3)]. The wind is supposed blowing from the left to the right of the diagram.

Curves 1 and 2 correspond respectively to the conditions of figs. 3 and 4, and reference to expressions 3 and 4 shows that curve 2 represents a rate of evaporation one half that represented by curve 1. In order to illustrate the influence of surface temperature on the rate of evaporation, curves 3 and 4 have been drawn, corresponding to the same conditions as the first two curves, but with surface temperature 283° a. (50° F.) instead of 297° a. (75·2° F.). Such a surface temperature would apply, at some period of the year, to the North Sea, and fig. 6 shows that, under the conditions of curve 3, the mean depth of water evaporated from a stretch 1000 kilom. in length would be of the order of 6×10^2 mm. year^{-1}. Under the conditions of curve 4, it would be only half this. Utilising expression 5, it is found that, under the conditions of curve 3, the air, after traversing 1000 kilom. of the North Sea, would have gained enough water vapour to

produce, if precipitated instantaneously, a depth of rain of the order of 4 mm. Again, this would be halved under the conditions of curve 4. Curve 5 replaces curve 1 if the wind speed is changed from 5 m. sec.$^{-1}$ (11·2 miles hour^{-1}) to 10 m. sec.$^{-1}$, while curve 6 replaces curve 1 if the value of K is reduced to 10^2. The dependence of the rate of evaporation from a body of water on the value of K is well illustrated by this example.



Showing the mean depth of evaporation per unit time, under various conditions, from stretches of water extending any distance in the direction of the wind, [computed from expression (4)].

11. Murray* has estimated the mean annual rainfall on the land of the globe as equivalent to $8{\cdot}4\times10^2$ mm. (upper dotted line in fig. 6), and that only a fraction 1/4·5 of this, finds its way to the oceans by rivers. This fraction spread over the oceans amounts† to $7{\cdot}4\times10$ mm. (lower dotted line in fig. 6), and must be the excess of evaporation above precipitation over the oceans, if we assume that the mean annual evaporation over the whole globe equals the mean annual precipitation. The fact that the dotted lines repre-

* J. Murray, 'Scottish Geographical Magazine,' February, 1887.

† Taking the ratio of land surface to water surface as 1·45 to 3·67. See 'Computer's Handbook,' Meteorological Office, 223, Introduction. London, 1916.

senting these quantities in fig. 6 fall where they do, when plotted on the same scale as the curves representing the rates of evaporation from large stretches of ocean, under selected sets of possible conditions, is of great interest, and shows, at least, that the numerical quantities employed in these examples are of the right order of magnitude.

Some General Conclusions.

12. Let us now compare the distribution of water vapour in the air over a water surface, and also the rate of evaporation from it, under different conditions as to wind, surface temperature and turbulence.

Consider, firstly, the relative humidity "near" the surface as given by expression (2).

$$H_0 = 1-(1-H_0')\,\psi(\tau) \quad \text{where} \quad \tau = A\alpha T_0\,(1+cv)\sqrt{(y/Kv)}.$$

Since $\psi(\tau)$ is a function which decreases as τ increases (see fig. 1), it is evident that H_0, the relative humidity "near" the surface must increase as τ increases. Other quantities remaining constant, τ increases as y increases, hence the relative humidity "near" the surface increases with increased distance from the origin, as is to be expected. This is illustrated in figs. 3 and 4. Again, considering conditions at an assigned distance y from the origin, a greater value of T_0 implies a greater value of τ, so that a higher surface temperature implies a higher relative humidity. It is to be noted, however, that τ is only altered in the ratio of the absolute temperatures, so that the increase in relative humidity at a given point "near" the surface, corresponding to a higher surface temperature, is not very great for the ordinary range of sea surface temperatures which occur in nature. Thus, conditions being the same as in fig 3, the relative humidity at a distance of 171 kilom. from the origin, corresponding to a value of τ equal to 2, is 74·5 per cent., while if the surface temperature were 283° a., instead of 297° a., the value of τ would become 1·91 and the relative humidity consequently 73·5 per cent., a difference of only 1 per cent. The rate of evaporation from the strip of water extending from the origin 171 kilom. down the wind would, however, be very different in the two cases, as is seen by referring to curves 1 and 3 of fig. 6. In fact, it is the greater evaporation, accompanying the higher surface temperature, which leaves the relative humidity at an assigned distance from the origin, very little changed.

