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Gravity-driven horizontal locomotion: theory and experiment

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Gravity is usually considered not to contribute net energy to horizontal locomotion, the force of gravity being at right angles to the velocity vector. However, when walking or running, humans essentially rotate around the support foot while falling forward during the support phase of the stride. The body thereby acquires angular momentum and rotational energy that are entirely due to the force of gravity, which may be coupled into the forward motion as a source of propulsion upon landing on the other foot. A theory is developed to determine the magnitude and nature of these effects of gravity, showing that more than 10% of the energy needed for running can be obtained from the field of gravity. Likewise, at a particular optimum velocity, walking may become entirely driven by gravity-induced angular momentum without any muscular effort. Experiments with athletes running or walking on a treadmill—after appropriate training—are consistent with the theory: angular momentum-rich locomotion techniques would have applications in sports, competition running and the mechanics of walking, and in the making of leg prostheses and exoskeletons, and might help to explain how humans can outrun larger prey animals to exhaustion during hours of hunting pursuits.

1. Introduction

Consider a runner moving horizontally forward at a velocity v from a vertical posture (figure 1). In the field of gravity, the body is subject to an external, downwards force $F_g = Mg$, where M is the body mass and $g = 9.81\,\mathrm{m\,s^{-2}}$ is the acceleration of gravity on Earth. Proceeding from an upright position and into an angle θ_m with the vertical, the centre of mass becomes

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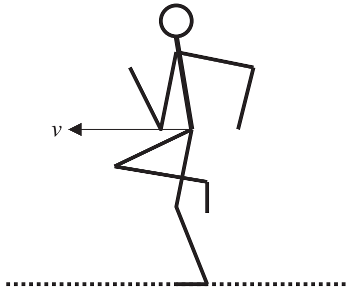

Figure 1. A runner landing 'on top of the stride'; having changed the positions of the legs while in the air, the whole body takes part in the forward rotation when on the ground.

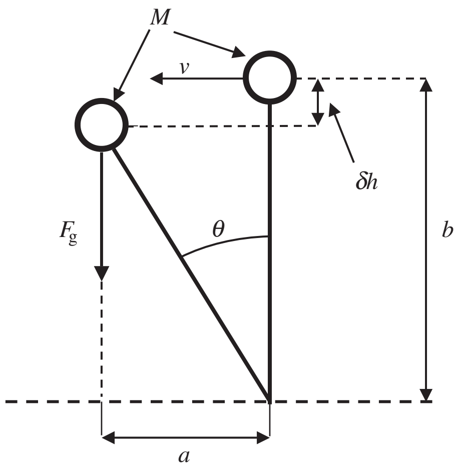

Figure 2. The runner—for simplicity represented as a mass M located at the centre of mass—rotating forward into an angle θ with the vertical, where v is the runner's horizontal velocity, b is the height above ground of the runner's centre of mass, $F_g = Mg$ is the force of gravity acting upon the body, $a = b\sin\theta$ is the arm through which the force of gravity exerts a torque on the body around the point of ground support and δh is the distance that the centre of mass becomes lowered through this rotation.

lowered a distance $\delta h = b(1 - \cos\theta_m)$, where b (assumed constant) is the radius of rotation for the body's centre of mass around the support foot (figure 2). An amount of gravitational energy

$$\delta E_h = F_g \delta h = Mgb(1 - \cos\theta_m) \tag{1.1}$$

is thereby lost on the body. By extending the leg, the height above ground is restored and a matching amount of energy is provided by muscular force. During each single stride, therefore, the runner's net change of energy in the field of gravity is zero. This is common to all bipedal locomotion [1–3] and contributes to the prevailing conception that no net energy for horizontal walking and running can be gained from the field of gravity.

