Symmetrical Taylor impact studies of copper

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An integrated investigation of rod-on-rod (symmetric Taylor) impact of annealed copper was conducted using the single-stage gas-gun facility at the Cavendish Laboratory as a validation study of the Armstrong–Zerilli constitutive model, as modified by Goldthorpe. Two main techniques were used for obtaining data from the experiments: high-speed photography (up to 20 million frames s⁻¹ framing rate) and a velocity interferometer system for any reflector (VISAR). The symmetric configuration was used to minimize friction effects and eliminate target indentation seen in classic Taylor tests, where a rod is fired against a massive target block. However, the need for coaxial alignment of the two rods made the experiments considerably more challenging to perform than the classic case. The propagation of plasticity along the rods was monitored using high-speed photography and VISAR. It was found to propagate with a logarithmically decelerating velocity. The rod profiles and VISAR traces can be understood in terms of material properties such as strain hardening. No asymmetry between the responses of the two rods involved (moving and stationary) was observed within the resolution of the techniques employed. A modified Armstrong–Zerilli material model for copper predicted intermediate profiles well, but slightly overestimated the material strength.

Keywords: Taylor impact; high-speed photography; numerical modelling

1. Introduction

Taylor (1946) first proposed the normal impact of a rod against a rigid surface as a method of estimating the yield strength of materials from measurements of the initial and final lengths of the rod, along with the length of the deformed section. Since then, more accurate methods have also been developed for measuring the dynamic yield stress, such as the split Hopkinson pressure bar (Taylor 1946; Volterra 1948; Kolsky 1949). However, the Taylor technique remains in use today as a useful test of constitutive relations at high strain rates and large strains. The method is used as a benchmark test to check and modify the material models used within numerical simulations. The research reported in this paper was part of a validation study of the Armstrong–Zerilli model for copper, as modified by Gould & Goldthorpe (2000).

The main experimental improvements involve the recording of intermediate deformation profiles by high-speed photography, demonstrated initially by Briscoe & Hutchings (1976, 1978) for polymers, combined with measurements of...
the displacements of the free end of the rods using velocity interferometry. The series of intermediate profiles can be compared with simulations, effectively increasing the usefulness of the test by incorporating information about strain rates and plastic strain magnitudes during the process.

2. Theory

The plastic deformation undergone by a rigid-plastic cylindrical rod normally striking a massive rigid anvil was analysed by Taylor and colleagues (Taylor 1946, 1948; Carrington & Gayler 1948; Whiffin 1948) in order to develop a simple method of estimating the dynamic compressive yield strength of the projectile material. It was already known that the dynamic tensile and shear yield stresses were larger than their static values (Luerssen & Greene 1933; Itihara 1935a–d, 1936; Mann 1935). Taylor’s analysis was one-dimensional, i.e. no account was taken of material movement normal to the axis of the cylinder. More sophisticated analyses of the cylinder impact problem were subsequently published by various authors (Lee & Tupper 1954; Hawkyard 1969; Johnson 1972), which took into account, for example, multiple interactions of the plastic front with elastic waves (Hohler & Stilp 1990). However, the beauty of Taylor’s analysis is that simple length measurements of the cylinder can give results accurate to approximately 10 per cent using the following formula:

\[
\frac{\sigma_y}{\rho_p V^2} = \frac{L - X}{2(L - L_1)} \frac{1}{\ln(L/X)},
\]

where \(\sigma_y\) is the yield stress; \(\rho_p\) is the density; \(V\) is the impact velocity; \(L\) is the initial length; \(L_1\) is the final length; and \(X\) is the length of the undeformed rear section of the cylinder (figure 1).
The derivation of equation (2.1) and its subsequent refinements will not be given here, but may be found in Taylor (1946, 1948), Johnson (1972) and Chapman et al. (2005b).

An estimate of the global strain rate may be made as follows: the reduction in length, \((L - L_1)\), is confined to the material whose initial length was \((L - X)\) (figure 1). Therefore, the total strain in the portion that yielded is \((L - L_1)/(L - X)\). This strain develops in a time \(T\) given by \(2(L - L_1)/V\). Hence, the overall strain rate is given by

\[
\frac{\partial e}{\partial t} = \frac{L - L_1}{L - X} \left( \frac{1}{2} \frac{V}{L - L_1} \right) = \frac{1}{2} \frac{V}{L - X},
\]

(2.2)

Taylor (1948) and Whiffin (1948) calculated that the global strain rate was approximately \(10^4\) s\(^{-1}\) in their experiments. However, the strain rate varies both with time and position throughout the cylinder.