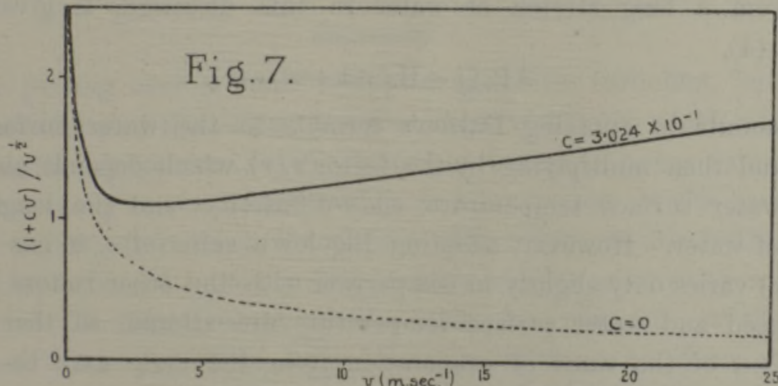
As regards turbulence, it is seen that a decrease in the value of K implies an increase in τ and a corresponding increase in the value of the relative humidity at a given point near the surface. The effect, too, may be large, for as has been indicated, K may vary over a large range. A rather large value,

10^4 C.G.S. units, was adopted for K in the examples of figs. 3 and 4. Suppose K decreased to 10^2 C.G.S. units, then τ is multiplied by 10, so that the point in fig. 3, distant 171 kilom. from the origin, which there corresponds to $\tau = 2$ now corresponds to $\tau = 20$, and the relative humidity increases from 74·5 to 97·2 per cent. The effect on the rate of evaporation from strips of various lengths is seen by comparing curves 1 and 6 in fig. 6.

The speed of the air v enters the expression for τ in the form $(1+cv)/v^{\frac{1}{2}}$. This function of v is plotted in fig. 7 for Bigelow's value of c and for the value $c = 0$. When $c = 0$ it is seen to decrease as v increases, so that τ decreases and consequently the relative humidity at a given point "near" the surface decreases. The effect of c, which is positive, is, however, to counteract this and, if Bigelow's value is correct, to reverse it for wind speeds above about 3 m. sec.$^{-1}$, an increase in the relative humidity at a given distance from the origin corresponding to an increased wind speed. It is here appropriate to emphasise the twofold manner in which the wind speed enters into the problem. If c were zero, then, as has been seen, the greater the wind speed the less would be the relative humidity H_0 at every point "near" the surface and hence, by reference to the evaporation formula which may then be written $E = AP_0(1-H_0)$, the greater the evaporation. In this effect the wind speed merely operates by controlling the rate at which drier air is carried along to replace that rendered humid by evaporation and account is taken of it in the mathematical analysis. However, all experiments, such as those of Bigelow and Fitzgerald, designed to study the rate of evaporation from an element of a large water surface point to a value of c, far different from zero. Thus at a given instant, at any point of a water surface, for a given value of the surface temperature and of the vapour pressure at the dew point of the air "near" the surface, a greater rate of evaporation corresponds to a higher wind speed "near" the surface. This is an experimental result and the factor $(1+cv)$ in the evaporation formula must be regarded as taking account, empirically, of the effect of the wind speed "near" the surface on the process of the diffusion of water vapour between the actual surface of the water and the point defined earlier as "near" the surface.

So far each of the conditions affecting evaporation has been varied independently of the others. The effect of an increase in K, corresponding to an increase in v, should be considered though there are no grounds for supposing that K can be expressed directly as a function of v. Suppose, purely for the sake of illustration, that K is proportional to v, then $\tau \propto (1+cv)/v$, *i.e.*, $\tau \propto 1/v + c$, and consequently an increase in v definitely implies a decrease in τ and hence a decrease in the relative humidity "near" the surface at a given distance from the origin.