However, the energy budget of a moving body does not alone suffice to represent and analyse the complexities of locomotion. The laws of physics imply that energy and momentum are conserved for a moving body [4], so that energy and momentum both need to be taken into account for any complete description of bipedal motion. On this basis, the overall physics of walking and running is analysed in terms of external and internal energy and momentum, features that will be shared by all physical models of human locomotion. Angular momentum generated by the force of gravity, as a walker or runner rotates around the support foot when on the ground, thus may be coupled into the forward motion upon landing on the other foot. Technical elements of locomotion are described that will further such angular momentum transfer, from which the magnitudes of these gravity effects on running and walking can be calculated. Energy savings of 10% or more in running may be achievable, while horizontal walking may become entirely driven by angular momentum generated by the field of gravity at a certain low velocity. Experiments carried out with athletes on treadmills to test the theory show the expected effects of gravity-generated angular momentum in running, and the near-perfect correspondence between calculated and measured optimum velocities for angular momentum-driven walking, without muscular input into the forward motion when walking at the optimum velocity even when carrying heavy loads on shoulders.

2. Angular momentum

With horizontal velocity v, the human depicted in figure 1 enters a gyration around the point of ground support, with an initial angular velocity $\omega = v/b$ and angular momentum $L = I\omega$ that are entirely associated with the external forward motion, where I is the moment of inertia of the moving body around the ground contact point. As seen from figure 2, at any instant t during the gyration the force of gravity F_g exerts a torque $N(t) = F_g a = Mgb\sin\theta$ on the body through an arm $a = b\sin\theta$ around the support foot, where the angle θ increases with time as $\theta = vt/b$, from $t = 0$ (vertical position) to $t = t_m$ (at maximum angle θ_m), assuming that v and b are constant through this time. The torque $N(t)$ thereby introduces angular momentum into the body at a rate $dL/dt = N(t)$ during this forward rotation, so that a finite, *internal* amount of angular momentum

$$\delta L = \int_0^{t_m} N(t)\mathrm{d}t = \int_0^{\theta_m} N(\theta)\frac{b}{v}\mathrm{d}\theta = \frac{Mgb^2}{v}(1-\cos\theta_m) \tag{2.1}$$

becomes established in the body by the force of gravity when θ varies from zero to θ_m. The body thereby obtains a corresponding angular velocity $\delta\omega = \delta L/I = (Mgb^2/Iv)(1-\cos\theta_m)$, and an amount of rotational energy δE_{rot} is acquired by the gyrating body, with

$$\delta E_{rot} = \delta((1/2)I\omega^2) = I\omega\delta\omega = Mgb(1-\cos\theta_m). \tag{2.2}$$

Like δL, δE_{rot} is intrinsic to the body, i.e. not connected with the forward motion. It will be observed that $\delta E_{rot} = \delta E_h$, as one would expect; the body falls in the field of gravity, loses gravitational energy and receives a matching amount of kinetic energy. Furthermore, and according to the conservation laws of physics [4], δL and δE_{rot} will be conserved when the support foot leaves the ground. With suitable locomotion techniques, δL and δE_{rot} may thereby be carried by the body through the intermediate phase, with the potential to change the energy and velocity of the body's motion upon landing on the opposite foot, similar to what is regularly applied with great success on cricket, tennis and table tennis balls. This course of events is entirely general and will be valid for walking as well as for running.

3. Running

The runner leaves the ground with rotational energy δE_{rot} and angular momentum δL that are superimposed on the forward motion characterized by velocity v, momentum Mv and kinetic energy $E_v = (1/2)Mv^2$. Having left the ground, the state of motion cannot change (ignoring air resistance), so that δL and δE_{rot} are conserved throughout the flight phase and into the following stride, with δL preserved as a free rotation around the body's centre of mass. As the runner lands for the next stride that free rotation changes into a bound gyration around the contact foot, whereby the runner's internal angular momentum and energy of rotation become coupled into the external forward motion. Instantly upon landing, therefore, and due to the conservation of angular momentum, the runner's forward momentum Mv increases by an amount $\delta L/b$, and an amount of energy δE_{rot} is added to the external kinetic energy E_v. This completes the stride sequence, and the runner enters the subsequent stride with a potential gain of kinetic energy $\delta E_v = \delta E_{rot}$. In effect, and through the very act of landing, the runner may be able to *increase*

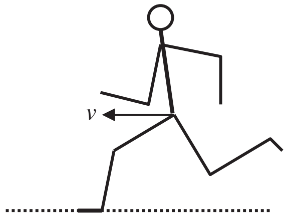

Figure 3. A runner landing 'behind the stride' and having to change the positions of the legs while on the ground, thereby rotating the recovery leg around the hip in the negative direction to the body's overall forward rotation.

