Edward W. Taylor

Description of Plates 6 and 7.

Plate 6. Epithelial cells. Objective 4 mm. \( \lambda = 5461 \text{A} \). 475 x

(a) Transmitted light 12%. Phase difference +\( \frac{1}{4} \lambda \).
(b) Transmitted light 3%. Phase difference +\( \frac{1}{4} \lambda \).
(c) Transmitted light 0%. Dark ground.
(d) Transmitted light 3%. Phase difference -\( \frac{1}{4} \lambda \).
(e) Transmitted light 12%. Phase difference -\( \frac{1}{4} \lambda \).

Plate 7. Caryophanus latum. Objective 4 mm. 375 x

(f) Positive phase contrast. Large amplitude. Phase difference +\( \frac{1}{4} \lambda \).
(g) Positive phase contrast. Small amplitude. Phase difference +\( \frac{1}{4} \lambda \).
(h) Transmitted light. Narrow cone. Appearance with substage condenser nearly closed.
(i) Negative phase contrast. Large amplitude. Phase difference -\( \frac{1}{4} \lambda \).
(j) Negative phase contrast. Small amplitude. Phase difference -\( \frac{1}{4} \lambda \).

Cosmic-ray bursts and shower spread under large thicknesses of lead

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(Communicated by H. S. W. Massey, F.R.S.—Received 21 December 1946)

The frequency of occurrence of cosmic-ray bursts under large thicknesses of lead and iron has been measured as a function of the thickness, using a small ionization chamber with and without shielding material round its sides. Experiments have also been carried out with two small chambers side by side, the frequency of occurrence of bursts simultaneously in both chambers being observed under different thicknesses of lead.

While the burst-rate—thickness curves are, on the whole, fairly flat beyond the first maximum, there are indications of a second maximum at large thicknesses in the case of lead. From the experimental results information has also been obtained about the mean and the maximum angular spread of showers, and the increase in shower spread with increasing size of shower.

Introduction

In a previous paper from this laboratory (Mohr & Stafford 1944, hereafter referred to as paper A), the rate of burst production in large thicknesses of lead and iron was measured as a function of the thickness, and a pronounced ‘second maximum’ was found to occur for bursts of from 40 to 70 particles. The chamber used was an upright cylinder with a collecting volume of 221. and ratio of length to diameter of 2.6.

This result was sufficiently unexpected to make it worth while repeating the experiments with a smaller chamber of collecting volume 1.11., and ratio of length
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to diameter of 1.8. Observations have also been made on the simultaneous occurrence of bursts in two similar chambers placed side by side below the burst-producing material.

The experiments with coincidence chambers, together with experiments on the decrease in burst rate with increase in height of the burst-producing material, have provided an estimate of the average angular spread of the showers which gave rise to the bursts.* This information is of value in interpreting experiments made with ionization chambers, since such chambers register only part of the total ionization produced in a cosmic-ray shower. It should also be of value when a more complete theory of the angular spread of showers is available.

Apparatus

The two ionization chambers used were of duralumin of wall thickness 1 cm., and contained dry air at a pressure of 90 atm. The electrodes, of thin aluminium, were approximately in the form of concentric cylinders of mean diameter 10.0 and 4.0 cm. respectively with hemispherical ends, the length of the outer electrode being 18.0 cm. The mean path length of a ray through either chamber when placed with its axis horizontal was calculated to be 7.2 cm., and with its axis vertical 9.7 cm.

The potential applied to the electrodes was 600 V, obtained from a number of Exide 10 V high-tension units. The ionization currents from the two chambers were each amplified with an electrometer valve, the output currents of the valves being passed through Moll short-period galvanometers, whose movements were recorded photographically side by side on the same rotating sheet of photostat paper. The two amplifiers were adjusted to almost equal sensitivity, and a kick of 1.0 mm. on the film was found to correspond to a burst containing approximately 10 particles when the chamber axis was horizontal, and about 7 or 8 particles when the chamber axis was vertical.

