Ozonosphere temperatures under radiation equilibrium

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In two previous papers (Gowan 1928, 1930) calculating the effect of ozone on the temperature of the upper atmosphere, the height of the centre of gravity of the ozone layer was assumed to be about 45 km. Later, more reliable measurements were made (Gotz, Meetham & Dobson 1934) lowering this figure to 22 km., and also supplying the vertical distribution up to a little beyond 45 km. The results of the necessary revision of calculations were given to the Oxford Conference on Atmospheric Ozone (Gowan 1936). Strong (1939) and Summerfield (1941) have given a revision of the infra-red absorption of ozone and have taken the pressure into account. Elsasser (1942) and Dobson (1942) have discussed the absorption of water vapour in the atmosphere and Elsasser has justified the use of the square-root law for the pressure effect throughout the atmosphere for this vapour in the infra-red. The present paper gives the results of a further revised calculation making use of the new data, for latitude 50° N.

Method

The foundation of the method is the primary assumption that radiation equilibrium is closely approached about midday in the stratosphere as a whole, and that what convection exists is too slow to prevent the attainment of this equilibrium but is nevertheless sufficient to keep the composition substantially constant. The following equation represents this equilibrium in the form most convenient for computation:

\[ \int_{0}^{\infty} K'_\lambda S_\lambda d\lambda + \int_{0}^{\infty} K_\lambda E_\lambda d\lambda + \int_{0}^{\infty} K_\lambda A_\lambda d\lambda = 2 \int_{0}^{\infty} K_\lambda B_{\lambda T_0} d\lambda. \]

This equation applied to any horizontal layer where, for wave-length \( \lambda \),

- \( K'_\lambda \) is the fractional absorption for radiation in a parallel beam,
- \( K_\lambda \) is the same for diffuse radiation,
- \( S_\lambda \) is the solar radiation reaching the layer,
- \( E_\lambda \) is the earth and troposphere radiation reaching the layer,
- \( A_\lambda \) is the radiation to the layer from the rest of the stratosphere,
- \( B_{\lambda T_0} \) is the black-body radiation at \( T_0 \)° K, which temperature has to be assumed for each layer at the beginning of the computation.

Graphical integration is necessary because of the irregular variation of \( K'_\lambda \) and \( K_\lambda \). Successive approximation is necessary because \( A_\lambda \) depends on the temperature of
the entire ozonosphere except the layer being considered. For given latitude, season, amount of ozone, and amount of water vapour only the term containing \( A_\lambda \) needs recalculation at each stage of the successive approximation process.

Nine layers have been chosen: from 11 to 15 km., then every 5 up to 55 km. The distribution of water vapour, carbon dioxide, and ozone in the nine layers was calculated on the basis of a preliminary estimate of temperature for average conditions at latitude 50° N. These were then considered as 'optical layers', i.e. as containing this constant amount of gas. Only small changes in height resulted from the final temperatures being different from the first estimate. These have been neglected in the region considered.

**DATA ON QUANTITIES**

In table 1 are shown the amounts of absorbing material in the nine layers. For oxygen and carbon dioxide the units of thickness are centimetres, in all the other columns they are microns. For ozone and oxygen and carbon dioxide the amounts refer to normal temperature and pressure; for water vapour they are the equivalent thickness of liquid (compare Brunt & Kapur (1938) and Jaumotte (1936)).

**Table 1**

| layer | \( A_\lambda \) (cm.
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>0-200 cm.</td>
<td>0-280 cm.</td>
</tr>
<tr>
<td>0-10%</td>
<td>40%</td>
</tr>
<tr>
<td>(cm.)</td>
<td></td>
</tr>
<tr>
<td>km.)</td>
<td></td>
</tr>
<tr>
<td>height</td>
<td></td>
</tr>
<tr>
<td>boundary</td>
<td></td>
</tr>
<tr>
<td>(mm. Hg)</td>
<td></td>
</tr>
<tr>
<td>pressure</td>
<td></td>
</tr>
<tr>
<td>total</td>
<td></td>
</tr>
<tr>
<td>total</td>
<td></td>
</tr>
<tr>
<td>50-55</td>
<td></td>
</tr>
<tr>
<td>45-50</td>
<td></td>
</tr>
<tr>
<td>40-45</td>
<td></td>
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<td>35-40</td>
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<td>30-35</td>
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<td>25-30</td>
<td></td>
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<tr>
<td>20-25</td>
<td></td>
</tr>
<tr>
<td>15-20</td>
<td></td>
</tr>
<tr>
<td>11-15</td>
<td></td>
</tr>
</tbody>
</table>