the velocity of motion thanks to angular momentum generated by the force of gravity in the preceding stride. The velocity increment δv can be found from $\delta(Mv) = \delta L/b$ as

$$\delta v = \left(\frac{gb}{v}\right)(1 - \cos\theta_m). \tag{3.1}$$

In order to take advantage of the field of gravity, the runner needs to practise a technique whereby he lands 'on top of the stride', with the support leg in a position close to vertical and with the recovery leg already in front, as in figure 1. To avoid the whole body rotating during the flight phase, the runner's legs would be kept in a net forward rotation while in the air, which serves to carry the angular momentum through the flight phase. The runner thereby changes the positions of the legs while in the air, with the upper body in a steady, upright position and the recovery leg moving from behind to the front, before landing as in figure 1. The rationale behind this can be seen from comparing the runner in figure 1 with the one depicted in figure 3, who is landing in the conventional manner with the support leg in front and the recovery leg well behind. The runner then needs to rotate the recovery leg around the hip, from a rearward into a forward position while on the ground, thereby working into the body an angular momentum that is in the opposite direction to δL and may easily negate the latter. As a result, the runner shown in figure 3 is left with little or no net internal angular momentum for each stride; the runner essentially uses muscular energy to counteract what gravity provides for free. On the other hand, thanks to the recovery leg being already in a forward position at landing, the runner in figure 1 introduces a minimum of negative angular momentum into the body while on the ground, and the gravity-induced gains can be exploited to their full effect.

With suitable techniques of running, therefore, gravity-supplied energy δE_{rot} and angular momentum δL may, for each stride, relieve runners from engaging an equal amount of energy by muscular force, so as to maintain velocity with less effort or else increase the running speed accordingly. The magnitude of this effect can be quite impressive. Consider the size of the rotational energy compared with the kinetic energy of the forward motion,

$$\frac{\delta E_{rot}}{E_v} = \frac{(Mgb)(1 - \cos\theta_m)}{(1/2)Mv^2} = \left(\frac{2gb}{v^2}\right)(1 - \cos\theta_m). \tag{3.2}$$

With $b = 0.9$ m, $g = 9.81\,\mathrm{m\,s^{-2}}$ and $v = 6\,\mathrm{m\,s^{-1}}$ (long-distance speed), ratios of 6.6%, 8.9%, 11.5% and 14.4% result for θ_m values equal to 30°, 35°, 40° and 45°, respectively, which are more pronounced than for sprint speeds ($10\,\mathrm{m\,s^{-1}}$) with similar ratios of 2.4%, 3.3%, 4.1% and 5.2% for the same values of θ_m. Corresponding relative increases in running speeds would be half of these, as can also be seen from equation (3.1), with $\delta v/v = (gb/v^2)(1 - \cos\theta_m)$, in the range 3.3–7.3% for distance running and 1.2–2.6% for sprinting, equivalent to 1–2 min on a 10 000 m run and 0.2–0.3 s over 100 m. For any runner, δE_{rot} is the same for all velocities of motion (although θ_m may change slightly with the runner's velocity) and explains why these effects are more pronounced at lower velocities of motion. Tall runners have a potential advantage over smaller runners due to their larger radius of rotation. Running at altitude with lower g would reduce the effects of gravity only marginally.

4. Walking

It follows from the generality of the present theory that the relations derived here are valid for any velocity, and thus would describe walking as well as running. Moreover, the effects of δL are larger at low velocities, as seen above, and so should be even more prominent in walking. Walking at low velocity is of special interest as the ratio $\delta E_{\mathrm{rot}}/E_v$ will then be near its maximum value (due to the factor $1/v^2$). In particular, it becomes possible to ask if δE_{rot} and the associated angular momentum δL may enable horizontal walking without any muscular input, and, if so, at what velocity? In other words, is it feasible to walk without using one's muscles for propulsion, by applying a similar technique for walking as was described above for running, to be entirely powered by the force of gravity? δE_{rot} would then be the only energy input into the forward motion, so that at a certain velocity v_{g} it follows from equation (3.2) that

$$\frac{\delta E_{\mathrm{rot}}}{E_v} = \left(\frac{2gb}{v_g^2}\right)(1 - \cos\theta_{\mathrm{m}}) = 1, \tag{4.1}$$