3. Material

XM C103 copper was supplied by DERA Fort Halstead. This copper was over 99 per cent pure and has a face-centred cubic (FCC) structure. The longitudinal and transverse sound speeds of the XM copper are taken from the LASL shock Hugoniot data book (Marsh 1980). Standard elastic relations were then used to calculate the moduli, bulk sound speed and ‘rod sound speed’. The yield strengths were measured in-house (figure 2) using a split Hopkinson pressure bar at a strain rate of \(7 \times 10^3\) s\(^{-1}\), which is close to the estimated global strain rate for Taylor impact \((10^4\) s\(^{-1}\)).

4. Experimental

The Taylor impact experiments were all performed in the ‘symmetrical’ configuration, where one rod is fired coaxially at an identical stationary rod (Erlich et al. 1982; Erlich 1985). The rods were 10 mm in diameter and 10
diameters (i.e. 100 mm) long. Symmetric impact at a velocity $V$ is equivalent to classical impact at a velocity $V/2$. This method overcomes two defects of the ‘classical’ configuration: (i) the fact that no target is perfectly rigid and (ii) frictional effects at the specimen–target interface. Both these effects are eliminated because the ends of the two rods deform identically. These advantages come at the cost of having to ensure very precise rod alignment, similar to that required for performing plate impact.

Two different methods of mounting the stationary target rod were tried. Figure 3a shows the rod resting lightly upon notched supports, leaving the rear end of the rod unconstrained. This design was eventually rejected due to the possible interference of the mount with the lateral deformation of the rod. A second design was therefore developed (figure 3b). In this configuration, the rod was rigidly mounted with lateral constraint around the rear end, but with the rear surface effectively free. Since the plastic front does not usually reach the rear of the rod during the period of observation (approx. 100 µs) and the elastic stress wave is able to reflect from the rear, this mounting design was seen as the least invasive for our purposes. The set-up would not be ideal if final rod dimensions were required.

Figure 3. Two mounts used for the symmetrical Taylor impact target rods. (a) The mount used in the initial trial design: (i) side view and (ii) rear view. (b) The mount used in the experiments. Neither mount is ideal in the sense of putting no constraint on the deformation or motion of the rod. However, any effects on the lateral deformation of the target rod produced by the second mount occur after the end of the high-speed photographic sequences presented. The space machined behind the rod in the second mount is to allow the movement of the rod.
The projectile rod was mounted rigidly within a polycarbonate sabot, leaving the rear surface of the rod free except for a small lip around its perimeter. The alignment was checked before firing by placing the projectile in the end of the barrel and then mounting the target rod in the impact chamber. The rods were then brought together so that any mismatch could be corrected directly. To avoid the movement of the target rod after the completion of this process, the sabot and rod were drawn back down the barrel to the launch position using a vacuum pump. To maintain the alignment during collision, the impact between the rods occurred while the sabot was still partly in the gun barrel. This avoids the yaw or pitch inevitably produced by sabot strippers.

An Ultranac FS501 image converter camera was used for all these experiments (Riches & Garfield 1993). It is capable of taking 24 frames, each frame being individually programmable via a PC with respect to exposure and interframe times. Illumination was provided by a Bowen professional flash unit (Bowens Mono 400D), which takes 100 $\mu$s to rise to peak illumination but then provides constant illumination for approximately 500 $\mu$s. The assembly consists of a wire coiled around a circular tube, in front of a metallic reflector. The flashes were used in a backlighting configuration, giving a silhouette of the specimen. A Fresnel lens was used to ensure approximately parallel illumination and the light diffused using sheets of translucent tracing paper to provide more even lighting. These sheets are also useful for altering the intensity of illumination without changing the aperture, which would affect the depth of focus as well. Since the camera was placed alongside the gun, it observes the impact chamber through a front-silvered mirror. As the specimen is a long thin rod, a dove prism was used to rotate the image by 90° to correspond with the 'portrait' aspect ratio of the Ultranac frames (long axis vertical). Care must be taken with dove prisms, however, as misalignment can cause a shear distortion. An alternative method of fitting the experiment into the frames would be to operate the camera on its side.

As high accuracy was required in measuring the rod diameter from the photographs, a transparent sheet, on which was printed a square grid with a line spacing of 2 mm, was placed on the far side of the rod from the camera at a distance of 5–10 mm, i.e. far enough behind that the deforming rod would not displace it during the time the sequence was being captured. Since the grid and the rod axes were not in the same vertical plane, it was not possible to scale the frames directly using the actual grid spacing. However, the spacing in the plane of the rod axis can be obtained by examining the frames that contain the undeformed rod, the diameter of which is known. This comparison between the rod and the grid was then used to scale all the frames. The use of a grid overcomes any slight magnification differences (up to 1%) between frames due to distortion produced by the camera (Melin 1995) by providing an independent measure in each frame.