Observations

The rate of burst production was measured for different thicknesses of lead and iron above a single chamber, placed with its axis vertical and with a square enclosure of iron or lead surrounding it, as in figure 1a. An enclosure was used when making the observations described in paper A, in order to prevent soft showers from the walls of the room causing bursts, and it seemed at that time that part of the burst rays were produced in the enclosure by shower particles originating in the burst-producing material above the chamber. The effect of using an enclosure round the sides of the chamber was therefore investigated in the present work. Experiments were then carried out on the spread of showers, and the results indicated that showers from the walls of the room and more remote parts of the ceiling could not be producing

* As is usual, we use the term ‘shower’ to denote the totality of particles produced simultaneously in a given region, and ‘burst’ to denote just those particles in the shower which pass through the ionization chamber.
a significant number of bursts, and the burst rate was accordingly measured for different thicknesses of lead without any enclosure. In these runs two chambers were used, being placed side by side with their axes horizontal, as shown in figure 1b, and the mean of the results obtained with each chamber was taken. The pairs of curves in figure 2 embody the results of runs whose times totalled 1870,* 980, 1450 and 920 hr. respectively.

![Figure 1](http://rspa.royalsocietypublishing.org/download)

**Figure 1.** Disposition of ionization chambers and burst-producing material.

With the two chambers arranged as in figure 1b, a number of bursts were recorded coincidentally in both chambers, and from the coincidence rate information was obtained about the angular spread of showers. Further data on shower spread was collected by observing the burst rate with the burst-producing material placed at two different heights above the chambers as shown by the broken lines in figure 1b.

Finally, by observing the variation of the burst rate as the burst-producing material was displaced sideways by different amounts, in the manner indicated in figure 1c, the extent to which the axes of showers are inclined to the vertical was studied.

**Results**

(a) Shape of Rossi curve. In figure 2 are shown the observed values of the burst rate for different thicknesses of burst-producing material above the chamber. The limits of error refer to the standard deviation, and the smoothest possible curve has been drawn through each set of points.

The curves show differences in shape characteristic of the material above the chamber. In particular, the curve for kicks greater than 1.2 mm. for iron above and round the chamber shows a very broad 'first maximum'.

* As both chambers were recording simultaneously for this series, the effective time is thus 3740 hr.
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It is seen that curves can be drawn so as to pass within the limits of error of most of the points without showing any very marked 'second maximum' such as appears in the curves in paper A. The latter curves were obtained with a large and rather long chamber, and were consequently for much larger bursts than in the present work

![Graph](http://rspa.royalsocietypublishing.org/)

**Figure 2.** Rate of burst production for different thicknesses of material above chamber. The top two curves are for lead above chamber, no enclosure; the next two for lead above, and lead enclosing sides of, chambers; the next two for lead above, and iron round, chamber; the bottom two for iron above, and iron round, chamber.

and for a different 'geometry'. A gradual increase in the burst rate with increasing thickness is, however, suggested by the present values for kicks greater than 1.2 mm. with lead above the chamber and no surrounding material; this is indicated in the figure by the broken curve. The corresponding curves for lead and for iron surrounding the chamber may also follow the broken line rather than the full line, but the limits of error are not small enough to make this reasonably certain.
Supporting evidence for a second maximum for lead is to be found in the curve, shown in figure 3, for the variation with thickness of the rate of simultaneous occurrence of kicks greater than 0.3 mm. in both chambers. Although the number of coincidences is small, totalling 111 for the data in the figure, the curve is surprisingly similar in shape to the burst-rate curve for the larger kicks from lead (no enclosure), both showing the gradual rise towards a second maximum at large thicknesses. The close similarity of the two curves is to be expected if the coincidences are due to the larger showers which—as we shall see—have the greatest angular spread.

![Graph showing coincidence rate for kicks greater than 0.3 mm. in both chambers simultaneously, for different thicknesses of lead placed above the chamber, as shown in figure 1b.]

The evidence therefore tends to favour the existence of a second maximum, the existence of which was attributed in paper A to the occurrence of multiple processes involving the creation of mesotrons. The maximum obtained here, however, appears to lie at about 60 cm., whereas in paper A the maximum for lead occurred at 20 cm. If the present maximum is real and not due to statistical fluctuations, the shape of the Rossi curve must be dependent on the size of the bursts measured or on the shape and geometrical arrangement of the chamber and burst-producing material. The geometrical arrangement seems to have been, to some extent, a determining factor as to whether or not a second maximum in the Rossi curve is obtained with counter measurements.