which suggests that gravity-based horizontal walking would indeed be possible, at a velocity

$$v_{\mathrm{g}} = [2gb(1 - \cos\theta_{\mathrm{m}})]^{1/2} \approx (gb)^{1/2}\theta_{\mathrm{m}}, \tag{4.2}$$

where $\cos\theta_{\mathrm{m}}$ has been expanded into lowest order in θ_{m} without loss of accuracy for small θ_{m}. The existence of such angular momentum-driven horizontal walking is a new concept and would be a robust result as it derives directly from the conservation of energy and angular momentum. Walking at higher velocities than v_{g} will require muscular energy to increase the speed, while walking at lower velocities would need muscular force to retard the motion, as δE_{rot} would then be larger than E_v and gravity would tend to increase the speed towards v_{g}. Precisely at the velocity v_{g}, therefore, a minimum should occur in the energy requirements for walking. Furthermore, as v_{g} is independent of the body mass and increases with larger b, it might even be conceivable at velocity v_{g} to carry loads high up on the body, which increases the radius of gyration—and therefore the velocity, too—without demanding any additional muscular energy. These predictions suggest experiments that would provide a crucial test of the theory.

5. Experiments

Experiments on running and walking were carried out in the physiology laboratory at the University of Kuopio, Finland; before that, one of us (S.O.K.) had for some years coached a number of athletes (in Norway) on running according to the present theory, but no measurements had ever been made. The experimental equipment included a treadmill built into the floor, a strap from the ceiling to help prevent runners from falling off the treadmill, and instruments to measure the volume of exhaled air from runners and the associated concentrations of oxygen, with runners wearing a breathing mask connected through a hose to the instrument station. Volumes of air and O_2 were recorded two to four times every minute. The main test person (TP) was a 49-year-old veteran male runner who had over the years become able to do relatively well on angular momentum-assisted running, in addition to conventional running. Runs were made alternately with one or the other technique in pairs, and then repeated at different velocities, each run typically lasting 6 min with similar lengths of resting periods between runs.

(a) Running

Figure 4 shows the energy input per unit time for TP, measured as oxygen volume per minute and presented as a function of velocity for conventional running and angular momentum-rich running techniques, respectively; curves for exhaled air were similar but steeper. The curves are essentially linear and parallel, and reflect the constant, velocity-independent reduction in energy demands experienced by TP when using the angular momentum-rich running technique, as suggested by the theory (equation (2.2)). This would imply that the maximum rotation angle θ_{m} was more or

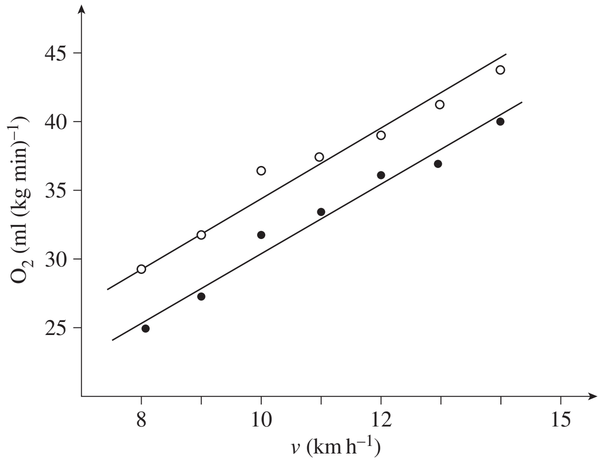

Figure 4. Oxygen consumption in running. Open circles, conventional technique; filled circles, angular momentum-rich technique.

less the same at all velocities: what increased for higher velocity was mainly the stride frequency, the runner's stride remained the same. The measured energy savings range from 16% at 9 km h^{-1} to 10% at 14 km h^{-1}, which would be about 1/2 to 2/3 of TP's maximum energy savings, as calculated from equation (3.2) for his $b = 0.97$ m (half of body height) and with θ_m of his stride estimated at somewhat less than 30°.