The polaroid photographs produced by the Ultranac were digitally scanned and enlarged for accurate measurement. The profiles of the rods were then traced onto transparencies along with fiducial points in the grid. These transparencies were then scanned and analysed using a software package that uses the fiducial points to create a Cartesian grid, which then allows the profiles to be encoded as a set of coordinate data points. The measurement error in the stated impact velocities is $\pm 5\text{ m s}^{-1}$.
5. Results and discussion

(a) High-speed photography

Figure 4 presents the impact between two copper rods. The stationary target rod fills most of the frame, while the projectile rod moves into view from the right. The rods impact in the third frame, labelled $0 \, \mu s$, with no visible misalignment.

Figure 4. Rod-on-rod impact of 100 mm long, 10 mm diameter copper rods at $395 \, m \, s^{-1}$. The rod mount is the same as illustrated in figure 3b. The zero of time is taken as the moment of impact. The triangular shapes seen in the corners of frames 1, 3, 22 and 24 are the edges of the fibre-optic bundle on the Ultranac camera.
For misalignment to be detectable on these frames, the angle between the rod axes would have to be above 2° and their lateral displacement above 0.2 mm. The interface between the rods can be seen moving to the left in successive frames. At 8 \( \mu \)s after impact, the rod diameters at the interface have expanded by 40 per cent, and at 32 \( \mu \)s by 80 per cent. Initially, the profiles seem to be simple concave curves but, at approximately 50 \( \mu \)s, two distinct zones of deformation are apparent. That is, approximately 4 mm distant from the interface, there is a noticeable discontinuity in slope from which the profiles slope away more gently, forming ‘shoulders’. This change in profile may relate to the arrival of a release wave from the rear end of the rod. This release wave will be mostly reflected from the plastic front as a compression wave. The target and projectile profiles remain symmetrical, confirming the initial good alignment. The plastic deformation progresses beyond the field of view before the end of the sequence.

Figure 5 shows a selection of the profiles extracted from these frames. An estimation of the random error introduced into the measured diameter during tracing and data capture is \( \pm 0.2 \) mm (2\%) or less. In figure 5a, the complete profiles are shown with the x-coordinates being measured from the impact plane. Note that the x- and y-axes are not equally proportioned: they are compressed in the horizontal direction. The development of convex shoulders is easily visible in the profiles, situated approximately 4 mm from the interface on the 68 \( \mu \)s profile in figure 5a. In figure 5b, the profiles of the target and projectile rods have been overlaid so as to show up any asymmetry more clearly. However, of the minor differences that can be seen, neither rod deforms consistently more than the other. Therefore, the different methods of mounting the target and projectile rods do not seem to affect the deformation to the level of resolution of the high-speed photographs. So the assumption that the impact is symmetrical in the centre-of-momentum frame appears to be valid.

The horizontal movement of the interface is quite steady (figure 6). The velocity calculated from the gradient of this plot is 206\( \pm \)2 m s\(^{-1}\), a little more than the expected value of one-half of 395\( \pm \)5 m s\(^{-1}\).
After the end of the sequence shown in figure 4, the deformation at the interface begins to slow, while the shoulders continue to expand outwards, resulting in the major fraction of the target rod reaching approximately one final level of permanent strain (figure 7). The projectile rod was damaged during deceleration and so its dimensions are not depicted. Much of the length has a fairly constant radius of 7 mm (40% strain), except for the impact end and the constrained rear end. The final length (55 mm) was just over half of the original length (100 mm). Also shown in figure 7 are the profiles of two rods (projectile and stationary) recovered from another experiment performed at a lower impact velocity of 284 m s$^{-1}$. These rods exhibit similar profiles with values for the peak radius of 7.4 mm, 6.2 mm for the shoulder radius and 30 mm for the length lost. The projectile and target profiles deviate near their rear ends, which were constrained in different ways. It should be

Figure 6. Plot of the horizontal position of the interface between the two copper rods whose impact is shown in figure 4.

Figure 7. Final profiles of three copper rods recovered from rod-on-rod impacts. Filled square, 395 m s$^{-1}$ target; open diamond, 284 m s$^{-1}$ target; filled diamond, 284 m s$^{-1}$ projectile.
repeated that the constraints and the lack of undeformed rod portions mean that it would not be possible to use measurements on these rods to perform a traditional Taylor calculation of yield strength.

These experiments showed evidence of the propagation of a plastic front. As copper is ductile, there was a pronounced maximum in the lateral deformation at the impact interface. This was later followed by a more uniform lateral deformation along the rest of the length of the rods. Fracture was not observed. The existence of bulges or shoulders that are distinct from the deformation close to the impact interface is thought to be a sign of strain hardening (Gould & Goldthorpe 2000).