(b) Shower spread—core of shower. The angular spread of particles near the centre of a shower was investigated by placing lead as burst-producing material at heights of 11 and 24 cm. respectively above the centres of the chambers, as shown in figure 1b. Increasing the height of the lead naturally gives a smaller burst rate, but this must be interpreted as due to a reduction in the fraction of the shower particles which pass through the chamber to produce a kick.

In deducing the angular spread of the shower from the reduction in size of kick resulting from an increase in height of the lead, it was assumed that the shower
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particles diverge from a single point close to the bottom of the shower-producing material, and that the distribution in angle of the shower particles is Gaussian in two mutually perpendicular horizontal directions. Account was taken of the diminishing length of path of a shower particle in the chamber as the particle comes near to grazing the outer electrode, by taking a calculated effective radius of 4.3 cm. and effective length of 16.6 cm. for the outer electrode in place of the geometrical radius of 5.0 cm. and geometrical length of 18.0 cm.

With 20 cm. of lead above the chambers, it was found that the smaller bursts (corresponding to kicks between 0.4 and 1.2 mm.) were diminished to 0.63 of their size when the height of the lead was increased from 11 to 24 cm. A quite small correction has now to be applied to take account of the decrease in the angle subtended at the chamber by the lead when the height of the latter is increased, this correction being obtained from the experimental data discussed in § (d) below. The result then indicates that, in a shower of approximately 6 to 20 particles, half the particles lie on the average within a cone whose semi-vertical angle is about 26°. The corresponding value for 2 cm. of lead was found to be about 29°.

(c) Shower spread—periphery of shower. Information about the angular spread of particles at the periphery of showers of different sizes was obtained from a consideration of the coincidence rate for simultaneous bursts in two chambers. For this purpose the results for different thicknesses of lead were combined in order to reduce statistical variations due to the comparatively small number of coincidences observed for each thickness.

In figure 4 the coincidence rate is compared with the burst rate for all kicks greater than a given size s. By differentiating the curves at each point, curves were derived for the coincidence rate and the burst rate for kicks of a given size s. The derived curves will not be shown in an additional figure, as they are of similar shape to those in figure 4, and the considerations below lead to the same general conclusions whether they are applied to the derived curves or to the curves of figure 4. This is to be expected, because the contribution to the total number of kicks greater than a given size s comes largely from kicks not much greater in size than s.

Consider first the right-hand end of curve II. This corresponds to showers giving large bursts in one chamber and small bursts (most probably) in the other chamber, and the shower is therefore most likely to be coming from that region of the lead just above one chamber. A comparison of corresponding points on curves I and II then shows that, of all the large showers occurring above one chamber, just under one-half cause a barely perceptible burst in the other chamber, i.e. just under one-half of the larger showers have their periphery on the surface of a cone whose semi-vertical angle is about 36°. The remaining half of the showers will have a somewhat smaller angular spread. If one calculates the mean angle between the shower particles and the axis of the shower, assuming that the distribution in angle (in two dimensions) is Gaussian right out to the periphery of the shower, one finds that, on the average, half the shower particles lie within a cone whose semi-vertical angle is about 20°. This value is smaller than that obtained from the considerations in § (b) for the
smaller bursts, viz. 26°, but the results of the present section can be reconciled with an angle of 26° or more, if one assumes that the distribution in angle of the shower particles falls off near the periphery of the shower faster than in a Gaussian distribution, i.e. that there is a sharper cut-off.

Consider now the right-hand end of curve III. This corresponds to showers giving large bursts simultaneously in both chambers, and these showers are therefore coming from points in the lead almost equidistant from both chambers. It is difficult to deduce accurately the size, and hence the angular spread, of such showers from the observed results, but the observations are at least not inconsistent with the deductions previously made.

![Graph](http://rspa.royalsocietypublishing.org/)

**Figure 4.** Curve I shows the number of kicks per hour greater than a given size, $s$, in one chamber; curve II, number of kicks per hour greater than a given size, $s$, in one chamber when a kick greater than 0.3 mm. occurs simultaneously in the other chamber; curve III, number of kicks per hour greater than a given size, $s$, in both chambers simultaneously.