Five top regional runners (three males, two females, mostly distance runners) were tested on the treadmill, instructed and trained for 3–4 days on the angular momentum-rich running technique and then tested again. Before instructions they all showed variants of conventional 'jumping' techniques with a rather open landing as in figure 3. From a visual point of view, the instructions helped all of them to improve their running technique, in so far as they were able to land with their recovery knee in a more forward position. However, four of them showed either similar or even significantly increased energies when running at the same speed after the sessions. The fifth one (a male sprinter), who started with a pronounced jumping technique, managed to adopt the angular momentum-rich running technique when coached on the treadmill, so as to realize 10% energy savings at 14 km h^{-1} (with θ_m estimated at 30–35°, corresponding to a theoretical maximum of 16–21% energy savings), while enthusiastically exclaiming 'I'm flying!'; his measured ml O_2/kg · min before and after training sessions essentially coincided with those for TP in figure 4, as did his percentage energy savings. This shows that even over such a short time the instructions and training programme were effective, at least for the sprinter. Experience shows that distance runners—even though their techniques may appear to have improved—normally need more coaching time to practise and automate a relaxed angular momentum-conserving running technique. It would appear that there is more to running than meets the eye.

(b) Walking

Walking differs from running in having a doubly supported phase and no flight phase; normal walking also is not constrained by physiological capacity and so can be considered from a purely kinematic point of view. After some initial practising TP was instructed to breathe as evenly as possible (to minimize signal variations) when set to walk at velocities ranging from 0.7 m s^{-1} to 1.5 m s^{-1}. The walking technique involved short steps of about 30 cm, corresponding to a maximum gyration angle of $\theta_m = 17°$, and landing on a vertical and slightly bent leg into the doubly supported phase with the knee of the recovery leg somewhat in front, as in figure 5.

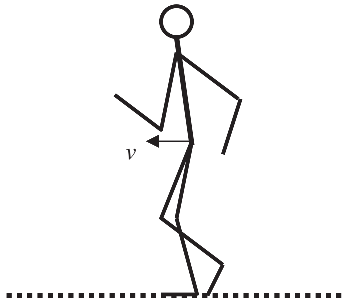

Figure 5. Landing in gravity-driven walking.

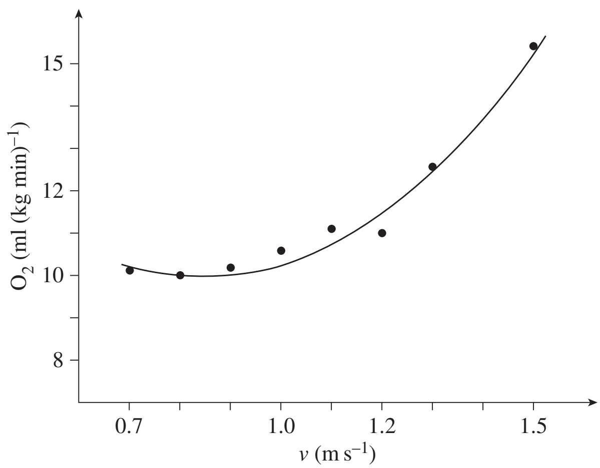

Figure 6. Oxygen consumption in gravity-driven walking.

Walking was continuous with changes of velocity being made every 6 min; measurements of exhaled air and O_2 consumption were made every 15 s. Up to the maximum velocity of 1.5 m s^{-1}, differences in energy demands were too small to be felt by TP. Curves that contain the results of measurements are shown in figures 6 and 7. To determine whether the rise of effort towards the end of the walk could be due to TP becoming tired, TP was also set to walk at 1 m s^{-1} for 40 min, which produced an essentially flat curve.

The curve for exhaled air (figure 7) shows the energy minimum predicted by the theory, at a velocity of about 0.9 m s^{-1}. Unfortunately, the oxygen measurements (figure 6) are affected by the limited resolution of the O_2 sensor and so do not significantly resolve such small differences in signal; only a weak minimum below a velocity of 1 m s^{-1} could at best be suggested. It will be observed from the curves, however, that the relative changes in signals, from the minima around 0.9 m s^{-1} to their highest levels at 1.5 m s^{-1}, are essentially the same (*ca* 50%) for the two curves, consistent with angular momentum-driven walking being entirely kinematic and not constrained by physiologial capacity. The measured exhalation of air, therefore, would be in one-to-one correspondence with oxygen demands at those low velocities. From this we conclude that the curve for exhaled air gives a true measure of the oxygen consumption and thus reflects directly the energy budget of the slowly walking TP. With $\theta_m = 17°$ and $b = 0.97$ m, equation (4.2) predicts $v_g = 0.92$ m s^{-1} for TP, in excellent agreement with the measurements.