(b) Propagation of plasticity

In order to gain a more quantitative measure of how plasticity propagated in the rods than is given by plotting their profiles, it was decided to follow the progress of particular lateral strain levels along the rods. The first step towards this was to determine the average radial strain at the interface as a function of time (figure 8).

The lateral interface strain can be seen to rise sharply early on (up to 20 μs). The strain stops increasing at approximately 44 μs. This is probably due to strong strain hardening. However, the rest of the rod continues to shorten and increase in diameter, leading to the more uniform diameter found in the recovered rods. If the differences between the lateral interface strains in consecutive frames are divided by the interframe times, the average lateral strain rate as a function of time can be estimated (figure 9). Although the initial strain rate is indeed high (approx. 5–9×10⁴ s⁻¹), these values are just the average over the first 4 μs, so the strain rate at the instant of impact will be much higher. The strain rate was found to drop to approximately 2×10⁴ s⁻¹ by 8–10 μs, and then fall off more slowly, reaching zero sometime between 40 and 50 μs.

Figure 10 shows the progression of two levels of lateral strain, 2 per cent (5.1 mm in radius) and 4 per cent (5.2 mm in radius) along the target copper rod. The accuracy of these measurements is limited by the resolution of the
photographs and the tracing, calibration and data capture. Together, these errors amount to approximately \( \pm 0.2 \text{ mm} \) in the diameter measurements (i.e. approx. 2%). Therefore, it should only be possible to follow the motion of lateral deformations of 0.2 mm or greater in the rod diameter (or 0.1 mm in the radius). So the positions of two rod radii, 5.1 and 5.2 mm, were measured to determine whether or not the 2 per cent level was reliable, but the behaviour was found to be similar for both and roughly linear. An additional possible uncertainty is due to errors in measuring the position of the interface from which distances were measured. So the fact that the plot of the movement of the two strain values (figure 10) shows a steady behaviour is a good sign that these measurements are reliable. The slopes of the curves give a velocity of \( 380 \pm 20 \text{ m s}^{-1} \) for the 2 per cent strain case and \( 300 \pm 10 \text{ m s}^{-1} \) for the 4 per cent case. Conventional analysis of an elastoplastic or elastic-strain-hardening metal shows that smaller plastic strains

![Figure 9](http://rspa.royalsocietypublishing.org/)  
**Figure 9.** Average radial strain rate between frames for the interface between the copper rods shown in figure 4.

![Figure 10](http://rspa.royalsocietypublishing.org/)  
**Figure 10.** Plot of the movement down the copper rod shown in figure 4 of two levels of deformation as a function of time. Filled square, 5.1 mm rod radius; open square, 5.2 mm rod radius.
propagate more quickly than larger ones (Duwez & Clark 1947; von Kármán & Duwez 1950; Taylor 1958). This may change if the impact velocity is greater than the velocity of propagation of the plastic front, in which case a shock wave forms. However, in the case reported here, the effective impact velocity is approximately \(200 \text{ m s}^{-1}\) in the centre-of-momentum frame, i.e. well below the measured values for the plastic front velocity.

Several analyses of Taylor impact have tried to take account of how elastic stress waves reverberate back and forth in a rod during impact, reflecting as a tensile wave from the rear of the rod and as a compressive wave when they meet the oncoming plastic front (Johnson 1972; Hohler & Stilp 1990). In reality, the reflection off the plastic front will only be partial. Moreover, as plastic fronts are not sharp but diffuse, their position for calculation purposes is uncertain. For example, Wilkins & Guinan (1973) asserted plastic fronts may be closer to the impact interface than the lateral strain would suggest. Ignoring these complications, figure 11 shows how often elastic stress waves could interact with the plastic front during the duration of the high-speed photographic sequence using the elastic wave velocity measured using ultrasonic transducers. The speed used was that of the speed of sound in the rods (table 1).

This exercise implies that elastic waves travelling up and down the rods could interact with the plastic front at most once during the time the high-speed sequences were recorded. Measurements were also made of the movement of larger lateral strains and these are plotted in figure 12. The manner in which plasticity propagates in wires and rods was first outlined by Lenskii (1949) and Taylor (1958). They asserted that the propagation velocity of a stress level is determined by the local slope of the stress–strain curve. The typical shape of the stress–strain curve for a metal rod, where material is not constrained from moving outwards, is concave downwards. In a material such as annealed copper, there is no definite yield point (see, for example, figure 2).