13. Similarly, the effect of changes in the conditions, on the humidity at heights above the surface, may be readily found from expression 1, through the medium of τ and σ, in conjunction with fig. 1. One point worthy of notice is that, for a given value of σ, the function $\chi(\sigma, \tau)$ does not depart appreciably from unity until τ reaches a value given by $\tau = \sigma/2$, as is at once evident from fig. 1. This means that, at the height corresponding to σ, the humidity does not begin to be affected by the rising water vapour until the air reaches a distance from the origin corresponding to a value of τ equal to $\sigma/2$. In the example of fig. 4, for instance, the humidity at height 740 m. ($\sigma = 4$) is first affected when the air has travelled a distance of about 171 kilom. ($\tau = 2$) from land. The relation $\tau = \sigma/2$ is equivalent to $z = 4\sqrt{(\mathrm{K}y/v)} = 4\sqrt{(\mathrm{K}t)}$, and does not involve the surface temperature T_0. In his investigation of the formation of fog over the Newfoundland Banks, Taylor* found that, when air flows over colder water, the height to which cooling extends is given by $z = 2\sqrt{(\mathrm{K}t)}$ in the present notation. This height appears to disagree with the above result by being only half as great. Taylor, however, was concerned with the thickness of the "inversion" of temperature, and reference to the diagram on p. 35 of the 'Manual of Meteorology'† shows that cooling commences to be appreciable at twice this height, or where $z = 4\sqrt{(\mathrm{K}t)}$ in agreement with the present result.



Representing the function $(1+cv)/v^{\frac{1}{2}}$

14. The effect of changes in the conditions, on the mean depth of evaporation from strips of water extending from the origin any distance in the direction of the wind, has been considered incidentally in the preceding remarks. It may be studied by considering expression 4 in relation to fig. 2. As has been seen, Bigelow's estimate of the coefficient c implies that τ increases slowly as v increases, for values of v above about

* G. I. Taylor, "On Eddy-Motion in the Atmosphere," 'Phil. Trans.,' A, vol. 215, p. 1.

† Sir Napier Shaw, 'Manual of Meteorology,' Part IV. Camb. Univ. Press, 1919.

3 m. sec.$^{-1}$. Hence $\nu(\tau)$ decreases (see fig. 2). However, it is evident from the manner in which v enters the expression for E′, that, in spite of this, the rate of evaporation, $E'\nu(\tau)$, increases as v increases. This is exemplified by curves 1 and 5 of fig. 6. Exactly the same argument applies to the case of surface temperature, and the greater evaporation accompanying a greater surface temperature is shown by curves 1 and 2 as compared with 3 and 4 in fig. 6. The coefficient of eddy-diffusivity (K) enters the expression for the rate of evaporation solely through τ, and, since τ decreases as K increases, this is accompanied by an increased rate of evaporation. Since K may vary over a wide range, its effect on the evaporation may be very great. An example is furnished by curves 1 and 6 of fig. 6.

15. These last remarks furnish examples of Brooks's problem, which originated this enquiry. It was, as mentioned earlier, the comparison of the evaporation from a large expanse of water of uniform surface temperature, under different conditions as to wind and temperature. He made a rough comparison by applying Dalton's formula to the whole area, assuming H_0, the relative humidity of the air "near" the surface, to have the same value everywhere, and to be the same in the two cases compared. In this paper it has been shown how H_0 varies in the direction of the wind, and that, under the conditions laid down, the mean depth of evaporation per unit time, from a long stretch of water in this direction, is given by the formula (4),

$$AP_0(1-H_0')(1+cv)\,\nu(\tau).$$

This amounts to applying Dalton's formula to the water surface at the origin, and then multiplying by the factor $\nu(\tau)$, which depends on the wind speed, water surface temperature, eddy-diffusivity, and the length of the stretch of water. However, adopting Bigelow's value of c, it has been seen that $\nu(\tau)$ varies only slightly in comparison with the other factors, when the wind speed and water surface temperature are altered, so that a rough comparison of the rates of evaporation from the strip may be made by comparing the rates of evaporation at the origin, as given by Dalton's formula. This, of course, supposes that the eddy-diffusivity, which may affect $\nu(\tau)$ considerably, remains unaltered.