The water vapour was calculated as proportional to the total pressure (i.e. completely mixed in the stratosphere) starting with 10 and 40 % relative humidity at the tropopause. The ozone was computed from the curves given by Gotz et al. (1934). Carbon dioxide was calculated as proportional to the total pressure. The proportion found by Paneth (1937), 0.03 % by volume was used.

**DATA ON ABSORPTIONS**

The absorption of solar energy in oxygen has only been taken into account for wave-lengths longer than 1750 \( \AA \). It appears to be of little importance below 55 km. but shows signs of becoming important at greater heights. If calculations at such heights are attempted the effects of the strong absorption band at 1450 \( \AA \) should be included. Gotz (1936) has made some calculations of the absorption in oxygen.
He also gives a table showing total absorbed solar energy of wave-length less than 3400 A. in 0-200 cm. of ozone. The values are in fair agreement with the solar totals obtained during the calculations, though the conditions are not identical. The absorption coefficients for oxygen used in this paper are given by Granath (1929).

The absorption coefficients for ozone are: in the ultra-violet from Fabry & Buisson (1913), Lambert, Dejardin & Chalonge (1927), Lauchli (1929); in the visible from Colange (1927); and in the infra-red from Summerfield (1941). Summerfield has verified that the pressure effect is important only in the infra-red, and the necessary corrections have been made for the main band at 9-6 µ from his thesis, on the basis of a band width of 1-0 µ.

Elsasser (1942) concludes that the half-width of the individual lines comprising an infra-red band is proportional to the square root of the total pressure, and that for water vapour the lines are still without much overlap at atmospheric pressure. A blend of the coefficients of Fowle (1917) and Hettner (1918) has been used in all previous calculations. As a check the amounts of water vapour in the layers were considered as at atmospheric pressure, and the emissivity calculated at 20°C. The emissivities as a percentage of black were compared with the observed results of figure 29 in Elsasser's monograph, wherever the amounts of water vapour coincided. The agreement was good, and the absorptions which gave this agreement were used in one set of temperature calculations, which are therefore not adjusted for the pressure effect. Subsequently another set of temperature calculations was completed using the absorptions adjusted for the pressure at the lower boundary of the layer concerned.

For the upper layers an extrapolation was necessary, and for this purpose a replot on log-log paper was made. The curve is slightly concave downwards, but for the extrapolation a straight line was used to give the absorptions for the upper layers. These are therefore somewhat too high, but the pressure correction makes their influence on the calculated temperature rather small. In cases where water vapour is not corrected for pressure the 10 and 40 % values of water vapour would be slightly low for the upper three layers.

The emission of the 14 µ band (whose width is 2-0 µ) for carbon dioxide was taken from the curve on p. 80 of Elsasser's monograph. The curve was replotted on log-log paper which makes it concave downwards. For the small quantities involved in the upper layers an extrapolation is necessary. This was done on the basis of a straight line, which gives too large absorptions and emissions. Even so the carbon dioxide contribution, whether corrected or not corrected for the pressure effect by the square-root law, turned out to be negligible above 30 km. There may be a certain overlapping in absorption and emission for carbon dioxide and water vapour about 15 to 16 µ. This has been neglected, each constituent being treated individually.