Now note how the curves II and III fall further and further below curve I for decreasing size of shower. This shows clearly that the peripheral spread of showers decreases notably with their size, though quantitative deductions are difficult to make.

The above deductions have been made on the assumption that the shower particles have their origin at a single point and that this point is near the bottom of the lead. The effect of either assumption being untrue for some of the showers would be to reduce the calculated mean angle of spread of the shower particles. It must be emphasized, also, that the values given are average values, because showers with
greater and smaller angular spread may sometimes occur, as is clear from Wilson chamber photographs of showers.

(d) Direction of axis of shower. The extent to which the axes of showers deviate from the vertical was studied by observing the variation of burst rate when the burst-producing material was displaced sideways by different amounts, as shown in figure 1c. The experiments were carried out only with 2 cm. of lead, the thickness giving greatest shower production; with greater thicknesses it is difficult to be sure that part of the observed effect is not due to showers from the upper or lower edge of the lead layer.

In deducing an average value for the inclination of the axis of the shower to the vertical, it was assumed that the distribution in angle of the shower axes was Gaussian. Allowance was made for the finite length of the lead layer, and for the finite angular spread of the shower. It was then found that, for showers of from 6 to 20 particles, half the showers have their axes making angles of less than about 30° with the vertical.

**DISCUSSION**

The shape of the burst-rate—thickness curve for large thicknesses and the significance of a possible second maximum in the curve have been discussed in paper A, and it is scarcely necessary to add to what has been further said in §(a) above.

The angular spread of showers has been discussed by Bhabha & Heitler (1937), who have made an approximate estimate of the mean deflexion of shower particles based on the cascade theory of shower production. If there are 2n stages in the production of a shower particle of energy $E$, the mean deflexion of the particle is less than $(2n)^{1/2}mc^2/E$. The value of $n$ for a given thickness of burst-producing material may be obtained from a knowledge of the total number of particles in the shower.

In the present experiments under 2 cm. of lead, a mean deflexion of 29° was observed for showers of from 6 to 20 particles. For such showers the mean value of $n$, from the cascade theory, is from 2 to 3. The energy $E$ of each shower particle must exceed $8mc^2$ (Heitler 1944), the absolute minimum required for the particle to penetrate the 1 cm. thickness of aluminium wall of the ionization chamber. Most of the bursts observed will consist of particles whose energies do not greatly exceed this minimum value of $8mc^2$, for higher energy showers would require for their production electrons of greater energy, and their frequency of occurrence will be smaller. Substituting the values $n = 3$ and $E = 8mc^2$ in the above formula gives a mean deflexion of 17°, which is much less than the observed value of about 29°; moreover, taking larger values for $E$ gives even smaller values for the calculated mean deflexion.

The discrepancy might be explained as follows. As the energy of the quanta produced in the cascade process decreases towards the value $10mc^2$, the cross-section
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for Compton scattering in lead becomes comparable with the cross-section for pair production (Heitler 1944). The effect of such scattering in the lead will be to increase the angular divergence of the emerging shower particles.

Let us now consider the angular spread of showers under 20 cm. of lead. Under this and greater thicknesses most of the showers will have been initiated by mesotrons. A mesotron will give rise to a fast electron by a head-on collision or by emission of a high-energy quantum, and this electron will in turn produce a cascade shower. Now a shower of, say, 10 electrons emerging from the lead may be either (i) the remnant of a large shower having begun its growth high up in the lead, but having been partly absorbed before emerging from the lead, or (ii) a shower having begun its growth 2 cm. from the bottom of the lead, and having just built up to its observed size of 10 particles. The second type of shower is the most likely to occur, since it is produced by less energetic particles, whose frequency of occurrence will be correspondingly greater. The angular spread of the observed shower should therefore be given by the same formula as was applied above to showers under 2 cm. of lead, with about the same values of \( n \) and \( E \). In this way may be explained the approximate equality of angular spread observed for showers of the same size under 2 cm. and under 20 cm. of lead.

In conclusion we wish to acknowledge our indebtedness to the University of Cape Town for a further research grant for the purchase of apparatus for this and other work on cosmic rays. We also thank Mr R. D. Linton of the workshop staff for the readiness with which he gave any necessary technical assistance.

References