The actual cases compared by Brooks are (1), $v = 20$ m.p.h. (8·9 m. sec.$^{-1}$), $T_0 = 80°$ F. (299·7° a.), and (2) $v = 25$ m.p.h. (11·2 m. sec.$^{-1}$), $T_0 = 75°$ F.) (296·9° a.), the relative humidity at all points being assumed to be 50 per cent. If we suppose the relative humidity of 50 per cent. to apply to the origin and K and y to be the same in both cases, and distinguish between the cases by suffixes 1 and 2, then

$$\tau_2/\tau_1 = 1{\cdot}05 \quad \text{since} \quad \tau = A\alpha T_0(1+cv)\sqrt{(y/Kv)}.$$

If R denote the rate of evaporation from a stretch of water extending from the origin down wind, then from (4)

$$R_2/R_1 = 1{\cdot}01 \times \nu(\tau_2)/\nu(\tau_1).$$

Now if $\tau_2 = \gamma\tau_1$, where γ is a constant greater than unity, then it may be shown that, as τ_1 increases from 0 to ∞, $\nu(\tau_2)/\nu(\tau_1)$ decreases from unity to $1/\gamma$, whatever the value of K may be. In the present instance $\gamma = 1{\cdot}05$, so that as the length of the stretch of water is increased from 0 to ∞, R_2/R_1 decreases from 1·01 to 0·96, and this is independent of the value of K, provided it is the same in both cases. Thus, whatever the length of the strip, the evaporation is very little different in the two cases—the conclusion reached by Brooks.

16. It has been shown that the rate of evaporation, $E'\nu(\tau)$, from a stretch of water increases with the wind speed. Nevertheless, the water gained by a column of air in passing a given distance over the water surface, decreases as the wind speed increases. For a column of air, on base of unit area, the quantity referred to is, from (5), $E'tv(\tau)$, which may be written $AP_0(1-H'_0)(1/v+c)\,y\nu(\tau)$. Now, for values of v greater than 3 m. sec.$^{-1}$, $\nu(\tau)$ has been shown to decrease as v increases, and the other factor involving v clearly does so, with the result stated.

Summary.

17. Air passing over a water surface is generally turbulent, but in the immediate vicinity of the surface is a thin superficial layer, which mixes only slowly with the air above. In this layer the humidity passes rapidly from saturation actually at the water surface to a lower value at the base of the turbulent region, after which the change with height is slow. The region where the lapse rate of humidity first becomes small is referred to as "near" the surface. To take account of the molecular processes at the water surface and in the superficial layer of air in contact with it, an empirical formula is chosen, expressing the rate of evaporation per unit area in terms of the temperature of the water surface and the vapour pressure and speed of the air "near" the surface. Taking this formula as expressing the boundary condition at the surface, the problem of the upward stirring of water vapour in a current of air, of uniform speed, moving over a water surface of uniform temperature, is dealt with by mathematical analysis, and the resulting distribution of water vapour and rate of evaporation from large stretches of water, obtained. The turbulence is supposed uniformly distributed throughout the current of air above the level defined as "near" the surface, the coefficient of eddy-diffusivity in this region being taken as independent of

height. Certain approximations are made, in that the rate of change with height of the air speed and density are neglected.

Some particular cases are worked out, but in view of the approximations made and some uncertainty as to the value of the constants in the empirical evaporation formula, and indeed as to its form, the results give an indication of the order of magnitude, rather than precise numerical estimates, when applied to natural conditions.

Finally, the effect of the wind speed, the water surface temperature, and the eddy-diffusivity, on the evaporation from a large stretch of water and the distribution of water vapour above it, is examined by varying each individually within the natural range of values. The effect of eddy-diffusivity in particular, which may vary through a large range, is found to be very pronounced.

I wish to take this opportunity of expressing my thanks to Lt.-Col. E. Gold, F.R.S., both for suggesting this problem and for his helpful criticism and advice during the course of the work.

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*A Study of Catalytic Actions at Solid Surfaces.** VI.—*Surface Area and Specific Nature of a Catalyst: two Independent Factors controlling the Resultant Activity.*

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Catalytic action is determined by two primary factors, the presence of a mechanically suitable surface at which the process takes place, and the simultaneous presence of the chemical agent (catalyst) by which the change is promoted.

The work of Langmuir,† Bancroft,‡ Taylor§ and others upon the phenomena of adsorption has emphasised anew the importance of surface with reference to catalysis. When a suitable surface is present, however, no catalytic action will occur unless on that surface (whether it be formed by an inert

* Part V, 'Roy. Soc. Proc.,' A, vol. 98, pp. 27–40 (1920).

† 'J. Amer. Chem. Soc.,' vol. 38, p. 2221 (1916); vol. 39, p. 1848 (1917); vol. 40, p. 1361 (1918).

‡ Presidential Address, American Electrochemical Society, 1920.

§ 'J. Ind. Eng. Chem.,' vol. 13, p. 75 (1921).