There is an effect of temperature on absorption. It is smaller than the pressure effect, the numerical values are uncertain, and it adds an extra complication to the successive approximation scheme used. In all cases calculated this temperature effect has been neglected.
DATA ON ENERGY ENTERING THE OZONOSPHERE

The solar energy outside the atmosphere in the region of ozone absorption is not known with great accuracy. Coblentz & Stair (1936, 1943), Stair & Hand (1939), Pettit (1940) and Hunter (1943) have studied and discussed the matter. Some results give 4000° K, most 6000° K, and some even higher temperatures for the region around the short-wave limit of the solar spectrum. Most of the sets of calculations were done on the basis of 6000° K, but for comparison two are included, based on the lower figure.

The combined radiation of the earth's surface and the troposphere is required for $E_A$ at 11 km. This has been computed for the many wave-lengths necessary, using observed average values of temperature, humidity and ozone concentration (Napier Shaw 1928).

RESULTS

In table 2 are summarized the conditions governing each separate calculation, and the temperatures obtained for each of the nine layers, with an estimated error of about 3° K. The temperatures given in plain figures were obtained with ozone and carbon dioxide absorption corrected for pressure in the infra-red, but water vapour not so corrected. The temperatures in italics were obtained with the water vapour corrected for the pressure effect. This pressure correction is large, particularly in the upper layers, but it affects both sides of the equation, and so the temperature differences are not greater than 20%.

### Table 2. Calculated air temperatures in °K

<table>
<thead>
<tr>
<th>season</th>
<th>summer</th>
<th>summer</th>
<th>summer</th>
<th>summer</th>
<th>winter</th>
<th>summer</th>
</tr>
</thead>
<tbody>
<tr>
<td>amt. $O_3$</td>
<td>0-280</td>
<td>0-280</td>
<td>0-280</td>
<td>0-200</td>
<td>0-280</td>
<td>0-280</td>
</tr>
<tr>
<td>amt. $H_2O$ (%)</td>
<td>0</td>
<td>10</td>
<td>40</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>solar temp. (°K)</td>
<td>6000</td>
<td>6000</td>
<td>6000</td>
<td>6000</td>
<td>6000</td>
<td>4000</td>
</tr>
</tbody>
</table>

ht. in km.

| 50-55 | 452 | 415 448 | 344 441 | 410 445 | 406 439 | 323 347 |
| 45-50 | 429 | 410 424 | 361 421 | 409 422 | 364 375 | 321 333 |
| 40-45 | 399 | 385 397 | 350 394 | 382 390 | 305 314 | 311 319 |
| 35-40 | 335 | 324 332 | 295 327 | 320 330 | 278 285 | 291 301 |
| 30-35 | 296 | 285 295 | 262 291 | 281 292 | 262 273 | 272 282 |
| 25-30 | 275 | 258 272 | 244 265 | 257 269 | 246 256 | 252 266 |
| 20-25 | 254 | 241 249 | 240 239 | 239 247 | 230 238 | 236 245 |
| 15-20 | 239 | 223 232 | 232 221 | 229 225 | 221 221 | 229 229 |
| 11-15 | 228 | 218 217 | 211 209 | 215 208 | 208 209 | 217 215 |

Most of these data are plotted in figure 1 where the pressure correction has been applied, and in figure 2 where the water-vapour absorption is uncorrected for the pressure effect. Figure 2 therefore can be compared with previous results to show the influence of the new data on the infra-red absorption of ozone. For convenience
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Figure 1. All absorptions corrected for pressure.

Figure 2. H₂O absorption not corrected for pressure.
in comparing the two figures it should be noted that the curve marked zero in each is the same, and represents the limit in which water vapour is negligible above 11 km.

**DISCUSSION**

In each figure are given some temperature results obtained with sounding balloons. The values for Munich (latitude 48° N) are an average for summer, no time of the flights being specified. The Seattle (latitude 47° N) values are taken from the *U.S. Monthly Weather Review*, and refer to flights starting about 9.30 p.m. local standard time. Two curves are shown for Omaha (latitude 41° N), one an average for July 1938, and the other a single flight on 21 July 1938. All flights started late in the day, about 6 p.m. The curve for Ellendale (latitude 46° N) is displaced 20° to the right to avoid overlapping (Lennahan 1938). None of these curves is strictly comparable with the calculated values, since the attainment of radiation equilibrium at midday has been assumed for the calculations. Failure to attain this equilibrium, or cooling from it by 6 p.m. or by 10 p.m., would certainly make the calculated temperatures higher than those observed.