Because the slope decreases with strain for concave-down stress–strain curves, higher levels of stress or strain propagate more slowly than lower levels. Thus, an initially sharp mechanical pulse will tend to disperse during propagation.
Table 1. Physical properties of the copper used. (All the data were taken from or derived from the data in the *LASL shock Hugoniot data* book (Marsh 1980) or supplied by QinetiQ, except for the dynamic yield strength which was measured in house.)

<table>
<thead>
<tr>
<th>property</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>density ($\rho$, kg m$^{-3}$)</td>
<td>8924±1</td>
</tr>
<tr>
<td>longitudinal sound speed ($c_L$, m s$^{-1}$)</td>
<td>4760±5</td>
</tr>
<tr>
<td>transverse sound speed ($c_T$, m s$^{-1}$)</td>
<td>2330±5</td>
</tr>
<tr>
<td>sound speed in a rod ($c_R$, m s$^{-1}$)</td>
<td>3820±20</td>
</tr>
<tr>
<td>bulk sound speed ($c_B$, m s$^{-1}$)</td>
<td>3930±10</td>
</tr>
<tr>
<td>Young’s modulus ($E$, GPa)</td>
<td>130.1±0.7</td>
</tr>
<tr>
<td>shear modulus ($\mu$, GPa)</td>
<td>48.5±0.1</td>
</tr>
<tr>
<td>bulk modulus ($K$, GPa)</td>
<td>137.6±0.5</td>
</tr>
<tr>
<td>Poisson’s ratio ($\nu$)</td>
<td>0.342±0.006</td>
</tr>
<tr>
<td>yield stress ($\sigma_y$, GPa)</td>
<td>0.4±0.1</td>
</tr>
</tbody>
</table>

Figure 12. Plots of the propagation of various strain levels in the (*a,b*) target and (*c,d*) projectile copper rods. The strain levels are indicated on the plots by the deformed radii. For clarity, the small strains (*a,c*) radii: filled square, 5.5 mm; open diamond, 6 mm; filled circle, 6.5 mm; up triangle, 7 mm; down triangle, 7.5 mm) are plotted separately from the large strains (*b,d*) radii: filled square, 7.0 mm; open diamond, 7.5 mm; filled circle, 8 mm; up triangle, 8.5 mm; down triangle, 9 mm; open square, 9.5 mm). A radius of 5.5 mm corresponds to a plastic strain of 10%, 7.5 mm corresponds to 50% and 9.5 mm to 90%.
(the opposite phenomenon to shock wave formation). However, an impact velocity greater than the propagation velocity of plasticity will, at least initially, localize the plastic front as a shock wave near the impact site.

A plastic wave attenuates as it propagates because the energy is converted into plastic work. Note that the slower, higher strain components will move through material that has already been deformed by the low-strain components moving ahead.

The families of curves plotted in figure 12 are generally consistent with the behaviour described above. Curve fits (dotted lines) have been superimposed to reveal the general shapes. The strain–time plots for the larger lateral strains (40% and above) have a shape consistent with the stress–strain curve for annealed copper (figure 2) as the velocities of propagation gradually decrease from their initial values, the deceleration itself being greatest early on (figure 12b, d). These shapes are best fitted by logarithmic curves. The small strain levels (up to 30%) in both the target and projectile rods accelerate after an initial deceleration (figure 12a, c). The velocity of each strain level remains less than that of the next smallest strain level throughout the deformation, a consequence of the rod profile being a single curve (without bulges). However, the plastic zone disperses with time as the higher strain levels decelerate the fastest and the curves move apart.

(c) Velocity interferometry using a VISAR

A velocity interferometer system for any reflector (VISAR; Barker & Hollenbach 1972) was used to measure the motion of the rear of the target rods (figure 13). The velocity change $\Delta v$ of the rear of the rod measured using the VISAR can be converted to the stress $\sigma$ at the rear by the relation

$$\sigma = \rho c_R \Delta v / 2,$$

where $\rho$ is the rod density and $c_R$ is the elastic wave speed in the rod.
The VISAR is a non-invasive technique, but it relies on the accurate knowledge of material densities and wave speeds in order to give accurate stress measurements. The technique is more sensitive than gauges for small stress levels (such as in the early stages of deformation) and can continue providing data after a tensile release wave has returned along the rod, unlike embedded stress gauges that are destroyed under these conditions. However, the VISAR record is limited in duration by the space available for the rear of the rod to move before reaching the probe. The VISAR is sensitive to dispersion effects if the rod it is observing is too short, as elastic waves have to propagate a certain distance (above 10 diameters) before approximating to a longitudinal wave (same amplitude over a cross section) (Davies & Hunter 1963; Lindholm 1964; Kennedy & Jones 1969; Gorham 1980; Safford 1988; Field et al. 1994). The copper rods used were 10 diameters long.

