A Suggestion as to the Origin of Black Body Radiation.

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(Received October 15,—Read November 13, 1913.)

The discussion on "Radiation" in Section A at the recent British Association meeting in Birmingham was one of profound interest. Some of the remarks made then suggested the investigation which I now describe.

Planck's formula* for the emission function E_λ is

$$C^2 h\lambda^{-5}/(e^{Ch/k\lambda T}-1).$$

This formula represents the observations for short and long waves at various temperatures with considerable closeness; and, if it is the correct expression, it is held by some to prove that the classical equations of dynamics and electrodynamics are at fault. As I do not think the classical equations are in much danger if properly applied, I have endeavoured to trace the dynamical explanation of the experimental data. There must be many formulæ which will express the data as well as Planck's form. In searching for such an expression of dynamical form, I went back to the equations for the motion of a charged sphere which I established on dynamical principles.† A clue to a solution very soon appeared, and without further remark, at present, I will state the formula that was tried.

If the emission within a range $\delta\lambda$ is expressed by

$$\phi(\lambda, \theta)\, d\lambda,$$

it will be remembered that, in order to satisfy Stefan's and Wien's laws, the expression $\theta^{-5}\phi(\lambda, \theta)$ must be a function of the product $\lambda\theta$.

Accordingly, I selected the formula

$$\phi(\lambda, \theta) = k\theta^5 \frac{\lambda^4\theta^4}{\{(\lambda^2\theta^2-a^2)^2+b^2\lambda^2\theta^2\}^2}$$

as a function which satisfies the following conditions:—

(1) It gives Stefan's law that the total radiation varies as θ^4, since

$$\int_0^\infty \frac{x^4}{\{(x^2-a^2)^2+b^2x^2\}^2}\,dx = \tfrac{1}{4}\pi/b^3 \qquad (a \neq 0 \text{ and } b+).$$

(2) It gives Wien's law that the maximum radiation at any temperature occurs when $\lambda_m\theta$ = constant. In our formula

$$\lambda_m\theta = a.$$

* 'Theorie d. Wärmestrahlung,' 1906, p. 157.

† 'Phil. Trans.,' 1910, p. 152.

(3) It gives the condition that the maximum radiation at any temperature for this wave-length λ_m varies as θ^5. In our formula

$$\phi_{\max.} = k\theta^5/b^4.$$

(4) It gives Lord Rayleigh's formula for long waves

$$\phi = k\theta/\lambda^4.$$

These conditions are required by the experimental work of Kurlbaum, Wien, Paschen, Lummer and Pringsheim, and Rubens.

The remaining point is therefore whether the formula will fit with the results of Paschen or Lummer and Pringsheim for short wave-lengths.

I take Lummer and Pringsheim's* data as typical.

It soon appeared that one might take $b^2 = 4a^2$, and then ϕ takes the simpler form

$$\phi = k\theta^5\left(\frac{\lambda\theta}{\lambda^2\theta^2+a^2}\right)^4.$$

We have at once from the data $\lambda_m\theta = a = 2940$ where the units are λ_m in microns (10^{-4} cm.), and θ in centigrade degrees absolute.

In computing the ordinates for any specified value of θ, it is convenient to note that for any wave-length λ equal to $n\lambda_m$ or $(1/n)\,\lambda_m$, the function ϕ varies as $(n+n^{-1})^{-4}$. The following are the numerical values of the function $f(n) = 16\,(n+n^{-1})^{-4}$:—

n.	$f(n)$.	n.	$f(n)$.
1·0	1·0	2·0	0·4096
1·1	0·9820	2·5	0·2262
1·2	0·9260	3·0	0·1292
1·3	0·8728	4·0	0·04904
1·4	0·8007	5·0	0·02188
1·5	0·7260	6·0	0·01106
1·6	0·6528	7·0	0·00615
1·7	0·5836	8·0	0·00367
1·8	0·5197	9·0	0·00232
1·9	0·4617	10·0	0·00154

I take the experimental curve of Lummer and Pringsheim† at temperature 1450° absolute, for which λ_m is practically 2 μ, and choosing the scale to give a maximum ordinate of 73 units as shown in their diagram, the curve from our formula was calculated, and is shown in fig. 1. It fits the whole experimental curve excellently.

We have thus obtained a formula which fits all the data as well as, if not better than, Planck's. It is empirical, and there may be many others. But

* 'Verh. d. Deutsch. Phys. Gesell.,' 1899, p. 213.

† *Loc. cit.*, p. 217.

in form it suggests what we may expect from a dynamical system with very heavy damping.