In another experiment, TP was set to walk at a constant velocity of 3.5 km h^{-1} (0.97 m s^{-1}), with loads on his shoulders being increased from 0 to 25 kg at 4 min intervals while TP kept walking continuously. The resulting curve is shown in figure 8 and is quite remarkable: as the

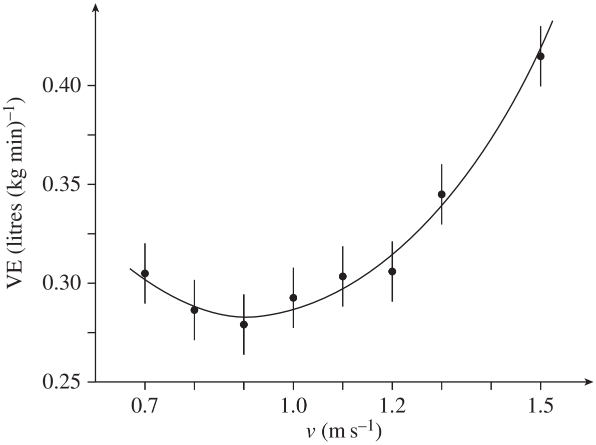

Figure 7. Exhaled air in gravity-driven walking.

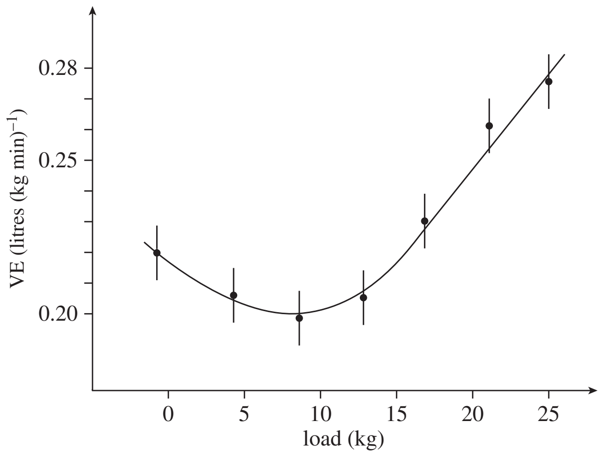

Figure 8. Exhaled air in gravity-driven walking with loads.

loads are increased from zero, TP actually uses less and less energy until a minimum is reached at about 9 kg of load, and only from then on do the energy demands (measured as exhaled air) start to increase. The curve can be understood as follows: with body mass $M = 88$ kg and a mass m carried on his shoulders (approximately at a height of $2b$), TP has a combined centre of mass at $b_m = [(M + 2m)/(M + m)]b$ above ground. A load of 9 kg on top of TP thus increases his arm of rotation to $b_m = 1.06$ m, with a corresponding angular momentum-driven velocity $v_g = 0.96\,\mathrm{m\,s^{-1}}$ from equation (4.2), which is essentially the velocity at which he was walking. The unloaded TP has an optimum angular momentum-driven velocity of $v_g = 0.92\,\mathrm{m\,s^{-1}}$. Consequently, as he starts to walk unloaded at velocity $0.97\,\mathrm{m\,s^{-1}}$, which is higher than the initial optimum velocity of $0.92\,\mathrm{m\,s^{-1}}$, he must use muscular force to drive himself forward. However, with increasing loads on his shoulders his optimum velocity also increases, and so TP needs to put in less and less muscular force to maintain the walking velocity, until the loads are sufficiently high to make his optimum angular momentum-driven velocity v_g equal to his actual walking velocity. With further loads on his shoulders, TP's v_g continues to increase beyond $0.97\,\mathrm{m\,s^{-1}}$, but, confined to walking at $0.97\,\mathrm{m\,s^{-1}}$, he now walks more slowly than the increasing optimum velocity and so has to use more and more muscular force to brake against the effect of gravity, which would otherwise drive him to walk faster and faster as the loads increase; this results in the minimum of the curve.