Stress gauges are simpler to work with and relatively cheap, particularly in comparison with the cost of purchasing a VISAR system, and thus often preferable as long as their insertion does not disrupt the phenomena being observed. An additional positive feature of gauges is their ability to detect the exact time of impact if electromagnetic radiation is emitted. A comparison between the two techniques was performed for a few reverse ballistic experiments (to be published) and the results largely confirm each other. For Taylor impact, it is possible that an embedded gauge could affect the transmission of plasticity. This could be tested for by comparing two experiments performed under the same conditions, except for there being a gauge in one but not the other.

Figure 13 presents the VISAR traces recorded during three symmetrical impacts of copper rods (designated C1, C2 and C3). The experiment whose high-speed photographic sequence was analysed above was shot C1. The impact velocities for shots C2 and C3 were 284 and 290 m s\(^{-1}\), respectively, i.e. slower than shot C1 (395 m s\(^{-1}\)). These values must be divided by two to obtain the effective impact velocities in the centre-of-momentum frame. The three traces all demonstrate marked work-hardening because, after slowly reaching an initial sloping plateau of 0.002 and 0.008 mm µs\(^{-1}\), which correspond to 34–140 MPa using equation (5.1), the particle velocity continues to rise for approximately 220 µs, gradually at first and then more rapidly after approximately 180 µs, to values an order of magnitude higher. The three examples presented vary in the maximum particle velocity attained, which is approximately 0.056, 0.064 and above 0.085 mm µs\(^{-1}\) for shots C1, C2 and C3, respectively, corresponding to stresses of 0.95, 1.1 and above 1.4 GPa. The plateau values are lower than the yield stress of 400±100 MPa, measured using a compression split Hopkinson pressure bar (figure 2), but this should only be taken as an estimate given the known strain-hardening properties of annealed copper. However, the final stress values calculated from the VISAR data exceed this estimate by as much as three to four times, although it should be pointed out that our knowledge of the other properties (e.g. density and wave speed) of the material becomes less certain as it deforms (Addessio et al. 1993; Mayes et al. 1993; Worswick & Pick 1995; Chapman et al. 2005a). There is no sign of any discrete, steep edges due to elastic wave pulses arriving repeatedly at the rod rear, as suggested by Johnson (1972) and Hohler & Stilp (1990), probably because such waves are too dispersed. In conclusion, the traces illustrate both the strong effect of strain hardening and the difficulties of characterizing such a material.

(d) Modelling of Taylor impact

The variety of stresses, strains and strain rates experienced during a Taylor impact experiment makes it useful as a rigorous benchmark test for modellers, particularly if data such as the development of rod profiles are obtained during impact using high-speed photography.

A two-dimensional Lagrangian hydrocode, DYNA-2D, was used to model copper shot C1. A modified path-dependent Armstrong–Zerilli constitutive model was used (Gould & Goldthorpe 2000; Walley et al. 2000). The following description of the model was first published in Church et al. (1999). As copper is an FCC metal, the mechanical behaviour of a specimen depends on its previous thermal and mechanical history. This means that a path-dependent constitutive relation must be used. Thus, an equation is needed that contains the state variable, an irreversible variable and various independent variables. Church et al. (1999) postulated an equation of the form

$$\bar{\sigma} = C_0 + f(\varepsilon) + \left(\frac{\mu_T}{\mu_0}\right)\sigma_t\phi(\dot{\varepsilon}, T),$$

where $\sigma_t$ is a strain-hardening function, which depends upon the strain rate and temperature at which deformation takes place (it therefore contains the history information that describes the path dependency of the flow stress); $\varepsilon$ (the plastic strain) is the irreversible variable; $\dot{\varepsilon}$ and $T$ (the strain rate and temperature) are independent variables; $C_0$ is the flow stress at zero state variable; and $\mu_T$ is the shear modulus at temperature $T$. The ratio $\mu_T/\mu_0$ can conveniently be replaced by $(1-aT)$, where $a$ is a known constant.

Experiments have shown that for FCC metals, $f(\varepsilon) = 0$. The function $\phi$ is taken to be of the Armstrong–Zerilli (1988) form

$$\phi = \exp\{T[-C_3 + C_4 \ln(\dot{\varepsilon})]\},$$

where $C_3$ and $C_4$ are constants. As $\sigma_t$ is the purely path-dependent part of the model, and since it describes the deformation history, it can be used as a state variable. The development of a representation for $\sigma_t$ requires some new techniques in constitutive modelling. The use of $\sigma_t$ as a state variable implies that it depends upon the previous deformation conditions but not on the strain rate and temperature at which it is measured. At any point along the rod, only the gradient of the $\sigma_t$ versus $\varepsilon$ curve is affected by the prevailing conditions of strain rate and temperature. If in two specimens of the same material, the same value of $\sigma_t$ is reached by two different deformation paths, then the two specimens will be in the same state.