The equation
$$(m+m')\ddot{\xi}-k\dddot{\xi} = X,$$
where
$$m' = \tfrac{2}{3}e^2/aC^2 \quad \text{and} \quad k = m'a/C,$$
for the motion of an electron, has been much used by Lorentz, to whom we

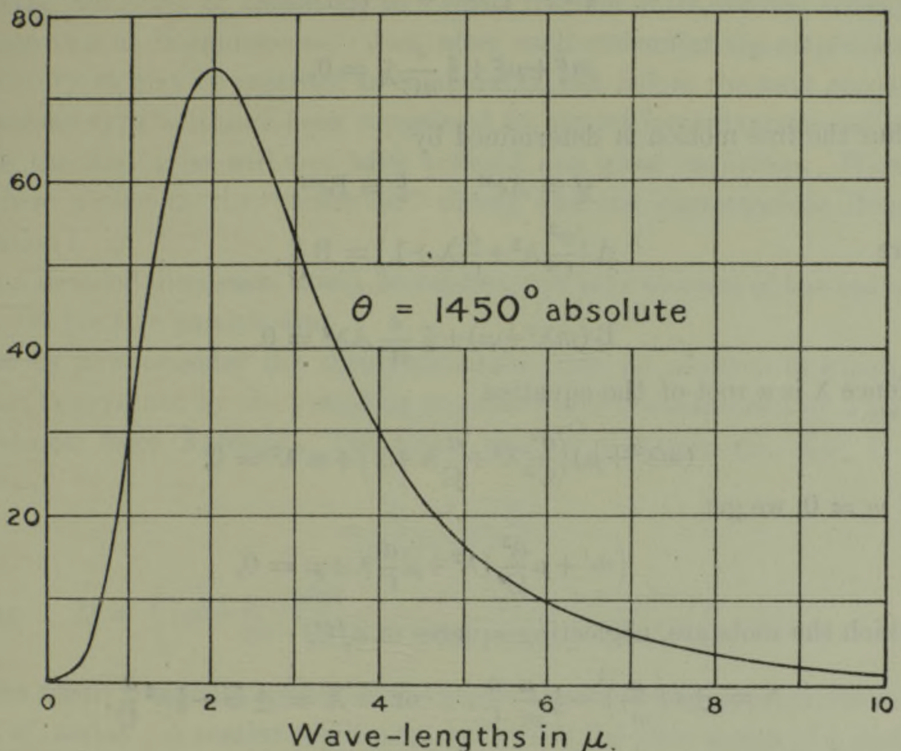

FIG. 1.

owe it, and by Planck. But it is only an approximate equation, although for many purposes a very good one.

I have shown* that more correct expressions for the motion are given by the two equations

$$\frac{a^2}{C^2}\ddot{\chi}+\frac{a}{C}\dot{\chi}+\chi-e\frac{\xi}{C} = 0.$$
$$m\ddot{\xi}+\tfrac{2}{3}\frac{e}{aC}\ddot{\chi} = X,$$

where χ is the value at $r = a$ of the function $\chi(Ct-r)$ which defines the state of the field in the æther, viz.,

$$(X, Y, Z) = \frac{C}{r^3}(-1, 0, 0)\left(r^2\chi''+r\chi'+\chi-e\frac{\xi}{C}\right)$$
$$+\frac{Cx}{r^5}(x, y, z)\left(r^2\chi''+3r\chi'+3\chi-3e\frac{\xi}{C}\right),$$
$$(\alpha, \beta, \gamma) = \frac{C}{r^3}(0, -z, y)(r\chi''+\chi').$$

* 'Phil. Trans.,' *loc. cit. ante.*

The equations were obtained first for a conductor, and were afterwards shown to be very nearly true for an insulating sphere.

Let us suppose that the restoring force is linear, so that $X = -\mu\xi$, and we then get

$$\frac{a^2}{C^2}\ddot{\chi}+\frac{a}{C}\dot{\chi}+\chi-e\frac{\xi}{C} = 0,$$

$$m\ddot{\xi}+\mu\xi+\tfrac{2}{3}\frac{e}{aC}\ddot{\chi} = 0.$$

Thus the free motion is determined by

$$\chi = Ae^{\lambda t}, \qquad \xi = Be^{\lambda t},$$

where
$$A\left(\frac{a^2}{C^2}\lambda^2+\frac{a}{C}\lambda+1\right) = B\frac{e}{C},$$

$$B(m\lambda^2+\mu)+\tfrac{2}{3}\frac{e}{aC}A\lambda^2 = 0.$$

Hence λ is a root of the equation

$$(m\lambda^2+\mu)\left(\frac{a^2}{C^2}\lambda^2+\frac{a}{C}\lambda+1\right)+m'\lambda^2 = 0.$$

If $m = 0$, we get

$$\left(m'+\mu\frac{a^2}{C^2}\right)\lambda^2+\mu\frac{a}{C}\lambda+\mu = 0,$$

of which the roots are, neglecting squares of a/C,

$$\lambda = \pm i\left(\frac{\mu}{m'}\right)^{\frac{1}{2}}-\tfrac{1}{2}\frac{\mu}{m'}\frac{a}{C}, \quad \text{or} \quad \lambda = \pm in-\tfrac{1}{2}n^2\frac{a}{C},$$

where n is the frequency.

This would agree with a calculation by Lorentz.

But experiments are against the supposition that m, the intrinsic mass of an electron, is zero, and Kaufmann's numbers suggest that m is of the same order as m'. For a positive particle it is generally agreed that m'/m is small.