(More than a year separated the measurements shown in figures 6 and 7 from those presented in figure 8, which could explain the lower energy demands for the walks shown in figure 8: TP was then generally in better physical condition.) The measurements suggest that, at $0.97\,\mathrm{m\,s^{-1}}$ velocity, TP could actually carry a 16 kg load—18% of his body mass—on his shoulders without using more energy than when walking without any load. It could possibly be that, if TP had been walking faster than $0.97\,\mathrm{m\,s^{-1}}$ from the start, the minimum of the curve would have been reached at even higher loads. TP might then have been able to carry significantly heavier weights on his shoulders without using more energy than when walking unloaded (although walking faster the whole curve would move upwards); this could open up interesting experiments and developments in optimizing human load-carrying capacity.

6. Discussion and conclusion

The present theory introduces conservation of angular momentum into the physical description of bipedal locomotion, as a mechanism to couple energy and momentum from one stride into the next. As a result, rotational energy and angular momentum, gained when a locomoting human falls forward in the field of gravity, may be carried into the subsequent stride to increase the forward drive. The theory thus offers new insights into the mechanisms of walking as well as running, with preferred locomotion techniques being rich in gravity-generated angular momentum. Experiments carried out with athletes trained to run according to the theory show the expected energy savings in angular momentum-rich running, exciting applications of which could be found in sports. In particular, the overall results in distance running might potentially be improved quite considerably: all top male sprinters appear to practise rather pure angular momentum-assisted techniques (probably without knowing), which could partially explain the development of world records over recent years, whereas only a handful of the best distance runners do (and, among these, more often women than men). However, in order to substantiate the results obtained here, and to learn in more detail what are the options and limitations offered by this theory of locomotion, these initial experiments should be extended to include a much larger group of runners. But this might not happen easily and it could take years for otherwise highly competent runners to master the angular momentum-rich running technique in as relaxed and effective manner as they do their current running. Alternatively, one could train two groups of runners according to similar physical regimes, with one (the control group) practising conventional running while the other group abandons their current running and concentrates on learning the new technique. Either option would require strong commitment from the runners, with substantial and long-term external supporting resources, both of which have been outside our current reach. And then not all runners will be able to master the new technique; on average maybe only half of them.

A new concept has been identified, in the shape of a velocity at which walking is entirely driven by angular momentum generated by the force of gravity. The experiments served to confirm that fully angular momentum-driven walking is indeed possible, and at essentially the exact velocity given by the theory; this provides strong experimental support for the entire theory and might be relevant for the development of more effective leg prostheses and exoskeletons. The theory also explains the otherwise baffling ability of East African women to carry heavy loads on their head without further muscular effort, as found by Maloiy *et al.* [5] when studying East African women walking on treadmills. For the unloaded women an energy minimum was measured at a velocity of $3\,\mathrm{km\,h^{-1}}$ ($0.83\,\mathrm{m\,s^{-1}}$), and at $3.25\,\mathrm{km\,h^{-1}}$ ($0.90\,\mathrm{m\,s^{-1}}$) when the women carried 20% of their body mass on their head. The women's body height and angles of gyration were not given. However, with a maximum gyration angle of $\theta_\mathrm{m} = 17^\circ$ (as in our experiments) and assuming $b = 0.8\,\mathrm{m}$, the theory predicts an energy minimum at $v_\mathrm{g} = 0.83\,\mathrm{m\,s^{-1}}$ for the unloaded walkers, exactly as measured. When the women carry a 20% load on their head, their arm of rotation increases to $b_\mathrm{m} = 0.93\,\mathrm{m}$ (from the same formula as for TP), with an energy minimum for walking at $v_\mathrm{g} = 0.89\,\mathrm{m\,s^{-1}}$ given by equation (4.2). Thus, there seems little reason to introduce complex *ad hoc* explanations for such feats [5,6]; the counterintuitive effects of East African walking found by Maloiy *et al*. follow naturally and in detail from the kinematics of angular momentum-driven walking, as a direct consequence of the conservation of angular momentum, and add to our own experiments in providing independent support for the theory presented here. Angular momentum-driven walking could find further applications in connection with current work on passive dynamic walking [7–9], in which computer and mechanical models are employed to study various modes of walking in the field of gravity. Until now those models have considered only the conservation of the potential energy of the walker, such as when walking slightly downhill. Extending the models to include the conservation of angular momentum would remove current limitations on passive dynamic walking, to offer new tools for theoretical work as well as for the construction of new and more complete mechanical models of walkers: our work differs from passive dynamic walking in that the latter is based on a variety of preferred models, whereas we consider the general physics of bipedal locomotion, which is not associated with any particular model. As another example of what might become possible, consider a walker moving at a velocity $v_g(\alpha)$ uphill with no muscular inputs, at an angle α with the horizontal. For that to be possible, the energy obtained through the conservation of angular momentum, $\delta E_{rot} = Mgb(1 - \cos\theta_m) \approx (1/2)Mgb\theta_m^2$, would have to support both the forward kinetic energy, $E_v = (1/2)Mv_g(\alpha)^2$, and the increase in potential energy of the body in the field of gravity, $\delta E_{pot} = Mg\delta h_\alpha$, where δh_α is the vertical displacement of the body for each stride. With stride length $a = b\sin\theta_m \approx b\theta_m$, as in figure 2, we have $\delta h_\alpha = a\sin\alpha \approx b\alpha\theta_m$, so that with $\delta E_{rot} = E_v + \delta E_{pot}$ we have the following:

$$(1/2)Mgb\theta_m^2 = (1/2)Mv_g(\alpha)^2 + Mgb\alpha\theta_m, \tag{6.1}$$

from which results $v_g(\alpha) = [gb\theta_m(\theta_m - 2\alpha)]^{1/2}$. This suggests that, for inclinations of $\alpha < (1/2)\theta_m$, it might be feasible to walk uphill without muscular effort, being fully driven by angular momentum generated by the field of gravity. For instance, with $\alpha = (1/4)\theta_m$ ($\approx 4^\circ$) we find $v_g(\alpha) = (gb/2)^{1/2}\theta_m = (1/2)^{1/2}v_g$, which for TP with $v_g = 0.92\,\mathrm{m\,s^{-1}}$ would imply $v_g(\alpha) = 0.65\,\mathrm{m\,s^{-1}}$, assuming that v_g and θ_m remain unaffected by the inclination; this might warrant further theoretical and experimental investigations. Contrary to running, angular momentum-driven walking by humans requires little instruction or exercise beyond what has been mentioned above, and so should be fairly easy to reproduce.

Current walking techniques in East Africa are indicative of a cultural adaptation to angular momentum-assisted long-distance locomotion, which may contribute to the East African dominance of distance running in recent decades. Intriguingly, at a running velocity of $3\,\mathrm{m\,s^{-1}}$, equation (3.2) indicates that 25–45% of the energy for running may be freely obtained from the field of gravity, which could be the mechanism that enables Bushmen of the Kalahari Desert to outrun larger prey animals to exhaustion over several hours of hunting pursuits [10], typically at $3\,\mathrm{m\,s^{-1}}$. As has been argued [11], a capability for endurance running by the early *Homo* genus in East Africa, developed through persistence hunting, may even have been crucial in making *H. sapiens* evolve into the successful species. Such running ability of early *Homo* is supported by morphological evidence from fossil hominins [12], and would have been greatly strengthened when and if angular momentum-assisted locomotion developed in East Africa during this timeline—as may well have been the case. Outside of Africa such advanced techniques of locomotion seem generally to be absent today, but may have been more widely practised in former times [10,11], although Sami people in Northern Scandinavia may apply related techniques (the 'Sami roll') for long-distance trekking, and the female way of walking often appears to include some measure of angular momentum assistance, as may be the case also for small children when learning how to walk.

Ethics. We confirm that informed consent was obtained from all subjects who took part in the experiments.
Data accessibility. The data as measured are given in figures 4 and 6–8.
Authors' contributions. S.O.K. developed the theory, instructed the athletes, took part in the experiments, made the initial data analysis and presentation, and drafted the manuscript; A.K. planned, organized and led the experiments, took part in the instructions of the athletes, helped in the data analysis and interpretation, and in drafting the manuscript. Both authors gave final approval for publication.

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