The effect of temperature is as follows: if, for example, deformation occurs at 298 K up to a particular plastic strain and then the temperature is changed to 123 K, the gradient will change to the value that it would possess at the same value of the state variable if deformation had been entirely at 123 K. If no further change in independent variable occurs, then the gradient will continue as would be expected at 123 K but at a higher value of plastic strain; that is, the indicated horizontal distance between the two curves will remain constant.

One further piece of information is required: if deformation is carried out at constant strain rate and temperature, then, at some strain, the state variable will reach a maximum value (saturate) due to the competition between dislocation generation due to stress and dislocation annihilation due to recovery.
The above conditions allow a formulation for $\sigma_t$ to be made. The properties of path dependency are adequately described by

$$
\left( \frac{\partial \sigma_t}{\partial \varepsilon} \right)_{\eta, \theta} = \theta \left\{ 1 - \frac{\sigma_t}{\eta} \right\}^\alpha,
$$

where $\eta$ is a function of $\dot{\varepsilon}$ and $T$ and represents the saturation value of the state variable. It obeys the equation

$$
\eta = k \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^{S[T/(1-aT)]},
$$

where $k$, $S$ and $\dot{\varepsilon}_0$ are constants. The $\theta$ is found to vary approximately linearly with strain rate. Equation (5.4) is empirical, as indeed are all descriptions of state variable behaviour. The advantages of this particular form are that it can be integrated and allows a non-iterative solution. Note that the derivative of $\sigma_t$ is a partial derivative because $\sigma_t$ varies not only with strain but also with strain rate and temperature. The values for the parameters of the model outlined above are given in Table 2.

Integrating equation (5.4) at constant strain rate and temperature between strain limits of 0 and $\varepsilon$ and state variable limits of 0 and $\sigma_t$, we obtain

$$
\frac{\sigma_t}{\eta} = 1 - \left( \theta \{ \alpha - 1 \} \frac{\varepsilon}{\eta} + 1 \right)^{1/(1-\alpha)}.
$$

Although this has been integrated at constant values of the independent variables, its form is such that it is valid for all strain rates and temperatures for which the behaviour described above holds. It represents a family of master curves, one for each strain rate.

Using the above formalism, the experiment was treated as a two-dimensional axisymmetric problem. It was modelled with and without the confinement produced by the end mounts, but this was found to make no obvious difference for the time interval of the high-speed sequence. Changing the friction coefficient at the impact interface between the two rods also had no discernable effect. This is expected as there should be no slip at the interface. A mesh resolution study was performed to ensure that convergence had been achieved. Mesh sizes of 0.5 and 0.25 mm were

Table 2. Parameters for the path-dependent constitutive model for copper. (From Gould & Goldthorpe 2000.)

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<thead>
<tr>
<th>parameter</th>
<th>value or expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_a$</td>
<td>50 MPa</td>
</tr>
<tr>
<td>$k$</td>
<td>1475 MPa</td>
</tr>
<tr>
<td>$S$</td>
<td>$4.77 \times 10^{-5}$ K$^{-1}$</td>
</tr>
<tr>
<td>$C3$</td>
<td>$3.6 \times 10^{-4}$ K$^{-1}$</td>
</tr>
<tr>
<td>$C4$</td>
<td>$2.2 \times 10^{-5}$ K$^{-1}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.88</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$1048 + 10.42 \ln(\dot{\varepsilon})$ MPa</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_0$</td>
<td>$2.4 \times 10^{11}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>289 MPa</td>
</tr>
</tbody>
</table>
used in the rod axis direction and mesh sizes ranging from 0.14 to 0.36 mm were tried in the radial direction. The final mesh size for the latter direction was settled on as 0.14 mm for the projectile and 0.17 mm for the target rod.

The mesh distortions for three times after impact (4, 20 and 68 μs) are presented in figure 14. Figure 14a (4 μs) shows a space opening up between the rods at the centre of the interface. High-speed optical photography cannot reveal whether or not this actually happens. However, an X-ray technique has recently shown a dimple, rather smaller than indicated in figure 14a, which does indeed form at the interface in symmetric Taylor impact in an aluminium alloy (Chapman et al. 2005a).