Ballard (1941) obtains a cooling of 10° K from 6 p.m. to 2 a.m. at the height of 25 km., based on a small number of observations. If a cooling of 15° K could be assumed from midday to midnight at 25 km., the observed temperatures would nearly agree with the calculated values. This agreement may imply that even below 30 km. the diurnal temperature changes are rapid enough to allow the attainment of radiation equilibrium.

The difference between observations over Seattle on the west coast and over Omaha in the great plains is very similar to the difference in temperature distributions for temperate and tropical latitudes, both in height of tropopause and lapse rate above it (Chiplonkar 1940; Jaumotte 1937). Hafer (1940) states that at Omaha in general the summer shows a large temperature inversion and the winter months little or none in the stratosphere. The same effects might be observed at greater heights over Ellendale.

It will be noticed that the pressure corrected curves for zero, 10 and 40% water vapour are very close together above 25 km., while this is not the case for the uncorrected ones. The crowding towards the zero curve in the higher layers is due to the fact that the pressure correction is equivalent to decreasing greatly the effective amount of water vapour in these layers.

A significant difference is evident from summer to winter below 25 km. The curves, however, show about 10° K difference near 20 km., and this is in agreement with balloon observations (Munich and Seattle). The calculations for winter conditions do not take into account any possible change from summer in the water-vapour content of the ozonosphere or in the vapour pressures in the troposphere. Another feature of the winter curves is the small change from summer in the temperature of the 50 to 55 km. layer. This is mainly due to the large zenith angle of the sun and the consequent long path of the parallel solar rays. For the shorter wave-lengths of the
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Ultra-violet ozone band the absorption is complete in this one layer, almost compensating for the greater area over which the energy is spread. In the next few layers down to 35 km, this depletion of energy in the top layer does not leave enough to compensate for the larger area, and the temperature differences are large. Still lower, below 30 km, the other terms in the equation become more important than the solar term and the temperature differences become small again.

The curves for reduced solar energy show that the temperature could still be above 300° K around 50 km, in spite of the 2000° K drop in the effective temperature for solar ultra-violet.

The curves for 0-200 cm of ozone have not been plotted. They show no change above 40 km, as would be expected, since the ozone decrease is in the layers below this height due largely to inaccuracy of determination above 40 km. A real decrease of ozone and temperature almost certainly occurs here. In the middle layers there is a tendency for the solar energy to be absorbed lower down (in contrast to the winter calculations) and a small decrease in temperature is shown from 15 to 30 km. This change is less than that expected by Dobson (1942).

Vassy & Vassy (1939) infer that the mean temperature of atmospheric ozone is low, but nevertheless there is a temperature increase in the upper part of the ozonosphere. The calculated results of the present paper agree, since the bulk of the ozone is below 30 km, where the average calculated temperature is about 240° K. Regener (1941) discusses the high temperature of the upper ozonosphere, and its explanation by the absorption of solar energy. He suggests that spectral methods may yet be used to determine the temperature distribution at great heights.

**Conclusion**

The results generally indicate that absorption of solar radiation by ozone will explain the existence of high temperatures around 35 to 50 km, though the ozone maximum is at 22 km. The small increase in the lower stratosphere also agrees in slope with the Omaha balloon observations (which may not be representative of the earth as a whole (Wigand 1931)), but the calculated temperatures are 15 to 20° K too high at 25 km. The assumptions regarding water vapour are reasonable, but in truth there may be irregular variations with height about which nothing is now known. The approximate agreement with directly observed temperatures serves to justify the primary assumption of radiation equilibrium as a good working hypothesis. For heights below 30 km, this aspect of the problem should repay further study, including an extensive analysis of balloon observations under widely different geographical and climatic conditions.
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