Hence, retaining m, we get two pairs of complex roots. The approximate values neglecting squares of a/C are

$$\lambda = \pm in-\tfrac{1}{2}\frac{a}{C}\frac{m'}{m+m'}n^2,$$

where
$$n^2 = \frac{\mu}{m+m'} \quad \text{and} \quad \lambda = \pm\frac{i}{2}\frac{C}{a}\left(3+\frac{4m'}{m}\right)^{\frac{1}{2}}-\tfrac{1}{2}\frac{C}{a}.$$

These equations may be regarded as applicable to the behaviour of a molecule made up of a heavy, and so comparatively stationary, positive particle with a single electron revolving round it.

The vibrations are of two types. In the first, which closely agrees with

Lorentz' result, the damping is for optical purposes small, and the reduction of amplitude in the time between two collisions of a molecule with its neighbours would be small. But in the second type we have something very different.

The frequency is enormous and the damping so great for optical purposes that the reduction of amplitude to a small fraction of its original value may be regarded as instantaneous. Just after each encounter the amplitudes of the two types may be regarded as comparable, but before the next encounter the second type will have been suppressed by almost instantaneous radiation, while the first type will not have suffered any great reduction. Have we not here a clue to the "quantum" theory and the characteristic Röntgen radiation ?

In a detailed discussion it will be necessary to take account of the radiation from the positive particle also.

Let us now consider the steady radiation from an electron in which the motion is governed by the preceding equations and is maintained by a purely mechanical force $X_0 \cos pt$. The mean rate of radiation, viz., $\overline{X_0 \dot{x}}$, I find to be

$$= \frac{1}{3m^2} \frac{e^2}{C^3} p^4 \frac{X_0^2}{D},$$

where $$D = \left\{\left(p^2 - \frac{\mu}{m}\right)\left(\frac{a^2p^2}{C^2} - 1 - \frac{m'}{m}\right) - \frac{m'}{m}\frac{\mu}{m}\right\}^2 + \frac{p^2a^2}{C^2}\left(\frac{\mu}{m} - p^2\right)^2.$$

This result is equally applicable to a positive particle with appropriate values of m, m', and a. A similar result must hold when the joint action of a positive and negative combination is considered, and m, m' are a combination of the intrinsic and electric inertia terms, while a would now be a linear quantity defining the radius of the orbit.

The coefficient of X_0^2 is, as far as p or λ^{-1} enters, simply a generalised form of

$$\left(\frac{\lambda}{\lambda^2 + \lambda_0^2}\right)^4,$$

and I have little doubt that it could be fitted with the data.

There are always three positive values of p^2 for which p^4/D is a maximum. These are in the vicinity of

$$p^2 = \frac{\mu}{m}, \quad p^2 = \frac{C}{a}\frac{\mu^{\frac{1}{2}}}{m^{\frac{1}{2}}}, \quad \text{and} \quad p^2 = \frac{C^2}{a^2}.$$

The corresponding maximum intensities are in the ratio $(m/m')^2$; 1 : 1 very nearly.

If we choose μ so that the first comes in the observed range of wave-lengths, the other two are in the ultra-violet, and if m'/m is small, they are of slight importance as compared with the first.

If, and only in so far as, Wien's and Stefan's laws are true, we require to suppose that a varies as θ^{-1} and μ varies as θ^2, and the expression is then a function of $\lambda\theta$. The reconciliation of these with each other and with the virial theorem of Clausius is a matter of difficulty, but as the experimental evidence is that neither law is strictly true, the matter may well rest for the present.

These considerations, and the present estimates of atomic magnitudes, lead me to suspect that black body radiation is determined not by the electron, nor by the positive particle alone, but by the joint action of the two. My conclusions are:—

(1) That the experimental data can be well represented by a formula of dynamical type, of which I have given one.

(2) That Newtonian dynamics and the electrodynamics of Larmor are capable of giving an explanation of that formula.

[*Note added October* 23, 1913.—Planck's radiation function has been applied by Einstein* and by Nernst† to the explanation of the variation of atomic heats with temperature. By similar reasoning I find that if the radiation function varies as

$$\theta^5\left(\frac{\lambda\theta}{\lambda^2\theta^2+a^2}\right)^4,$$

the typical term in the expression for the atomic heat varies as

$$\left(\frac{\theta^2}{\theta^2+\theta_0^2}\right)^4\left\{1+\frac{8\theta_0^2}{\theta^2+\theta_0^2}\right\},$$

where θ_0 is some definite temperature.

The following table gives the numerical values

θ/θ_0.	$f(\theta/\theta_0)$.	θ/θ_0.	$f(\theta/\theta_0)$.
0	0·000	4	1·153
1	0·312	5	1·112
2	1·065	10	1·037
3	1·181 maximum	∞	1·000

The result is similar to that required to explain the observations of Nernst.]

I would express my grateful thanks to Sir Joseph Larmor, who has discussed this paper with me, and by his suggestions has added so much to the manner of presenting the results.

* 'Ann. d. Phys.,' 1907, vol. 22, p. 184.

† 'Sitzber. d. Berl. Akad.,' 1911, p. 494.