The mesh elements themselves are not unduly distorted by this time, but this has begun to change by 20 μs (figure 14b), where the central elements are simultaneously stretched in the radial direction and compressed in the direction of travel. Extreme mesh element distortion can contribute to inaccuracy in the simulation, but, in this case, the coarser mesh confirms the rod profile with much
less element distortion. The simulation suggests, as proposed previously by Wilkins & Guinan (1973), that the elastic–plastic boundary is not planar, and is closer to the interface between the rods on the axis than at the rod surfaces. Also, by this time, the space between the rods has closed and there is instead a slight separation of the rods near the outer surfaces. The profiles of the rods are still strictly concave, unlike at 68 \(m\) \(s\) (figure 14c), where the development of convex shoulders has been successfully predicted. Here, the coarser mesh simulation has been chosen for presentation, so that the patterns of plastic flow are still visible. Even so, the mesh elements have been stretched considerably and are thin near the interface between the rods. The convex contours pervade all the way to the rod interiors.

The experimental and simulated rod profiles at the three times are compared in figure 15. The axis scale in the direction of travel has been compressed. The simulated rod profiles show the boundary between the rods initially travelling slightly more quickly than in the experiment (figure 15a,b) but equalizing by 68 \(\mu\)s (figure 15c). It is not known why this should be since the interface is expected to travel at a steady velocity, and appears to do so in figure 6. The model predicts the general shape of the profile quite well, particularly at 20 \(\mu\)s (figure 15b), although note that the small separation at the edge of the simulation interface (figure 14b) has not been depicted. It also predicts the shoulders in the profile at 68 \(\mu\)s (figure 15b). However, the amount of lateral strain is slightly underpredicted for these three times. This indicates that the constitutive model

Figure 15. Comparison of the experimental (thick line) and simulated (thin line) profiles at various times after the impact of the copper Taylor impact experiment shown in figure 3. (a) 4 \(\mu\)s, (b) 20 \(\mu\)s and (c) 68 \(\mu\)s.
used has overpredicted the strength of the copper at this strain rate. This is possibly due to the omission from the model of microstructural processes that might aid plastic flow (Carrington & Gayler 1948). Dislocation saturation is one possible explanation for the underprediction of plastic deformation.

The experimental profile for 4 μs (figure 15a) is asymmetric, with the projectile rod (positive x-values) showing more lateral plastic deformation than the target rod (negative x-values). By contrast, the radius of the target rod matches the simulation well at distances greater than 1 mm from the interface (the fit is even better if the peak strains are brought into alignment). The difference in the maximum radii between simulation and experiment is 0.3 mm (5%).

At 20 μs, the difference between the simulation and the experiment has reduced to 0.2 mm (2.5%), almost within the measurement error, but it increased again to 0.6 mm (6%) by 68 μs. The experimental profile at 20 μs is thus closest to the simulation. The simulated positions of the shoulders are accurate at 68 μs, though the plastic deformation decreases more quickly with distance than in the experiment.

In summary, the dynamic plastic deformation response during Taylor impact is complex. However, this complexity makes high-speed photography of Taylor impact an excellent test of constitutive models and hydrocodes, providing much more information than can be obtained from the recovered rod alone.

6. Conclusions

This study into the Taylor impact response of an annealed copper found that the velocity of propagation of plasticity depends on the magnitude of the strain. Small strains accelerated within the time window observed, whereas large strains decelerated at a logarithmic rate. It should be noted that modelling shows that the plastic front is not planar, but curved, so that the motion of small strain levels may not reflect well the movement of the overall plastic front.

Evidence for work-hardening was found in these experiments. First, profiles of the shape of the deforming rods show that they develop bulges in the diameter some distance from the impact interface. Second, the lateral strain rate at the interface between the two rods may decline, while further along the rod deformation continues. The VISAR traces also reveal changes in local yield strength through the way in which the maximum stress is reached (the long observation time VISAR allows is crucial in making this observation).

The diagnostics deployed gave sensitive and detailed profile–time data for a comparison with computer simulation in order to test constitutive models. A modified Armstrong–Zerilli model predicted intermediate profiles quite well, including the bulges further down the rods, but because it slightly overestimated the material strength, it underpredicted the maximum diameter by 2–6 per cent. This may have been due to a lack of failure modes in the model used. A more advanced path-dependent model has been developed to address such issues (Gould & Goldthorpe 2000). The rods were found to deform symmetrically within the experimental error.

Coaxial alignment is a major challenge in performing symmetric Taylor impact. However, there are several important benefits over the classic configuration. These arise because the interface between the rods remains planar and there is no slip. Hence, no matter what the friction coefficient between the rods.
rods may be, no shear stresses develop at the interface. In any future studies, the effect (if any) of embedded stress gauges on the propagation of plasticity should be determined using the VISAR. If they make little difference, gauges could be used in preference to the VISAR, as they are easier to use.

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